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Research Paper

Calibration of Fixed-Income Returns Segregated Fund Liability

Committee on Life Insurance Financial Reporting

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Memorandum

To: All Fellows, Affiliates, Associates, and Correspondents of the Canadian Institute of Actuaries

From: Bruce Langstroth, Chair
Practice Council

Alexis Gerbeau, Chair
Committee on Life Insurance Financial Reporting
Chair of the Designated Group

Date: April 11, 2014

Subject: **Research Paper: Calibration of Fixed-Income Returns for Segregated Fund Liability**

The Committee on Life Insurance Financial Reporting (CLIFR) has drafted this paper to provide support for an updated promulgation of calibration criteria for investment returns with respect to the valuation of segregated fund guarantees.

No calibration criteria for fixed-income returns for the valuation of segregated fund guarantees had been established until now. This report presents new calibration criteria for Canadian and U.S. broad-based fixed-income returns.

The structure of the calibration criteria provides the maximum values for the fixed-income index total return accumulation factors for the 2.5th, 5th, and 10th percentiles for the one-year, five-year, 10-year, and 20-year horizons. This is the same structure as used for the calibration criteria of equity returns. The calibration criteria also provide minimum values for the accumulation factors for the 90th, 95th, and 97.5th percentiles for the one-year horizon. Criteria are given for each of three initial benchmark yields: 3.95%, 5.60%, and 8.80%. The intent of the three initial yields is to ensure that models are appropriately calibrated for use in low, medium, and high initial yield environments.

Finally, qualitative guidance is provided for modelling fixed-income indices other than Canadian and U.S. broad-based fixed-income indices.

CLIFR intends to review updated experience from time to time, which could lead to revisions to the calibration criteria in the future.

CLIFR would like to acknowledge the contribution of the Working Group and thank the members for their efforts. Members of the Working Group at the time of publication include Alexis Gerbeau, Lynn Guo, Sara Lang, Martin Labelle, Michael McDonald, Brock McEwen,

Ricardo Mitchell, Steven Prince, Jim Snell, Dean Stamp, Maxime Turgeon-Rhéaume, Anthony Vaz, Chen Xing, and Chong Zheng. Many other people have contributed at various stages of the project.

In accordance with the Institute's Policy on Due Process for the Adoption of Guidance Material Other than Standards of Practice, this research paper has been prepared by the Committee on Life Insurance Financial Reporting, and has received the approval for distribution from the Practice Council on March 31, 2014.

If you have any questions or comments regarding this research paper, please contact Alexis Gerbeau, Chair, Designated Group, at his CIA Online Directory address, alexis.gerbeau@standardlife.ca.

BL, AG

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1. PURPOSE

Calibration criteria for equity returns for the valuation of segregated fund guarantees have existed since 2002 and updated calibration criteria were promulgated by the Actuarial Standards Board (ASB) in July 2012. Until now, no calibration criteria for fixed-income returns for the valuation of segregated fund guarantees have been established. As fixed-income assets were considered less volatile and risky than equity returns, providing calibration criteria for these assets was not a priority when the guidance for the valuation of segregated funds was first developed. However, the growth of the segregated fund business and the trend toward longer-term products and additional reset features increases the need for guidance for the modelling of fixed-income assets.

The scope of this research paper includes:

- Establishment of calibration criteria for the left tail of Canadian and U.S. broad-based fixed-income total returns at percentiles 2.5%, 5%, and 10%, for the one-, five-, 10-, and 20-year horizons, for each of three initial yields: 3.95%, 5.60%, and 8.80%;
- Establishment of calibration criteria for the right tail of Canadian and U.S. broad-based fixed-income total returns over a one-year period, for each of three initial yields: 3.95%, 5.60%, and 8.80%; and
- Guidance for modelling fixed-income funds other than Canadian and U.S. broad-based fixed-income indices.

The publication of this paper follows the publication by Canadian regulators of calibration criteria for investment returns applicable when calculating capital requirements for segregated funds with an internal model. The actuary is reminded that Canadian regulators' criteria apply to the calculation of capital requirements only, and that the criteria set out in this research paper apply to the calculation of insurance contract liabilities. However, the actuary may satisfy the Canadian regulators' criteria for calculating insurance contract liabilities, when these criteria are more stringent than those set out in this research paper.

2. SUMMARY

The criteria have been determined using monthly total returns from December 1979 to December 2012 for the DEX Universe Bond Index, and from January 1976 to December 2012 for the Barclays Capital U.S. Aggregate Bond Index. Government yields from January 1956 to December 2012 were used for both Canada and the U.S. Adjustments were made to ensure consistency with the guidance applicable to stochastic risk-free interest rates modelling provided by the Canadian Institute of Actuaries.

The method for deriving the calibration criteria consists of selecting appropriate benchmark yields, calibrating the relevant statistical models, and then using these models to simulate fixed-income index accumulation factors. Details on the method are provided in section 3.

2.1 Left-Tail Calibration Criteria

Two sets of calibration criteria have been established for the left tail of fixed-income return distributions: one applicable to Canadian broad-based fixed-income indices, and one applicable to U.S. broad-based fixed-income indices.

Calibration criteria for the left-tail percentiles have been set for the 2.5th, 5th, and 10th percentiles of the accumulation factors for the one-, five-, 10-, and 20-year horizons, for three initial yields:

3.95%, 5.60%, and 8.80%. The following tables present the maximum values for the left percentiles of the accumulation factors.

Canadian Broad-Based Fixed-Income Indices

Initial Yield	1-year			5-year			10-year			20-year		
	2.5th	5th	10th	2.5th	5th	10th	2.5th	5th	10th	2.5th	5th	10th
3.95%	0.99	1.00	1.01	1.11	1.13	1.16	1.32	1.35	1.39	1.82	1.90	1.99
5.60%	0.98	1.00	1.01	1.19	1.21	1.24	1.52	1.57	1.62	2.24	2.35	2.50
8.80%	1.00	1.02	1.04	1.38	1.42	1.46	2.00	2.06	2.15	3.29	3.53	3.86

U.S. Broad-Based Fixed-Income Indices

Initial Yield	1-year			5-year			10-year			20-year		
	2.5th	5th	10th	2.5th	5th	10th	2.5th	5th	10th	2.5th	5th	10th
3.95%	1.00	1.01	1.02	1.16	1.17	1.19	1.38	1.41	1.43	1.90	1.95	2.02
5.60%	1.00	1.01	1.02	1.24	1.25	1.27	1.58	1.61	1.64	2.27	2.37	2.49
8.80%	1.02	1.03	1.05	1.44	1.46	1.49	2.03	2.08	2.16	3.21	3.43	3.77

2.2 Right-Tail Calibration Criteria

Calibration criteria for the right-tail percentiles have been set for the 90th, 95th, and 97.5th percentiles of the accumulation factors for the one-year horizon, for three initial yields: 3.95%, 5.60%, and 8.80%. The following table presents the minimum values for the right-tail percentiles of the accumulation factors.

Canadian Broad-Based Fixed-Income Indices

Initial Yield	1-year		
	90th	95th	97.5th
3.95%	1.07	1.08	1.09
5.60%	1.10	1.11	1.12
8.80%	1.15	1.17	1.18

U.S. Broad-Based Fixed-Income Indices

Initial Yield	1-year		
	90th	95th	97.5th
3.95%	1.05	1.06	1.06
5.60%	1.08	1.09	1.10
8.80%	1.13	1.14	1.16

3. METHOD

Returns on fixed-income assets are largely driven by movements in interest rates. Data on Canadian broad-based fixed-income returns are only available since December 1979. Interest rates since December 1979 have followed a decreasing trend, which helped to inflate the returns on fixed-income assets since that date. The average return over that period is therefore not an appropriate benchmark for establishing a prospective expected return on fixed-income assets. This illustrates that statistical techniques cannot be applied to fixed-income returns over a historical period without considering the underlying path of interest rates during that period. As a result, the method used for developing calibration criteria for equity returns, which consists of fitting models directly to the observed equity returns over a certain historical period, cannot be applied to fixed-income returns.

Instead, the underlying risk-free interest rate must be simulated and used to project fixed-income index total returns, using models calibrated based on history. In this research paper, three components of fixed-income index total return were modelled:

- Benchmark total returns, estimated using simulated benchmark yields (i.e., the sum of correlated government yields and credit spreads) and a fixed-duration parameter;
- Other systematic factors, using a fixed parameter; and
- Residual noise, by simulating a noise term.

The detailed steps used to develop the calibration criteria for the Canadian and U.S. broad-based fixed-income indices are given below:

1) Select a benchmark yield appropriate for the index, where the benchmark yield is the sum of a government yield and a credit spread.
2) Calibrate the models:
   a. Calibrate the benchmark yield models:
      i. Select stochastic models for government yields and credit spreads;
      ii. Apply statistical techniques to estimate parameters based on history; and
      iii. Adjust the parameters to ensure consistency with the promulgated calibration criteria for risk-free rates; and
   b. Calibrate the fixed-income index total return models: perform a regression to estimate the duration, systematic factors, and residual noise volatility parameters using historical fixed-income index total return data and historical benchmark yield data.
3) Simulate fixed-income total returns and calculate accumulation factor percentiles:
   a. Using the models developed in step 2a, produce Monte Carlo simulations of correlated government yields and credit spreads (combined to form benchmark yields).
   b. Using the simulated benchmark yields from step 3a as inputs and the fixed-income total return model developed in step 2b, produce Monte Carlo simulations of bond fund total returns (the additional simulation at this step is for the residual noise).
   c. Calculate percentiles of bond fund accumulation factors over various horizons.
4) Establish calibration criteria for fixed-income returns based on results of step 3.

The process would be ideally performed for various choices of models in step 2a.

4. DATA

4.1 Sources

Historical government yield and credit spread data are needed to calibrate the benchmark yield model, and historical fixed-income index total return data are needed to calibrate the fixed-income total return model.

4.1.1 Canadian Broad-Based Fixed-Income Index Total Returns

The DEX Universe Bond Index is used to represent the Canadian broad-based bond index in order to derive the criteria. It is a broad-based fixed-income index of Canadian investment grade government and corporate fixed-income bonds with remaining terms of at least one year. The DEX Universe Bond Index is calculated daily by PC Bond Analytics (previously by Scotia Capital). As at December 2011, the index comprised approximately 1,150 issues and a total face amount of just over one trillion dollars. By face amount it was split approximately 45% federal, 28% provincial/municipal, and 27% corporate. Data for the DEX Universe Bond Index begin in December 1979. More information can be found at http://www.canadianbondindices.com.

Monthly data from December 1979 to December 2005 were obtained from the *Annual Handbook of Debt Market Indices* published by Scotia Capital in January 2006. Monthly data from January 2006 to December 2012 were obtained from Bloomberg.

4.1.2 Government Yields and Spreads in Canada

The monthly 10-year Government of Canada Benchmark Bond Yield was used in the calibration. Data since December 1934 are available.

The average spread for the DEX was calculated as the average yield for the DEX minus the average yield for the federal bonds in the index. The average yield for the total DEX Universe Bond Index and the average yield for the federal bonds in the total index were captured monthly from December 1979 to December 2012 using the same source as for the DEX returns. For some months, the federal average yield was not captured and linearly interpolation was used to derive missing values.

4.1.3 U.S. Broad-Based Fixed-Income Index Total Returns

The Barclays Capital U.S. Aggregate Bond Index is used to represent the U.S. broad-based bond index in order to derive the criteria. It is a broad-based benchmark that measures the investment grade, U.S. dollar-denominated, fixed-rate taxable bond market, including Treasuries, government-related and corporate securities, MBS, ABS, and CMBS with a remaining term of at least one year. The Barclays Capital U.S. Aggregate Index was created by Lehman Brothers in 1986, with index history backfilled to January 1, 1976. The Barclays Capital Aggregate Bond Index is an intermediate-term index. As of April 30, 2012, the index contained about 7,900 securities.

Monthly data from January 1976 to December 2012 were obtained from Bloomberg.

4.1.4 Government Yields and Spreads in the U.S.

The seven-year Federal Reserve Constant Maturity Treasury Rate was used in the calibration. Data since April 1941 are available.

The Option Adjusted Spread (OAS) measures the yield spread over U.S. treasuries for a bond or a portfolio. The OAS takes into account the possible changes in expected bond cash flows due to interest rate changes. The data for the OAS for the Barclays Capital U.S. Aggregate Bond Index have been captured monthly from August 1988 to December 2012 from Bloomberg.

4.2 Historical Periods

4.2.1 Benchmark Yield Calibration

The benchmark yield is the sum of a government yield and a credit spread.

The period chosen for estimating the initial parameters of the stochastic models for government yields is from January 1956 to December 2012. This is consistent with the period used for developing the calibration criteria for equity. The parameters are then adjusted to be consistent with the 2013 Calibration of Stochastic Risk-Free Interest Rate Models for CALM Valuation research paper, which uses the historical period from January 1936 to December 2012.

The period chosen for estimating the parameters of the stochastic models for spreads is the longest period of available data.

The correlation parameters of the stochastic models have been estimated using the longest common period of available data.

The following table summarizes the historical periods used for the parameters estimation.

Calibration of Benchmark Yields

Period	Canada		U.S.	
	Government Yield	Spread	Government Yield	Spread
From	Jan 1956	Dec 1979	Jan 1956	Aug 1988
To	Dec 2012	Dec 2012	Dec 2012	Dec 2012

4.2.2 Broad-Based Fixed-Income Index Total Return Calibration

The periods chosen for estimating the parameters of the regression between the benchmark yields and the returns of the fixed-income indices are the longest available periods.

The following table summarizes the historical periods used.

Calibration of Fixed-Income Indices

Period	Canada	U.S.
From	Dec 1979	Aug 1988
To	Dec 2012	Dec 2012

5. CHOICE OF BENCHMARK YIELDS (Step 1)

The benchmark yield is the sum of a government yield and a credit spread. The government yields and credit spreads chosen for determining the Canadian and U.S. broad-based fund criteria are described below.

5.1 Canada

5.1.1 Choice of Government Yield

As mentioned earlier, the DEX Universe Bond Index is a Canadian broad-based bond index. The average remaining term of the bonds in the index has averaged between nine and 10 years for many years so a term of 10 years for the government yield was chosen.

5.1.2 Choice of Credit Spread

The average spread for the DEX Universe Bond Index has been defined as the difference between the average yield for the total DEX Universe Bond Index and the average yield for the federal bonds in the total index at each month end.

5.2 U.S.

5.2.1 Choice of Government Yield

The Barclays Capital Aggregate Bond Index is a U.S. broad-based bond index. The average remaining term of the bonds in the index has averaged between six and eight years for many years with an average of seven years. Therefore, a term of seven years for the government yield was chosen.

5.2.2 Choice of Credit Spread

The credit spread for the Barclays Capital Aggregate Bond Index has been defined as the OAS for the index. The OAS measures the average yield spread over U.S. treasuries for the bonds constituting the index, adjusted for any option values.

6. CALIBRATING THE BENCHMARK YIELDS MODELS (Step 2a)

6.1 Choice of Models for Interest Rates and Credit Spreads

For modelling government yields, we selected the two stochastic model forms that were used to develop the interest rate calibration criteria published by the CIA: the Cox-Ingersoll-Ross (CIR) and Brennan-Schwartz (BS). The forms of these models are provided in appendix A, and more details are provided in the 2013 Calibration of Stochastic Risk-Free Interest Rate Models for CALM Valuation research paper.

The CIR model was selected to model the credit spreads. This model provides a good fit to the historical data.

6.2 Reference Points for Government Yields

The 2013 Calibration of Stochastic Risk-Free Interest Rate Models for CALM Valuation research paper establishes calibration criteria for both the Canadian long-term (at least 20 years) risk-free interest rate at the two-year, 10-year, and 60-year horizons and the Canadian short-term (one-year) risk-free interest rate at the two-year and 60-year horizons. These criteria are expected to take effect in 2014 and are presented in appendix B.

According to the guidance provided on page 2 of the research paper: “These calibration criteria are directly applicable to Canadian risk-free interest rates or instruments denominated in Canadian dollars, but could be adapted for the US and other developed countries.”

It is generally accepted that the Canadian criteria can be used for the U.S. economy without adjustment. Therefore, we used the Canadian criteria for the U.S. calibration.

As mentioned above, the 10-year and seven-year government yields have been chosen for modelling the DEX and Barclays indices respectively. There are no recommended quantitative calibration criteria for the seven-year and 10-year rates. Based on the guidance provided for medium-term rates in the same research paper, we derived reference points for the medium-term rates by using the criteria for the one-year and 30-year rates with some adjustments.

The reference points for the seven-year and 10-year rates are derived as the weighted average of the criteria for the short-term and long-term rates. The weights were determined by performing a regression on historical data from the longest common period with sum of the weight equal to one. The historical periods used in the regression are from January 1985 to December 2012 for the seven-year rate and from January 1980 to December 2012 for the 10-year rate. A regression is required due to the convexity of the yield curve (e.g., the 10-year rate is typically closer to the 30-year rate than it is to the one-year rate).

The seven-year rate is approximated by the one-year and 30-year rate using:

$$y_7 \approx 0.34 \times y_1 + 0.66 \times y_{30}$$

The 10-year rate is approximated by the one-year and 30-year rate using:

$$y_{10} \approx 0.19 \times y_1 + 0.81 \times y_{30}$$

The initial yields associated with the reference points were obtained by weighting the initial rates for the long-term and short-term criteria with the same regression coefficients as above. Then the interpolated left-tail and the right-tail percentile values were rounded up and down to the closest 5 basis points, respectively, and the interpolated initial rates were rounded to the nearest 5 basis points to get the final reference points. The above approach was applied to derive both the reference points at two-year horizon as well as at the 60-year horizon.

Thus, combining the two tails, we obtain the following reference points:

Reference Points for Canada 10-year Government Yield					
		2-Year Horizon			60-Year Horizon
Initial Rate		**3.60%**	**5.90%**	**8.80%**	**5.90%**
Left-Tail	2.5	2.50%	3.90%	6.10%	2.30%
	5	2.65%	4.20%	6.50%	2.45%
	10	2.85%	4.50%	6.95%	2.65%
Right-Tail	90	4.70%	7.40%	10.40%	10.00%
	95	5.10%	7.90%	11.00%	12.00%
	97.5	5.40%	8.30%	11.45%	13.50%

Reference Points for U.S. 7-year Government Yield					
		2-Year Horizon			60-Year Horizon
Initial Rate		**3.30%**	**5.65%**	**8.65%**	**5.65%**
Left-Tail	2.5	2.20%	3.65%	6.00%	2.00%
	5	2.35%	3.90%	6.40%	2.20%
	10	2.55%	4.25%	6.85%	2.35%
Right-Tail	90	4.40%	7.15%	10.30%	10.00%
	95	4.80%	7.60%	10.85%	12.00%
	97.5	5.05%	8.00%	11.30%	13.50%

6.3 Benchmark Yields Calibration

The benchmark yield is the sum of a government yield and a credit spread. When simulating benchmark yields, the simulated government yields and credit spreads were correlated.

For modelling government yields, we selected both the CIR and BS model forms. A Maximum Likelihood Estimation (MLE) analysis was done to get a first estimate of parameters for the CIR model forms. We then adjusted the calibrated parameters for both the CIR and BS model forms in order to satisfy the reference points for government yields presented in section 6.2 above. The approach for adjusting the parameters was first to adjust the volatility parameters to meet the reference points at the two-year horizon and then the mean reversion speed parameter to meet the reference points at the 60-year horizon. The adjusted parameters for both the CIR and BS government yield model forms are shown in the tables below for Canada and the U.S.

6.3.1 Government Yield Model Calibrations

10-Year Canada Government Yield Parameters

Annualized Parameter	CIR Adjusted	BS Adjusted
τ	6.12×10^{-2}	5.70×10^{-2}
α	4.25×10^{-2}	3.55×10^{-2}
σ	4.25×10^{-2}	1.555×10^{-1}
Rate of Mean Reversion[1]	23.5294	28.1690

Seven-Year U.S. Government Yield Parameters

Annualized Parameter	CIR Adjusted	BS Adjusted
τ	5.88×10^{-2}	5.72×10^{-2}
α	4.25×10^{-2}	3.50×10^{-2}
σ	3.87×10^{-2}	1.700×10^{-1}
Rate of Mean Reversion	23.5294	28.5714

6.3.2 Credit Spread Model Calibrations

For credit spreads, only the CIR model form was used. The MLE estimates were used to parameterize the models. The MLE estimates for Canadian and U.S. credit spreads are shown below.

DEX Universe Average Spread CIR Parameters (Canada)
(Dec 1979–Dec 2012)

Annualized Parameter	MLE
τ	4.1×10^{-3}
α	0.2657
σ	2.35×10^{-2}
Rate of Mean Reversion	3.7636

Barclays' Aggregate OAS CIR Parameters (U.S.)
(Aug 1988–Dec 2012)

Annualized Parameter	MLE
τ	5.8×10^{-3}
α	0.3444
σ	3.02×10^{-2}
Rate of Mean Reversion	2.9036

6.3.3 Correlation Parameter Calibrations

The correlation parameters were estimated as the correlation between the movements of historical monthly government yield and the movements of the historical monthly credit spread

[1] In the tables above, the rate of mean reversion in years is defined as $1/\alpha$.

over the common period. The estimated correlation parameters for Canada and the U.S. are shown below.

Correlation between Canadian Government Yield and Spread Changes (Dec 1979–Dec 2012)

Parameter	Estimated
P	-0.21

Correlation between U.S. Government Yield and Spread Changes (Aug 1988–Dec 2012)

Parameter	Estimated
ρ	-0.21

The government yield and credit spread models and correlation parameters were combined to simulate the benchmark yields. Simulation results, including those illustrating that the government yields satisfy the reference points in section 6.2, are presented in appendix C.

7. CALIBRATING FIXED-INCOME INDEX TOTAL RETURNS MODELS (Step 2b)

The relationship between fixed-income index total returns and benchmark total returns is expressed through the regression equation below ($\Delta t = \frac{1}{12}$).

$$TR_k^{BF} = s\Delta t + TR_k^{bench}(D) + \varepsilon_k \sqrt{\Delta t}\text{, where } StdDev(\varepsilon_k) = \sigma_{err}$$

The variable TR_k^{BF} is the fixed-income index total return at time step *k*. The variable $TR_k^{bench}(D)$ is the benchmark return, which is a function of the duration *D*.

The benchmark total returns represent the component of the fixed-income index total return that is explained by the movement in the benchmark yield. Benchmark total returns, $TR_k^{bench}(D)$, are calculated from the benchmark yields, y_k, using a duration approximation as follows:

$$TR_k^{bench}(D) = y_{k-1}\Delta t - D \cdot (y_k - y_{k-1})$$

The component of the fixed-income index total return not explained by the movement in the benchmark yield is called the residual return, X_k, and is defined as follows:

$$X_k = TR_k^{BF} - TR_k^{bench}(D) \sim N(s\Delta t, \sigma_{err}\sqrt{\Delta t})$$

The parameters *s* and *D* were estimated by performing a regression on the historical fixed-income index total return against the benchmark return over the common period. The volatility parameter of the residual return, σ_{err}, was calculated as the annualized standard deviation of X_k above. The parameter *D* (duration) represents the sensitivity of the benchmark return to changes in the benchmark yield. The parameter *s* represents the average spread in the fixed-income index total return due to systematic factors other than the benchmark yield. Finally, the standard deviation, σ_{err}, represents the noise in the residual fixed-income index total return.

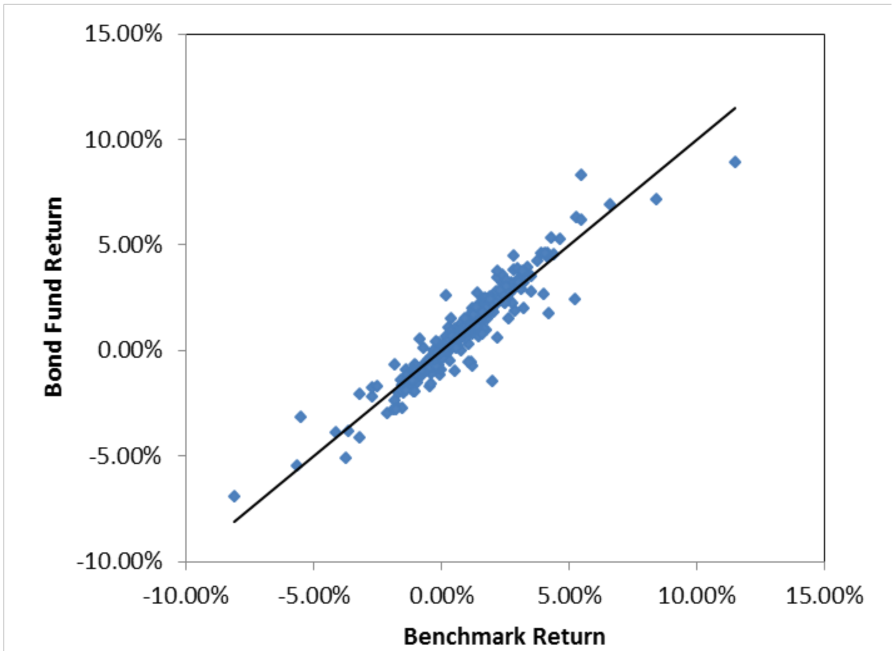
7.1 Canada

The following table presents the regression parameters obtained from MLE analysis.

DEX Universe—Coefficients and Residual Volatility (Dec 1979–Dec 2012)

Annualized Parameter	MLE	Units
s	1.9×10^{-3}	1/year
D	4.3571	years
σ_{err}	2.11×10^{-2}	$1/(\text{year})^{0.5}$

Scatter Plot of Monthly Canadian Fixed-Income Index Total Returns Against Benchmark Returns and Regression Line



The parameters obtained from regression analysis does not correspond exactly to those expected based on their physical interpretation. For example, one would have expected the parameter *D* to be closer to 6, which corresponds to the average duration of the DEX Universe Bond Index. This is explained by simplifying assumptions made by the model. For example, the model assumes a constant sensitivity of the index to the benchmark yield, while in reality this sensitivity varies over time as the constituents of the index change. Another example of simplification is the assumption that the index reacts linearly to changes to the benchmark yield, while in reality the index exhibits convexity. Additional parameters in the model would have been required in order to obtain more intuitive parameters. As in any modeling exercise, we have sought to respect the principle of parsimony. Additional testing was performed to ensure that the results were not overly dependent on models (see section 10).

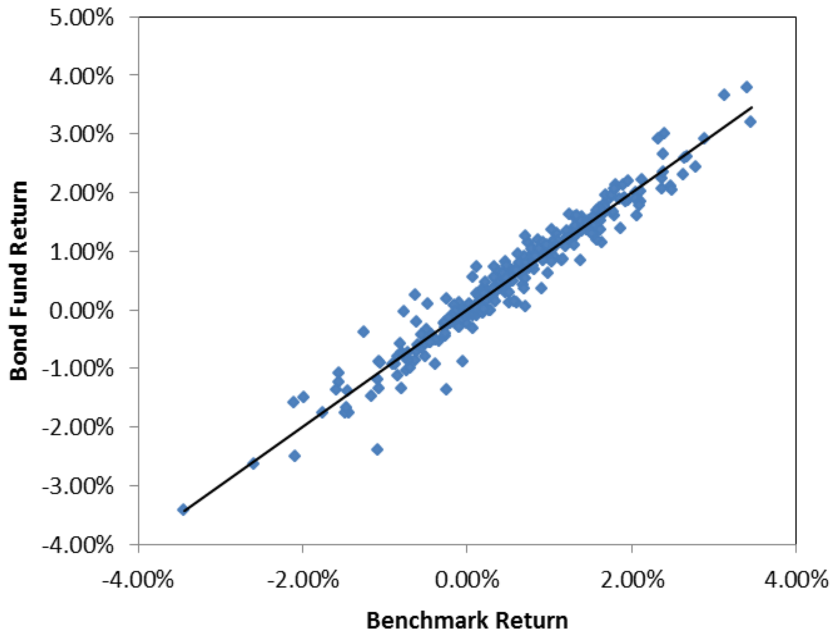
U.S.

The following table presents the parameters obtained from MLE analysis.

Barclays Aggregate—Coefficients and Residual Volatility (Aug 1988–Dec 2012)

Annualized Parameter	MLE	Units
s	1.1×10^{-3}	1/year
D	3.6614	years
σ_{err}	9.0×10^{-3}	$1/(\text{year})^{0.5}$

U.S. Scatter Plot of Monthly U.S. Fixed-Income Index Total Returns Against Benchmark Returns and Regression Line



8. SIMULATIONS OF FIXED-INCOME INDEX ACCUMULATION FACTORS (Step 3)

8.1 Choice of Initial Government Yields and Spreads

Simulations of total returns for fixed-income indices were produced for three levels of initial benchmark yields.

The initial government yields were chosen because they were convenient round numbers between the corresponding short-term and long-term initial yields chosen for the calibration of risk-free rates in the 2013 Calibration of Stochastic Risk-Free Interest Rate Models for CALM Valuation research paper. The following table presents the initial government yields (in bold).

Initial Government Yields			
	Short-Term	**Canadian 10-Year & US 7-Year**	Long-Term
Low Yield	2.00%	**3.00%**	4.00%
Medium Yield	4.50%	**5.25%**	6.25%
High Yield	8.00%	**8.50%**	9.00%

The initial credit spreads were chosen by investigating the relationship between the government yields and average spreads in Canadian history. The following table presents the initial average spreads and the resulting initial benchmark yields.

	Initial Government Yields	Initial Spreads	**Initial Benchmark Yields**
Low Yield	3.00%	0.95%	**3.95%**
Medium Yield	5.25%	0.35%	**5.60%**
High Yield	8.50%	0.30%	**8.80%**

The same initial government yields, credit spreads, and benchmark yields were used for the U.S. for simplicity.

8.2 Canada

The following tables show the left-tail percentiles of accumulation factors for various horizons and the right-tail percentiles of the accumulation factor at the one-year horizon obtained from Monte Carlo simulations using the calibrated stochastic models and regressions in sections 6 and 7 respectively.

DEX	1-year			5-year			10-year			20-year		
	2.5^{th}	5^{th}	10^{th}	2.5^{th}	5^{th}	10^{th}	2.5^{th}	5^{th}	10^{th}	2.5^{th}	5^{th}	10^{th}
y0 = 3.95%												
CIR	0.9760	0.9865	0.9993	1.0889	1.1108	1.1352	1.2661	1.3021	1.3446	1.6725	1.7535	1.8574
BS	0.9851	0.9955	1.0069	1.0989	1.1190	1.1436	1.2884	1.3177	1.3565	1.7268	1.8006	1.8945
y0 = 5.60%												
CIR	0.9778	0.9900	1.0059	1.1709	1.1987	1.2263	1.4762	1.5216	1.5745	2.1074	2.2324	2.4013
BS	0.9793	0.9928	1.0084	1.1655	1.1956	1.2263	1.4877	1.5308	1.5810	2.1463	2.2565	2.4020
y0 = 8.80%												
CIR	0.9997	1.0145	1.0334	1.3690	1.4050	1.4413	1.9623	2.0292	2.1109	3.1988	3.4387	3.7612
BS	0.9781	0.9976	1.0216	1.3189	1.3662	1.4196	1.9217	1.9951	2.0781	3.0734	3.2944	3.5778

DEX	1-year		
	90^{th}	95^{th}	97.5^{th}
y0 = 3.95%			
CIR	1.0858	1.0978	1.1083
BS	1.0812	1.0911	1.1001
y0 = 5.60%			
CIR	1.1105	1.1256	1.1377
BS	1.1110	1.1239	1.1366
y0 = 8.80%			
CIR	1.1631	1.1826	1.1976
BS	1.1746	1.1940	1.2109

8.3 U.S.

The following tables show the left-tail percentiles of accumulation factors for various horizons and right-tail percentiles of the accumulation factor at the one-year horizon obtained from Monte Carlo simulations using the calibrated models and regressions in sections 6 and 7 respectively.

Barclays	1-year			5-year			10-year			20-year		
	2.5th	5th	10th	2.5th	5th	10th	2.5th	5th	10th	2.5th	5th	10th
y0 = 3.95%												
CIR	0.9925	1.0001	1.0100	1.1349	1.1493	1.1652	1.3237	1.3478	1.3767	1.7124	1.7760	1.8760
BS	0.9999	1.0082	1.0166	1.1424	1.1556	1.1723	1.3487	1.3705	1.3948	1.8079	1.8581	1.9286
y0 = 5.60%												
CIR	0.9945	1.0041	1.0158	1.2203	1.2383	1.2586	1.5301	1.5630	1.6050	2.1179	2.2318	2.3977
BS	0.9906	1.0031	1.0163	1.2073	1.2301	1.2563	1.5418	1.5730	1.6073	2.1765	2.2775	2.3943
y0 = 8.80%												
CIR	1.0167	1.0287	1.0430	1.4216	1.4452	1.4730	1.9987	2.0482	2.1204	3.1165	3.3359	3.6732
BS	0.9877	1.0084	1.0288	1.3592	1.4040	1.4477	1.9544	2.0067	2.0714	3.0039	3.1857	3.4437

Barclays	1-year		
	90th	95th	97.5th
y0 = 3.95%			
CIR	1.0715	1.0803	1.0871
BS	1.0670	1.0732	1.0790
y0 = 5.60%			
CIR	1.0954	1.1055	1.1145
BS	1.0955	1.1048	1.1130
y0 = 8.80%			
CIR	1.1462	1.1598	1.1713
BS	1.1576	1.1727	1.1849

9. CRITERIA FOR BROAD-BASED FIXED-INCOME INDICES (Step 4)

9.1 Left-Tail Criteria

Two distinct sets of left-tail criteria have been established for the Canadian and U.S. broad-based fixed-income indices, as the results for the two countries were considered sufficiently different to deserve separate treatment.

The criteria have been determined by rounding-up the maximum of the percentiles obtained with the CIR and BS models, and by adding a buffer to these results.

The inclusion of the buffer is to allow for the model uncertainty and to allow actuaries to use models other than those used to develop the criteria. This is also consistent with the approach used to establish calibration criteria for equity returns.

The buffers vary with the time-horizon, and are as follows:

	1-year	5-year	10-year	20-year
Buffer	0.00	0.01	0.03	0.09

The rationale for increasing the buffer with the time-horizon is that the uncertainty in the percentile estimates increases with the horizon. The buffers approximately correspond to the impact on accumulation factors of an increase in the annual return of 0.20%.

The following tables present the maximum values for the left tail of accumulation factors for the 2.5th, 5th, and 10th percentiles, for the one-, five-, 10-, and 20-year horizons, for the three initial yield environments.

Canadian Broad-Based Fixed-Income Indices

Initial Yield	1-year			5-year			10-year			20-year		
	2.5th	5th	10th	2.5th	5th	10th	2.5th	5th	10th	2.5th	5th	10th
Low	0.99	1.00	1.01	1.11	1.13	1.16	1.32	1.35	1.39	1.82	1.90	1.99
Medium	0.98	1.00	1.01	1.19	1.21	1.24	1.52	1.57	1.62	2.24	2.35	2.50
High	1.00	1.02	1.04	1.38	1.42	1.46	2.00	2.06	2.15	3.29	3.53	3.86

U.S. Broad-Based Fixed-Income Indices

Initial Yield	1-year			5-year			10-year			20-year		
	2.5th	5th	10th	2.5th	5th	10th	2.5th	5th	10th	2.5th	5th	10th
Low	1.00	1.01	1.02	1.16	1.17	1.19	1.38	1.41	1.43	1.90	1.95	2.02
Medium	1.00	1.01	1.02	1.24	1.25	1.27	1.58	1.61	1.64	2.27	2.37	2.49
High	1.02	1.03	1.05	1.44	1.46	1.49	2.03	2.08	2.16	3.21	3.43	3.77

9.2 Right-Tail Criteria

Two distinct sets of right-tail criteria have been established for the Canadian and U.S. broad-based fixed-income indices, as the results for the two countries were considered sufficiently different to deserve separate treatment.

The right-tail criteria have been determined by rounding-down the minimum of the percentiles obtained with the CIR and BS models, and by subtracting a buffer of 0.01 to these results.

The following table presents the minimum values for the right tail of accumulation factors for the 90th, 95th, and 97.5th percentiles, for the one-year horizon, for the three initial yield environments.

Canadian Broad-Based Fixed-Income Indices

Initial Yield	1-year		
	90th	95th	97.5th
Low	1.07	1.08	1.09
Medium	1.10	1.11	1.12
High	1.15	1.17	1.18

U.S. Broad-Based Fixed-Income Indices

Initial Yield	1-year		
	90th	95th	97.5th
Low	1.05	1.06	1.06
Medium	1.08	1.09	1.10
High	1.13	1.14	1.16

9.3 Application of Criteria

Two alternatives are available for the application of the fixed-income calibration criteria. Alternative 1 would be used when fixed-income returns are not modelled by stochastically modelling interest rates (for example, when a regime-switching lognormal model form is used), and alternative 2 would be used when fixed-income returns are modelled by stochastically modelling interest rates.

Alternative 1

Under Alternative 1, a company would demonstrate that its valuation model meets the calibration criteria for the combination of government yield and spread prevailing at each valuation date. The company would first develop a criteria model that meets the calibration requirements for each of the three initial benchmark yields specified in section 8.1. A company would demonstrate this by verifying, for each of the three initial benchmark yields, that the left-tail percentiles of the scenarios generated by the criteria model are less than or equal to the left-tail criteria in section 9.1 and that the right-tail percentiles generated by the criteria model are greater than or equal to the right-tail criteria in section 9.2. The criteria model could be one of the two models presented in this document for developing the calibration criteria (i.e., the CIR or the BS model), but need not be.

At each valuation date, the company would then follow a two-step procedure:

1) Generate scenarios with the calibrated criteria model using the government yield and credit spread prevailing at the valuation date as the initial benchmark yield, and calculate the resulting left- and right-tail percentiles; and
2) Verify that the left-tail percentiles of the scenarios generated by the valuation model are lower than or equal to those derived from the criteria model in step 1, and that the right-tail percentiles of the valuation model are higher than or equal to those of the criteria in step 1.

Under alternative 1, the verification of the calibration for the criteria model is required only when the model parameters are updated. For the valuation model, the verification of the calibration is required at each valuation date.

Alternative 2

Under alternative 2, a company would demonstrate that its valuation model meets the calibration criteria for each of the three initial benchmark yields specified in section 8.1.

A company would demonstrate this by verifying, for each of the three initial benchmark yields, that the left-tail percentiles of the scenarios generated by the valuation model are less than or equal to the left-tail criteria in section 9.1 and that the right-tail percentiles generated by the valuation model are greater than or equal to the right-tail criteria in section 9.2.

For example, for the low-yield environment, the company would generate scenarios with an initial government yield of 3.00% and an initial average credit spread of 0.95%, for a total initial benchmark yield of 3.95%. Model results would then be used to demonstrate that the calibration criteria have been satisfied for the low-yield environment.

If a company models the government yield but not the credit spread, the company would verify that its valuation model meets the calibration criteria using the three initial government yields specified in section 8.1. For example, for the low-yield environment, the company would generate scenarios with an initial government yield of 3.00%.

Under alternative 2, the verification of the calibration is required only when the valuation model parameters are updated.

10. ADDITIONAL TESTING

The development of the calibration criteria for fixed-income returns presented in this document relies on a number of modelling choices and the application of judgment. It was felt that it was desirable to perform testing to ensure that the results were not overly dependent on models and judgment.

Areas where judgment was applied were identified, and tests were performed to assess the sensitivity of results. The additional tests included:

- Use of judgment in modifying the regression parameters based on their physical interpretation.
- Use of a first principles approach for mapping movements in the benchmark yield onto fixed-income returns instead of the regression technique described in section 8.
- Use of the BS model for modelling credit spreads instead of the CIR model.
- Use of a different approach for adjusting the parameters of the government yield model to meet calibration criteria for risk-free rates.
- Different choices of buffers.

Various combinations of the above alternatives were tested. The tests did show some variability in results, but the determination of calibration criteria was deemed not overly dependent on modelling choices.

11. GUIDANCE FOR OTHER THAN CANADIAN AND U.S. BROAD-BASED FIXED-INCOME INDICES

No numerical criteria are provided for fixed-income indices other than Canadian and U.S. broad-based fixed-income indices.

For fixed-income indices other than Canadian and U.S. broad-based, the actuary may consider applying the methodology that has been used to develop the criteria presented in this research paper. This would involve performing a regression of the historical index returns onto benchmark returns, as described in section 7. A money market index is an example of an index for which the actuary may consider applying this methodology.

For a high-yield fixed-income index, the actuary could consider mapping the index onto a combination of a broad-based fixed-income index and a broad-based equity index for the same geographic region. Other techniques could be considered.

The actuary would consider whether the index includes any embedded options, which can increase volatility.

Foreign exchange risk would be handled in a manner consistent with the handling of this risk in equity funds.

APPENDIX A: STOCHASTIC MODELS FOR INTEREST RATES

Cox-Ingersoll-Ross Model Form

We refer to a model form as the model is applied to project a single rate on an interest rate curve. The original CIR model is a short rate model, where the stochastic state variable is taken to be the instantaneous spot rate. The model, in its discrete form, is given by the equation.

$$r_t = (1-\alpha)r_{t-1} + \alpha\tau + \sigma\sqrt{r_{t-1}}\varepsilon_t,$$

where

τ is the mean-reversion level to which the process is reverting;

α is the mean-reversion speed, which would be between 0 and 1—a zero value would result in no mean-reversion while a one value would result in full reversion in next period;

σ is the volatility parameter; and

$\varepsilon_t \sim N(0,1)$.

Brennan-Schwartz Model Form

This model form is similar to the CIR model, with the difference that the stochastic process is now scaled by the interest rate. This model is represented by

$$r_t = (1-\alpha)r_{t-1} + \alpha\tau + \sigma r_{t-1}\varepsilon_t,$$

where

τ is the mean-reversion level to which the process is reverting;

α is the mean-reversion speed, which would be between 0 and 1—a zero value would result in no mean-reversion while a one value would result in full reversion in next period;

σ is the volatility parameter; and

$\varepsilon_t \sim N(0,1)$.

APPENDIX B: INTEREST RATE CALIBRATION CRITERIA

The 2013 Calibration of Stochastic Risk-Free Interest Rate Models for CALM Valuation research paper recommends the following calibration criteria for a Canadian long-term (at least 20 years) risk-free interest rate at the two-year and 60-year horizons:

Long-term Interest Rate Calibration Criteria for 2-Year and 60-Year Horizons				
Percentile	**2-Year Horizon**			**60-Year Horizon**
	Initial Rates			
	4%	6.25%	9%	6.25%
2.5	2.85%	4.25%	6.20%	2.60%
5	3.00%	4.50%	6.60%	2.80%
10	3.25%	4.80%	7.05%	3.00%
90	5.15%	7.80%	10.60%	10.00%
95	5.55%	8.30%	11.20%	12.00%
97.5	5.85%	8.70%	11.70%	13.50%

Also, a median between 4.5% and 6.75% would generally be expected and the rate of mean reversion would not be stronger than 14.5 years.

The same research paper recommends the following calibration criteria for a Canadian short-term (one-year) risk-free interest rate at the two-year and 60-year horizons:

Short-term Interest Rate Calibration Criteria for 2-Year and 60-Year Horizons				
Percentile	**2-Year Horizon**			**60-Year Horizon**
	Initial Rates			
	2%	4.50%	8%	4.50%
2.5	0.85%	2.35%	5.50%	0.80%
5	1.00%	2.70%	5.95%	0.90%
10	1.15%	3.10%	6.40%	1.00%
90	3.00%	5.90%	9.75%	10.00%
95	3.35%	6.30%	10.25%	12.00%
97.5	3.60%	6.65%	10.65%	13.50%

The research paper also provides calibration criteria for long-term risk free interest rates at the 10-year horizon, but they were not used as there are no equivalent criteria for the short-term interest rates.

APPENDIX C: SIMULATION RESULTS FOR BENCHMARK YIELDS CALIBRATION

The following are simulation results for Canada and the U.S. related to government yields, credit spreads, and benchmark yields. The government yield simulated percentiles satisfy the reference points. For credit spreads, there are currently no reference points developed, so historical percentiles are instead shown for comparison.

Canadian Simulation Results

Simulated Percentiles for Canadian 10-Year Government Yield with y0 = 5.90% at 60-Year Horizon

Percentiles	CIR Adjusted	BS Adjusted	Reference Points
2.5	1.61%	1.90%	2.25%
5	2.02%	2.18%	2.45%
10	2.59%	2.55%	2.60%
25	3.75%	3.35%	NA
50	5.58%	4.69%	NA
75	7.95%	6.21%	NA
90	10.31%	10.04%	10.00%
95	12.00%	12.84%	12.00%
97.5	13.53%	15.95%	13.50%

Simulated Percentiles for Canadian 10-Year Government Yield with y0 = 3.60% at Two-Year Horizon

Percentiles	CIR Adjusted	BS Adjusted	Reference Points
2.5	2.08%	2.42%	2.45%
5	2.34%	2.60%	2.60%
10	2.61%	2.81%	2.85%
25	3.13%	3.17%	NA
50	3.74%	3.67%	NA
75	4.42%	4.10%	NA
90	5.07%	4.81%	4.70%
95	5.49%	5.19%	5.10%
97.5	5.91%	5.52%	5.35%

Simulated Percentiles for Canadian 10-Year Government Yield with y0 = 5.90% at Two-Year Horizon

Percentiles	CIR Adjusted	BS Adjusted	Reference Points
2.5	3.68%	3.76%	3.90%
5	4.04%	4.05%	4.15%
10	4.40%	4.38%	4.50%
25	5.07%	4.97%	NA
50	5.86%	5.76%	NA
75	6.70%	6.46%	NA
90	7.51%	7.60%	7.40%
95	8.02%	8.20%	7.85%
97.5	8.53%	8.74%	8.25%

Simulated Percentiles for Canadian 10-Year Government Yield with y0 = 8.80% at Two-Year Horizon

Percentiles	CIR Adjusted	BS Adjusted	Reference Points
2.5	5.83%	5.46%	6.10%
5	6.28%	5.88%	6.50%
10	6.73%	6.37%	6.95%
25	7.56%	7.23%	NA
50	8.52%	8.40%	NA
75	9.54%	9.43%	NA
90	10.51%	11.11%	10.40%
95	11.11%	12.00%	11.00%
97.5	11.71%	12.80%	11.45%

Historical and Simulated Percentiles at 60-Year Horizon for DEX Universe Average Spread

Percentiles	MLE s0 = 0.35%	Historical (Dec 1979 to Dec 2012)
2.5	0.11%	0.14%
5	0.14%	0.15%
10	0.18%	0.17%
25	0.26%	0.23%
50	0.38%	0.31%
75	0.52%	0.42%
90	0.69%	0.71%
95	0.79%	0.77%
97.5	0.91%	0.85%

Simulated Percentiles for Canadian Benchmark Yield at 60-Year Horizon

Percentiles	CIR Adjusted	BS Adjusted
2.5	2.11%	2.33%
5	2.55%	2.60%
10	3.14%	2.96%
25	4.23%	3.76%
50	5.78%	5.04%
75	7.64%	7.07%
90	9.61%	10.25%
95	10.98%	13.00%
97.5	12.39%	15.98%

U.S. Simulation Results

Simulated Percentiles for U.S. Seven-Year Government Yield with y0 = 5.65% at 60-Year Horizon

Percentiles	CIR Adjusted	BS Adjusted	Reference Points
2.5	1.37%	1.72%	2.00%
5	1.75%	2.00%	2.20%
10	2.29%	2.34%	2.35%
25	3.44%	3.15%	NA
50	5.29%	4.51%	NA
75	7.74%	6.70%	NA
90	10.21%	10.30%	10.00%
95	12.00%	13.52%	12.00%
97.5	13.60%	17.18%	13.50%

Simulated Percentiles for U.S. Seven-Year Government Yield with y0 = 3.30% at Two-Year Horizon

Percentiles	CIR Adjusted	BS Adjusted	Reference Points
2.5	1.79%	2.15%	2.20%
5	2.04%	2.32%	2.35%
10	2.31%	2.52%	2.55%
25	2.82%	2.88%	NA
50	3.44%	3.38%	NA
75	4.12%	3.95%	NA
90	4.79%	4.54%	4.40%
95	5.22%	4.93%	4.80%
97.5	5.65%	5.27%	5.05%

Simulated Percentiles for U.S. Seven-Year Government Yield with y0 = 5.65% at Two-Year Horizon

Percentiles	CIR Adjusted	BS Adjusted	Reference Points
2.5	3.39%	3.46%	3.65%
5	3.74%	3.75%	3.90%
10	4.11%	4.09%	4.25%
25	4.79%	4.69%	NA
50	5.60%	5.51%	NA
75	6.47%	6.46%	NA
90	7.31%	7.45%	7.15%
95	7.84%	8.11%	7.60%
97.5	8.37%	8.69%	8.00%

Simulated Percentiles for U.S. Seven-Year Government Yield with y0 = 8.65% at Two-Year Horizon

Percentiles	CIR Adjusted	BS Adjusted	Reference Points
2.5	5.57%	5.14%	6.00%
5	6.03%	5.57%	6.40%
10	6.50%	6.08%	6.85%
25	7.35%	6.98%	NA
50	8.36%	8.23%	NA
75	9.42%	9.67%	NA
90	10.44%	11.17%	10.30%
95	11.07%	12.15%	10.85%
97.5	11.69%	13.04%	11.30%

Historical and Simulated Percentiles at 60-Year Horizon for OAS

Percentiles	MLE s0 = 0.35%	Historical (Aug 1988 to Dec 2012
2.5	0.17%	0.24%
5	0.21%	0.26%
10	0.26%	0.29%
25	0.38%	0.39%
50	0.54%	0.47%
75	0.74%	0.66%
90	0.95%	0.85%
95	1.10%	1.14%
97.5	1.25%	1.50%

Simulated Percentiles for U.S. Benchmark Yield at 60-Year Horizon

Percentiles	CIR Adjusted	BS Adjusted
2.5	2.04%	2.31%
5	2.48%	2.58%
10	3.06%	2.94%
25	4.17%	3.74%
50	5.77%	5.05%
75	7.72%	7.22%
90	9.80%	10.75%
95	11.26%	13.87%
97.5	12.73%	17.39%

APPENDIX D: REFERENCES

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