



Mortality Table

Canadian Insured Payout Mortality Table 2014 (CIP2014)

Annuitant Experience Subcommittee – Research Committee

February 2015

Document 215006

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1. Abstract

The Canadian Institute of Actuaries (CIA) has been collecting data on individual annuities on a seriatim basis since 1989, but no mortality table has been produced from the data. This paper sets out the construction of the first mortality table for Canadian payout annuities, CIP2014. It is based on data of the CIA Individual Annuitant Mortality Study for years of experience 2000–2011, but excluding data for policies with annual income of \$72,000 or more. The rates for ages younger and older than those for which there are sufficient data were obtained from the CPM2014 mortality tables (see section 16 for references).

This mortality table was prepared by R.C.W. (Bob) Howard, FCIA, FSA.

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3. Background

The Canadian Institute of Actuaries (CIA) has been collecting data on individual payout annuity mortality since the mid-1980s with seriatim data since 1989. There have been sufficient data to construct a table for many years, but the focus has been on monitoring experience rather than constructing a table.

The author requested and received access to the Individual Annuitant Mortality Study (IAMS) data collected by the Annuitant Experience Subcommittee of the CIA's Research Committee for the purpose of constructing the table presented here. It is the first table to be based on Canadian annuitant data and to be approved for publication by the Research Committee and Member Services Council.

4. Definitions

 q_x^y means the probability that a person, age x nearest birthday at the beginning of calendar year y, will die before reaching the end of the calendar year. Note that both x and y are defined at the beginning of the one-year period.

 I_x^y means the improvement rate in mortality for persons aged x nearest birthday at the start of calendar year y-1 to those aged x at the start of calendar year y. In this case x is constant through the one-year period, and y is defined at the end of the period.

Thus $q_x^y = q_x^{y-1}(1 - I_x^y)$

However, this definition yields a rather odd application if one desires improvement for a partial year, for example, at the middle of year y-1. It is odd because the improvement factor is indexed for year y, but the mortality rate is indexed for year y-1.

$$q_x^{y-0.5} = q_x^{y-1} (1 - I_x^y)^{0.5}$$

In what follows, the actuarial symbols rarely appear. The definitions are presented to clarify the meaning of a mortality rate or improvement rate for a year. The same definitions were used in the construction of CPM2014 and CPM-B and in the Society of Actuaries (SOA) publications RP-2014 Mortality Tables (Exposure Draft) and Mortality Improvement Scale MP-2014 (Exposure Draft).

5. Overview of Method

There are five main steps for table construction:

- 1. Select the data to be used;
- 2. Adjust the policy year data to January 1, as is normally expected of annuitant tables;
- 3. Adjust the data for expected mortality improvement to January 1, 2014;
- 4. Graduate the raw data to obtain smoothed mortality rates; and
- 5. Extend the table to older and younger ages for which there is insufficient experience.

6. Data to Use

The subcommittee collects data separately for single life annuities, joint annuitants while both are alive, and survivors of joint annuities. Although all three types could be combined for constructing a table, there are concerns that the quality of joint data is not as good as single. In particular, there are often much longer delays in reporting deaths for joint annuitants, especially for females while the male is alive, compared to single annuitants. This concern is less when more years of experience are combined because the higher incurred but not reported (IBNR) of the most recent years represents less of the total experience.

The table is constructed from both single and joint data, adjusted for IBNR to 2011. Experience by single/joint/survivor on the new table is shown in section 14.

Data are collected by duration from issue. It would be possible to construct a select and ultimate table from the data. However, there are much fewer data for the earlier durations than for the ultimate, so that the rates for earlier durations would be unreliable.

Therefore, the table is constructed from data by attained age, without regard to duration from issue. Experience by duration on the new table is shown in section 14.

It is well known that mortality rates tend to decrease with increasing income. This fact has been observed for the individual annuitant data. It is reasonable not to include data for the largest sizes in constructing the table because the mortality rates tend to be much lower than for the bulk of the data and also because the volatility of the experience is much greater due to the small number of policies and variation in size.

Because most back-to-back annuities have no guarantee period, and because no guaranteed period is rare otherwise, the table is constructed on data with a guarantee period. However, because not all companies can distinguish between annuities with no guarantee period and those for which the guarantee period has expired, the data for the table include annuities both with and without a guarantee period after the first 10 policy years. Because after 10 years the effect of selection is much less, it is not likely a problem to include some back-to-backs at the higher durations.

Table 1 shows the amount of exposure and the actual to expected ratios (A/E) of the most recent 10 years of data split into bands of annualized income. The bands are 0–10,000, 10,000–20,000, ..., 100,000 and over. Note that the A/E is by income although the exposure is shown by policies. The expected table is the 1983 IAM Basic table (83Basic). The choice of table is not important. The point is to show how much mortality differs by income band.

Table 1. Policy years exposed and Actual/Expected by										
income for 2002-2011 for ranges of annualized income.										
Expected on 1983 IAM Basic table.										
Income	Poli	cies	Actual/E	xpected						
Range	Male Female		Male	Female						
0-10k	1,240,813	1,680,653	92%	86%						
10-20k	103,814	124,512	85%	80%						
20-30k	27,002	28,849	79%	73%						
30-40k	8,427	8,135	75%	75%						
40-50k	4,190	3,644	73%	67%						
50-60k	2,160	1,934	75%	76%						
60-70k	1,290	1,129	65%	77%						
70-80k	792	700	60%	48%						
80-90k	462	337	88%	54%						
90-100k	362	434	66%	34%						
100k+	2,028	1,790	33%	40%						
Total	1,391,339	1,852,115	85%	81%						

CIP2014 is constructed on data for policies with annualized income less than \$72,000 (\$6,000 per month). The choice is arbitrary. It seems to balance accepting more data and avoiding higher volatility.

Important note: all further references to IAMS data in this paper are by income and including policies with annualized income less than \$72,000 and with deaths adjusted for IBNR to 2011, unless explicitly stated otherwise.

The next issue to resolve is what ages and years of data to include. It is desirable to have a wide range of ages on which to base the table. The more years included, the more data to work with. However, there may be a concern if too many years are included that the experience will not be recent enough to be considered relevant.

Tables 2 and 3 show the ratio of standard deviation to the raw mortality rate for selected ages and for groups of years of experience. A reasonable criterion, but not the only legitimate one, is to include those ages for which the ratio is under 10%.

Table 2. Ratio of standard deviation to raw mortality rate by income - Male											
Age	1989-2011	1992-2011	1996-2011	2000-2011	2004-2011	2008-2011					
60	16%	17%	22%	36%	55%	78%					
65	7%	9%	12%	16%	21%	36%					
70	4%	5%	6%	8%	12%	18%					
75	3%	3%	4%	5%	6%	9%					
80	2%	2%	2%	3%	4%	6%					
85	2%	2%	2%	2%	3%	4%					
90	2%	2%	2%	2%	2%	3%					
95	3%	3%	3%	3%	3%	4%					
100	10%	10%	11%	11%	11%	12%					
105	37%	41%	52%	52%	51%	128%					
Table 3. Batio of standard deviation to raw mortality rate by income - Female											
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CIP2014 is constructed using data for ages 70–100 and for years of experience 2000–2011. However, in a later step the youngest three rates and the oldest two rates will be dropped. There are two reasons. First, all graduation methods are less reliable at the ends. Second, by dropping these rates, all those remaining have a ratio of standard deviation to mortality rate of less than 6%.

In summary, there is a total exposure of \$17.25 billion of annualized income in the IAMS data for years of experience 2000–2011. The exposure for ages 70–100 is 85.2% of the total. The exposure for ages 70–100 and for annualized income under \$72,000 is 80.1% of the total. The exposure for ages 70–100, for annualized income under \$72,000, and excluding annuities with no guaranteed period if in the first 10 policy years is 77.0% of the total. Thus the table is constructed from 77% of the exposure available for 2000–2011.

Tables 4 to 7 show the data included in table construction. The expected table for A/E and standard deviations is 83Basic projected to the year of experience on Scale AA.

Tables 4 and 5 are by quinquennial age groups. The A/E ratios suggest that the slope of the experience is steeper than 83Basic. Overall male experience is not far from expected, but female experience appears to be markedly less than the expected table.

Tables 6 and 7 are by year of experience. There is some indication that A/E declines over the range of years. If so, that would indicate that improvement on Scale AA is insufficient.

Table 4. D	Table 4. Data included in table construction by age groups - Male											
Expected on 1983 IAM Basic table with scale AA.												
	Ex	posed	De	eaths	Actual/E	xpected	Standard	Deviation				
Ages	Policies	Income	Policies	Income	Policies	Income	Policies	Income				
70-74	205,010	974,735,565	4,862	20,978,400	109.7%	99.9%	1.5%	2.6%				
75-79	324,393	1,460,980,032	12,885	55,014,601	107.7%	102.8%	0.9%	1.5%				
82-84	407,474	1,692,867,559	27,207	107,390,823	103.7%	99.2%	0.6%	1.0%				
85-89	330,098	1,257,203,120	36,894	136,581,029	108.4%	105.9%	0.5%	0.9%				
90-94	139,482	485,033,674	24,613	83,372,096	114.6%	112.0%	0.6%	1.1%				
95-100	27,114	85,165,812	7,299	22,544,484	120.8%	119.7%	1.1%	2.1%				
Total	1,433,571	5,955,985,761	113,761	425,881,433	109.2%	105.2%	0.3%	0.5%				

Table 5. D	Table 5. Data included in table construction by age groups - Female											
Expected on 1983 IAM Basic table with scale AA.												
	Ex	posed	De	aths	Actual/E	xpected	Standard	Deviation				
Ages	Policies	Income	Policies	Income	Policies	Income	Policies	Income				
70-74	240,448	1,095,142,298	3,494	15,154,970	101.0%	96.6%	1.7%	2.8%				
75-79	406,064	1,691,564,259	9,697	38,070,151	95.0%	90.2%	1.0%	1.6%				
82-84	536,153	2,048,172,630	22,567	80,921,857	94.6%	89.4%	0.6%	1.0%				
85-89	445,819	1,563,613,624	34,710	117,320,969	95.8%	92.9%	0.5%	0.8%				
90-94	199,020	646,190,530	27,005	84,760,497	98.9%	96.0%	0.6%	1.0%				
95-100	43,489	129,804,559	9,394	27,741,652	105.7%	104.9%	0.9%	1.8%				
Total	1,870,994	7,174,487,900	106,867	363,970,096	97.2%	93.4%	0.3%	0.5%				

Expected	Expected on 1983 IAM Basic table with scale AA.											
	Ex	posed	Deaths		Actual/E	Actual/Expected		Standard Deviation				
Year	Policies	Income	Policies	Income	Policies	Income	Policies	Income				
2000	104,857	375,181,417	6,899	21,932,035	110.8%	107.8%	1.2%	2.1%				
2001	129,897	479,780,685	7,854	26,351,476	97.2%	96.0%	1.1%	1.8%				
2002	132,328	511,413,676	9,548	33,787,571	111.1%	110.8%	1.0%	1.8%				
2003	133,457	525,322,630	10,231	36,263,585	113.3%	110.3%	1.0%	1.7%				
2004	134,877	542,998,879	10,485	37,820,877	110.9%	107.3%	1.0%	1.7%				
2005	128,791	531,989,270	10,499	38,483,928	112.5%	107.7%	1.0%	1.7%				
2006	128,084	545,528,802	10,278	39,059,150	107.6%	103.6%	1.0%	1.7%				
2007	121,265	526,842,150	10,065	38,722,395	107.8%	102.8%	1.0%	1.7%				
2008	114,687	511,330,072	9,998	39,766,728	109.8%	105.1%	1.0%	1.7%				
2009	108,322	485,180,538	9,931	39,124,816	112.3%	105.9%	1.0%	1.7%				
2010	101,602	467,792,225	9,278	37,684,554	109.2%	103.2%	1.0%	1.8%				
2011	95,404	452,625,417	8,695	36,884,318	106.8%	102.3%	1.0%	1.8%				
Total	1,433,571	5,955,985,761	113,761	425,881,433	109.2%	105.2%	0.3%	0.5%				

Table 6. Data included in table construction by year of experience - Male

 Table 7. Data included in table construction by year of experience - Female
Expected on 1983 IAM Basic table with scale AA.

-	Exposed		Deaths		Actual/Expected		Standard Deviation	
Year	Policies	Income	Policies	Income	Policies	Income	Policies	Income
2000	116,600	393,384,941	4,672	13,458,089	97.2%	92.1%	1.4%	2.4%
2001	151,454	514,863,971	5,551	16,263,402	81.8%	78.1%	1.2%	2.0%
2002	159,648	564,312,296	7,641	23,290,968	101.0%	96.6%	1.1%	1.9%
2003	165,309	595,683,534	8,381	26,742,872	99.5%	96.2%	1.0%	1.8%
2004	172,306	634,524,093	9,319	29,273,022	100.4%	93.6%	1.0%	1.7%
2005	169,983	644,692,758	9,687	32,869,999	100.4%	98.3%	1.0%	1.6%
2006	170,687	663,164,074	10,006	34,421,048	97.8%	94.6%	0.9%	1.6%
2007	164,791	652,973,189	10,310	35,690,036	98.9%	94.4%	0.9%	1.6%
2008	159,736	658,711,329	10,615	38,413,810	99.7%	95.9%	0.9%	1.5%
2009	153,126	626,053,329	10,485	37,416,408	98.0%	93.6%	0.9%	1.5%
2010	146,685	616,878,549	10,020	36,913,826	93.6%	89.9%	0.9%	1.5%
2011	140,669	609,245,835	10,181	39,216,615	95.1%	93.0%	0.9%	1.5%
Total	1,870,994	7,174,487,900	106,867	363,970,096	97.2%	93.4%	0.3%	0.5%

If the A/E ratios were all close to 100%, then the expected table would be judged an appropriate estimate of mortality. If the A/E ratios were largely the same for all ages and years, then a multiple of the expected table would be judged appropriate. Because the ratios vary, particularly by age, it is evident that a new table is needed.

7. Adjustment to January 1

The IAMS reports published by the CIA and the IAMS summarized data determine age as age nearest birthday on the policy anniversary. (The weighted average policy

anniversary is 0.5011 of the way through the calendar year. For all practical purposes anniversaries can be taken as occurring at mid-year on average.) A mortality table, to be readily adjusted for mortality improvement, would best have age determined as age nearest birthday on January 1.

The IAMS data is further complicated by the fact that some contributors submit data for policy years, but others for calendar years.

Figure 1 shows how the study periods compare. It is a Lexis diagram of ages and calendar years; horizontal lines represent exact ages, and vertical lines represent December 31 of the indicated year. The thick black line represents the study period that is desired, age x nearest birthday on January 1 of year y, continuing to the end of the calendar year. The red line represents an average year in a policy year study for year of experience y. It begins at the average policy anniversary in year y-1 for those then age x nearest birthday, and it continues to the policy anniversary in year y. The green line represents an average year of experience y. Age is determined as age nearest birthday on the preceding anniversary, but there are two half-year line segments to represent the year of experience. One runs from the anniversary in y to the end of the calendar year, and the other from the start of the calendar year to the anniversary in y. (The latter line segment is offset in the diagram to avoid overlapping the red.)



The green lines, on average, match with the desired black line. The red line occurs half a year earlier than desired. The correction is to apply half a year of mortality improvement to move the red line so that it will overlap the black.

8. Adjustment for Mortality Improvement

It is typical in constructing mortality tables to combine the data for all years included and then to characterize the resulting raw mortality rates are representing the middle year of the study period. That does not always work well. There must be approximately the same amount of exposure each year, and not just in total, but also at each age. The worst case is to have substantially different distributions by year for different ages. The IAMS data are a bad case. At age 75 the weighted average year of experience is very nearly 2005, and at age 95 almost 2008. An effective solution to the problem is to apply the expected effect of mortality improvement to move all data to January 1, 2014. Policy year data for the year of experience 2011 would require 3.5 years of improvement (half of the rate for 2011 to move from the middle of 2010 to the beginning of 2011 and a full year of improvement for 2012, 2013, and 2014). Calendar year data for the year of experience 2011 would require three years of improvement. For each year earlier of the data, one more year of improvement is required.

The adjustment need be made only to deaths. The exposure is fine as is.

But which improvement scale should be used? The recently published CPM-B is used because it is the most recent and because its historical part is based on Canadian annuitants (of the Canada Pension Plan and Québec Pension Plan).

9. Graduation

The premise of graduation is that there is an underlying, smooth mortality curve that is hidden by statistical fluctuations. The graduation process produces a smooth curve that is likely to be much closer to the real underlying curve than the raw mortality rates.

The method of graduation used is Whittaker-Henderson (WH), a commonly-used method. WH is computationally complex, but conceptually quite straightforward. The "elevator version" of WH is this: WH optimizes the balance between closeness of fit of the graduated data to the raw data and smoothness of the graduated data. Fit is measured by the sum of the squared difference between the graduated and raw data, usually weighted by another set of numbers, such as exposure. Smoothness is measured by the sum of the squared finite differences, of a specified order, in the graduated data. The standard expression to be optimized is given below.

$$\sum Wt(Grad - Raw)^2 + h\sum (\Delta^n Grad)^2$$

To use WH one must choose a set of weights and the parameters n, the order of difference in determining smoothness, and h, the factor balancing smoothness and fit. The weights used are the exposure by annualized income, normalized by multiplying by a factor so that the sum of the weights is 31, the number of rates being graduated. The reasonable candidates for n are 3 and 4, so that perfect smoothness would be represented by a quadratic and cubic, respectively.

Table 8 shows a number of tests of different values for n and h. The column headed "Fit" shows the sum of the squared difference between the raw and graduated mortality rates. The columns headed "3rd diff" and "4th diff" show the sum of the squares of the third and fourth differences of the graduated mortality rates.

Note that as h increases smoothness improves (a lower sum of squared finite differences) but fit worsens (a higher sum of weighted squared errors). The challenge is to find a value of h which yields a curve that is smooth enough without departing too much in fit. Also note that with n=4 the variability of fit is less than with n=3 and for the same value of h, fit is lower. For that reason n=4 is preferred in this case. The choice of h is arbitrary; any of 200, 500, and 1000 appear to provide adequate fit and sufficient

Sex	n	h	Fit	3rd Diff	4th Diff
М	3	100	1.34E-04	1.50E-07	1.90E-08
М	3	200	1.41E-04	9.52E-08	7.11E-09
М	3	500	1.51E-04	6.52E-08	2.78E-09
М	3	1000	1.61E-04	5.01E-08	1.66E-09
М	4	100	1.03E-04	2.37E-06	1.26E-07
М	4	200	1.13E-04	1.33E-06	5.67E-08
Μ	4	500	1.24E-04	5.85E-07	1.81E-08
Μ	4	1000	1.31E-04	3.23E-07	7.83E-09
F	3	100	6.79E-05	1.43E-07	1.15E-08
F	3	200	7.25E-05	1.10E-07	5.97E-09
F	3	500	8.03E-05	8.57E-08	3.23E-09
F	3	1000	9.19E-05	6.97E-08	2.17E-09
F	4	100	5.60E-05	9.41E-07	4.31E-08
F	4	200	5.86E-05	6.48E-07	2.40E-08
F	4	500	6.23E-05	3.78E-07	1.19E-08
F	4	1000	6.57E-05	2.50E-07	7.16E-09

smoothness. The author prefers squared 4th differences to be less than 1E-09 per number graduated, but there is no objective criterion for deciding how smooth is smooth enough. The desired degree of smoothness is obtained with *h*=500 or higher.

The graduation used *n*=4 and *h*=500 for both males and females.

There is a variation of WH presented by Walter Lowrie that could be considered. This variation specifies another parameter, a growth rate. Then perfect smoothness for order *n* is represented by an exponential plus a polynomial of degree *n*-2 rather than a polynomial of degree *n*-1 as in the normal form of WH. Testing showed that the raw mortality rates tended to be flatter at both the younger and older ages and steeper in the middle. This condition does not recommend Lowrie's variation. A test graduation with Lowrie's variation and a growth rate of 6% yielded a very slight improvement in fit but a little less smoothness. Accordingly, Lowrie's variation was not used.

Figures 2 and 3 present the results of the graduation in graphic form as ratios to 83Basic projected on scale AA to 2014. The figures show the graduated mortality rates as a blue line and the raw mortality rates as red diamonds. The red tick marks represent one standard deviation above and below the raw mortality rates. One would normally expect that the graduated rates would pass between the pairs of tick marks about 2/3 of the time and there should be very few instances of the graduated rate being more than two standard deviations away from the raw rate. In fact the male graduated rates are outside the tick marks only four times, at ages 75, 77, 86, and 88. Age 86 is the only instance of the graduated rate deviating from the raw by more than two standard deviations. For females the graduated rates are outside the tick marks only for ages 73, 83, and 94, and none of these have a differential of as much as two standard deviations.





Note that the slope of the graduated rates is far from level. This indicates that the slope of the new table will be significantly different from that of 83Basic projected to 2014.

It may seem surprising that the blue line, although graduated, does not appear to be smooth. The reason is that 83Basic is not very smooth, and it gets much rougher when it is projected on scale AA for many years beyond 1983.

10. Extension to Older Ages

A table only for ages 70–100 is not very useful, but there are not sufficient data in IAMS at other ages. Therefore, the graduated rates, which are the ages most used for payout annuities, will need to be extended using other sources.

The recently published CPM2014 serves this purpose. The rates for ages 106 and higher are taken from CPM2014. The rates for ages 99–105 are calculated by fitting a 4th degree polynomial to the rates for ages 96, 97, 98, 106, and 107. It should be noted that the study underlying CPM2014 did not have enough data over 100 either. It used the rates from a paper by Howard presented at the Living to 100 Symposium in 2011.



Figures 4 and 5 show the interpolation between the two table segments.



11. Extension to Younger Ages

The approach for the younger ages is similar. The youngest graduated rate to be retained is the rate for age 73. The rates for ages 18–65 are taken directly from CPM2014. The intervening rates are calculated by fitting a 5th degree polynomial to the rates for age 63, 64, 65, 73, 74, and 75. CPM2014 and the graduated rates are very close together around age 73 for males, and 5% apart for females. It would not be unreasonable to use 105% of CPM2014 for females, but 100% is used for consistency with the males.

The data underlying CPM2014 were credible below age 70, but not below age 55. Rates under 54 were set to a multiple of CIA9704 ultimate non-smoker, and rates for ages 54–60 were interpolated.

Figures 6 and 7 show the interpolation between the two table segments.



Note that CPM2014 begins at age 18, and the same is being done for CIP2014. It is unlikely that younger ages will be needed, but if they are an actuary could readily extend the table from an insurance mortality table.

12. Sensitivity to Improvement Scale

Because an improvement scale is used in adjusting deaths to 2014, it is relevant to ask whether the choice of improvement scale unduly influenced the result. To answer that question, this section looks at the sensitivity of the mortality table to the improvement scale by presenting the result from calculating with the same method using three improvement scales in addition to CPM-B which was used as described above.

The improvement scales are:

- 1. Scale AA, for historical reasons. It appears to be too low for the actual rates of improvement during the period covered by the mortality study.
- 2. The scale from the Committee on Life Insurance Financial Reporting. This scale also seems low for the study period, but it is widely used in Canada for annuity valuations.
- 3. MP-2014. This scale appears in an exposure draft published by the Society of Actuaries. Like CPM-B it is a two-dimensional scale. For the years 2000–2014 MP-2014 is lower than CPM-B for all except the highest ages, and higher than CPM-B for most female ages after 2005.

Table 9 show various mortality rates in the tables that result from using the different improvement scales with no change in method. The sensitivity in the mortality rates, at each age, to the choice of improvement scale is large. It is clearly important to make a good choice of improvement scale.

Table	Table 9. Mortality rates from using various										
impr	improvement scales in table construction.										
Sex	Factor	CPM-B	AA	CLIFR	MP-2014						
	$q_{\ 70}$	0.01296	0.01437	0.01500	0.01342						
	<i>q</i> 75	0.02182	0.02546	0.02652	0.02285						
ale	$q_{\ 80}$	0.03890	0.04605	0.04611	0.04024						
Σ	$q_{_{85}}$	0.07470	0.08349	0.08110	0.07286						
	q_{90}	0.13864	0.14425	0.13884	0.12826						
	q_{95}	0.23477	0.23344	0.22878	0.21398						
	$q_{\ 70}$	0.00934	0.01025	0.00996	0.00911						
	<i>q</i> 75	0.01502	0.01662	0.01616	0.01463						
Jale	$q_{\ 80}$	0.02571	0.02847	0.02786	0.02542						
Fen	q_{85}	0.05050	0.05563	0.05358	0.04901						
	$q_{\ 90}$	0.10238	0.10715	0.10249	0.09424						
	q_{95}	0.18671	0.18627	0.18222	0.16788						

However, the sensitivity is magnified by adjusting the tables to 2014. The differences seen will be at least the combined effect of improvement to 2014 from the approximate mid-point of the experience. (The weighted average year of experience is 2005.53 for males and 2005.75 for females.) Table 10 shows the same mortality rates as in table 9 but adjusted by mortality improvement on the indicated scale to 2006. Note that the same scale is used to adjust the experience and also the final mortality rates, as indicated in the column headings. When adjusted to 2006, the sensitivity to the improvement scale is much less, and not significant, with the possible exception of age 95.

Table	Table 10. Mortality rates as of 2006 from using various									
improvement scales in table construction.										
Sex	Factor	CPM-B	AA	CLIFR	MP-2014					
	q 70	0.01668	0.01621	0.01626	0.01624					
	<i>q</i> 75	0.02786	0.02850	0.02874	0.02820					
ale	$q_{\ 80}$	0.04928	0.04990	0.04997	0.04950					
Ξ	$q_{ 85}$	0.08743	0.08831	0.08788	0.08816					
	q 90	0.15000	0.14895	0.15047	0.15166					
	q_{95}	0.23502	0.23721	0.23814	0.24208					
	$q_{\ 70}$	0.01083	0.01067	0.01079	0.01106					
	<i>q</i> 75	0.01741	0.01772	0.01751	0.01751					
Jale	$q_{\ 80}$	0.02980	0.03012	0.03019	0.03023					
Fen	$q_{_{85}}$	0.05851	0.05837	0.05806	0.05881					
	q_{90}	0.11077	0.10975	0.11107	0.11270					
	q_{95}	0.18691	0.18927	0.18968	0.19429					

If an actuary believes that CPM-B is not an appropriate improvement scale for the intended use, either for future improvement or for recent history, the actuary could back off the CIP2014 mortality rates to 2006 using CPM-B and then project forward from 2006 on the other improvement scale.

13. Comparison to CPM2014 and Other Tables

Figures 8 and 9 show the ratio of CPM2014, 83Basic projected on AA to 2014 (83B@2014), and RP-2014 for healthy annuitants to the table just constructed, CIP2014.

It is interesting to note how close CIP2014 and CPM2014 are. It is not surprising that the slopes are similar because both represent recent Canadian experience, but it seems coincidental that the levels of the two tables are so close.

The difference between 83B@2014 and CIP2014 is important to note because 83Basic and scale AA are still in common use. The slopes of the two tables are quite different, although annuity values on the two bases may not be terribly far apart at some ages.

RP-2014 is not final at the time of writing; it is presented in an exposure draft of the SOA. Its slope is closer to 83B@2014 than to CIP2014. That may be a feature of both being based on U.S. data.

14. Experience Relative to CIP2014

Because CIP2014 could be used in pricing payout annuities, actuaries will want to know how experience relative to CIP2014 varies over various subsets of annuities. The following tables endeavour to provide that further information. In all cases the expected is calculated on CIP2014 projected to the appropriate year of experience on improvement scale CPM-B and adjusting for policy year or calendar year data. The tables are in pairs. The male table comes first, followed by the female.

All tables include years of experience 2000–2011, all ages, annuities both with and without a guarantee period, and an IBNR adjustment as of 2011. (IBNR adjustment factors are company-specific.) Except for the tables showing variation by size of income, the data include only annuities of less than \$6,000 of monthly income.

Tables 11 and 12 show the three annuitant types, single life annuitants (single), joint life annuitants while both are alive (joint), and surviving joint annuitants after the first death (survivor). The experience for survivor is much heavier than for joint, for both males and females. The difference between single and joint is much larger for females than for males, and the difference between joint and survivor is larger for males than for females.

Table 11.	Table 11. Experience by annuitant type - Male										
	Ex	posed	De	eaths	Actual/Expected		Standard Deviation				
Туре	Policies	Income	Policies	Income	Policies	Income	Policies	Income			
Single	930,148	3,771,621,337	66,664	225,904,897	103.1%	96.0%	0.4%	0.7%			
Joint	625,159	3,031,712,946	37,741	168,224,313	96.6%	96.4%	0.5%	0.8%			
Survivor	111,678	476,331,541	13,795	56,144,422	120.7%	120.0%	0.9%	1.4%			
Total	1,666,985	7,279,665,824	118,200	450,273,632	102.6%	98.6%	0.3%	0.5%			

Table 12.	Table 12. Experience by annuitant type - Female											
	Ex	posed	Deaths		Actual/Expected		Standard Deviation					
Туре	Policies	Income	Policies	Income	Policies	Income	Policies	Income				
Single	1,157,769	4,313,014,829	67,211	212,979,807	104.8%	99.7%	0.4%	0.7%				
Joint	619,522	2,976,843,498	18,553	79,148,468	93.7%	91.0%	0.7%	1.1%				
Survivor	393,516	1,613,538,236	25,143	94,987,121	105.9%	103.7%	0.6%	1.0%				
Total	2,170,807	8,903,396,562	110,907	387,115,397	103.0%	98.7%	0.3%	0.5%				

Tables 13 and 14 divide the data into three tax types: registered retirement savings plan (RRSP), registered pension plan (RPP), and non-registered. As might be expected the heaviest mortality is for RPP and the lightest for non-registered.

Table 13. Experience by tax type - Male										
	Exposed Deaths Actual/Expected Standard Dev							Deviation		
Туре	Policies	Income	Policies	Income	Policies	Income	Policies	Income		
RRSP	971,134	3,952,292,891	77,366	281,057,880	103.3%	101.6%	0.3%	0.6%		
RPP	154,555	986,217,080	7,902	43,551,027	112.8%	105.6%	1.1%	2.0%		
non-reg	541,295	2,341,155,853	32,931	125,664,725	99.1%	90.6%	0.5%	1.0%		
Total	1,666,985	7,279,665,824	118,200	450,273,632	102.6%	98.6%	0.3%	0.5%		

Table 14. Experience by tax type - Female										
	Exposed Deaths Actual/Expected Standard Devia							Deviation		
Туре	Policies	Income	Policies	Income	Policies	Income	Policies	Income		
RRSP	1,364,196	4,987,633,199	73,059	232,873,564	102.9%	100.0%	0.4%	0.6%		
RPP	172,631	992,796,330	5,270	25,407,603	110.8%	106.5%	1.4%	2.5%		
non-reg	633,979	2,922,967,033	32,578	128,834,230	102.0%	95.0%	0.5%	1.0%		
Total	2,170,807	8,903,396,562	110,907	387,115,397	103.0%	98.7%	0.3%	0.5%		

Tables 15 and 16 show separately the first seven policy years from issue and then all other durations combined (ultimate). The data should be interpreted with some caution; the standard deviations for the first seven years are much higher than for the ultimate. However, there is a strong indication that mortality is lower in the early policy years. There are not enough data to develop a select/ultimate mortality table as is done for individual insurance.

Table 15.	Table 15. Experience by duration from issue - Male								
Policy	Ex	posed	De	Deaths		Actual/Expected		Standard Deviation	
Year	Policies	Income	Policies	Income	Policies	Income	Policies	Income	
1	31,193	221,039,370	525	3,732,894	71.2%	61.1%	3.6%	5.6%	
2	30,131	202,420,710	648	4,496,381	83.3%	72.7%	3.5%	5.5%	
3	30,845	194,384,545	767	5,183,417	89.4%	80.7%	3.3%	5.3%	
4	32,498	191,733,104	848	5,136,987	88.8%	76.4%	3.1%	5.1%	
5	35,714	202,093,874	1,089	6,746,171	99.6%	92.1%	2.9%	4.7%	
6	38,747	206,790,860	1,209	7,096,213	97.7%	91.0%	2.8%	4.5%	
7	42,635	217,648,160	1,463	7,908,230	103.5%	93.4%	2.6%	4.2%	
Ultimate	1,425,222	5,843,555,202	111,650	409,973,340	103.3%	100.6%	0.3%	0.5%	
Total	1,666,985	7,279,665,824	118,200	450,273,632	102.6%	98.6%	0.3%	0.5%	

Table 16. Experience by duration from issue - Female									
Policy	Exposed		Deaths		Actual/Expected		Standard Deviation		
Year	Policies	Income	Policies	Income	Policies	Income	Policies	Income	
1	36,961	267,807,820	353	2,947,986	60.3%	56.9%	4.1%	6.0%	
2	36,459	251,481,199	491	3,990,287	76.1%	73.6%	3.9%	5.8%	
3	37,040	240,601,845	594	4,489,411	83.3%	79.1%	3.7%	5.6%	
4	38,989	238,788,504	698	5,235,994	87.4%	87.5%	3.5%	5.3%	
5	42,864	250,320,699	870	5,811,413	95.3%	89.5%	3.2%	5.0%	
6	46,237	255,761,322	986	6,018,512	95.7%	86.4%	3.0%	4.7%	
7	50,384	265,616,105	1,150	6,459,211	98.2%	85.9%	2.8%	4.4%	
Ultimate	1,881,873	7,133,019,068	105,763	352,162,583	103.9%	100.9%	0.3%	0.5%	
Total	2,170,807	8,903,396,562	110,907	387,115,397	103.0%	98.7%	0.3%	0.5%	

Tables 17–22 show the variation in the level of mortality by size. The tables show amounts in increments of \$1,000 of monthly income. Note that the vast majority of the experience is on policies of less than \$1,000 per month of income. These six tables, unlike the earlier ones, include all policies, not only those with income under \$6,000 per month. Tables 17 and 18 contain all data. Tables 19–20 contain RRSP data only, and Tables 21 and 22 contain non-registered data only.

Table 17. Experience by amount of monthly income - Male									
	Ex	posed	De	eaths	Actual/Ex	xpected	Standard	Standard Deviation	
Size	Policies	Income	Policies	Income	Policies	Income	Policies	Income	
0-1k	1,541,219	4,608,857,382	111,504	312,772,552	103.2%	101.7%	0.3%	0.4%	
1-2k	92,644	1,502,651,368	5,121	82,193,186	96.1%	95.7%	1.3%	1.3%	
2-3k	21,420	607,264,646	1,029	29,238,325	88.0%	87.9%	2.8%	2.8%	
3-4k	6,764	278,870,604	312	12,884,896	84.5%	84.6%	4.9%	4.9%	
4-5k	3,237	171,137,881	161	8,561,683	90.9%	91.4%	7.2%	7.2%	
5-6k	1,701	110,883,943	72	4,622,989	86.4%	85.6%	10.5%	10.5%	
6-7k	876	67,375,258	34	2,581,206	77.3%	77.3%	14.4%	14.4%	
7-8k	526	47,326,542	23	2,022,275	85.8%	85.2%	18.4%	18.4%	
8k +	2,321	453,819,211	60	9,395,409	53.9%	41.9%	9.0%	13.0%	
Total	1,670,708	7,848,186,835	118,316	464,272,522	102.6%	95.8%	0.3%	0.8%	

Table 18.	Table 18. Experience by amount of monthly income - Female								
	Exposed		Deaths		Actual/Expected		Standard	Deviation	
Size	Policies	Income	Policies	Income	Policies	Income	Policies	Income	
0-1k	2,033,346	6,113,195,123	105,813	287,230,256	103.5%	101.6%	0.3%	0.4%	
1-2k	105,821	1,700,326,180	4,049	64,044,735	94.5%	93.7%	1.5%	1.5%	
2-3k	21,525	606,324,818	724	20,500,026	86.6%	86.5%	3.3%	3.3%	
3-4k	5,824	238,760,198	180	7,263,341	86.6%	85.7%	6.7%	6.7%	
4-5k	2,884	153,159,077	93	4,919,191	89.6%	89.7%	9.5%	9.5%	
5-6k	1,408	91,631,167	48	3,157,849	88.7%	89.1%	12.9%	13.0%	
6-7k	780	59,999,271	15	1,176,888	53.5%	53.8%	18.0%	18.0%	
7-8k	551	50,148,630	10	915,044	58.6%	57.6%	23.1%	23.1%	
8k +	2,062	390,016,261	52	7,799,469	67.0%	51.9%	10.9%	14.3%	
Total	2,174,199	9,403,560,724	110,984	397,006,799	103.0%	96.6%	0.3%	0.7%	

In spite of the large standard deviations for the larger amount bands, there is a strong correlation between increasing size and decreasing mortality.

Table 19.	Experience	Experience by amount of monthly income - Male RRSP									
	Ex	posed	Deaths		Actual/E	Actual/Expected		Deviation			
Size	Policies	Income	Policies	Income	Policies	Income	Policies	Income			
0-1k	914,721	2,846,902,431	73,816	213,052,885	103.4%	102.0%	0.4%	0.5%			
1-2k	45,001	719,878,060	2,879	45,705,534	99.6%	99.6%	1.8%	1.8%			
2-3k	7,906	221,942,437	482	13,524,123	106.2%	105.9%	4.5%	4.5%			
3-4k	2,193	90,395,685	120	4,976,590	97.2%	97.8%	8.5%	8.6%			
4-5k	956	50,151,102	53	2,774,214	93.3%	93.7%	12.7%	12.7%			
5-6k	357	23,023,175	16	1,024,534	91.4%	90.7%	22.8%	22.9%			
6-7k	263	20,158,209	10	783,663	85.1%	85.8%	27.9%	28.0%			
7-8k	64	5,810,007	5	463,349	161.5%	164.1%	54.8%	54.8%			
8k +	223	32,923,501	10	1,210,817	65.2%	53.5%	23.4%	25.5%			
Total	971,684	4,011,184,608	77,391	283,515,709	103.3%	101.2%	0.3%	0.6%			

Table 20. Experience by amount of monthly income - Female RRSP									
	Ex	posed	Deaths		Actual/Expected		Standard Deviation		
Size	Policies	Income	Policies	Income	Policies	Income	Policies	Income	
0-1k	1,303,606	3,855,358,519	70,670	189,214,046	103.2%	101.0%	0.4%	0.5%	
1-2k	50,363	797,602,396	2,036	31,965,160	96.3%	95.9%	2.1%	2.1%	
2-3k	7,633	214,113,592	258	7,301,887	91.5%	91.7%	5.7%	5.8%	
3-4k	1,653	68,001,596	59	2,423,769	99.3%	98.9%	12.4%	12.5%	
4-5k	678	35,626,694	26	1,346,547	101.5%	101.7%	19.2%	19.2%	
5-6k	263	16,930,402	9	622,156	122.8%	124.1%	34.8%	34.8%	
6-7k	157	12,038,752	4	329,824	93.7%	96.8%	46.1%	46.2%	
7-8k	82	7,342,063	3	275,861	112.5%	113.6%	59.4%	59.4%	
8k +	129	18,727,349	5	1,186,899	111.7%	164.9%	44.4%	48.9%	
Total	1,364,564	5,025,741,363	73,071	234,666,148	102.9%	100.2%	0.4%	0.6%	

The RRSP data show much less variation in A/E ratio by income than do the total data.

Table 21.	Table 21. Experience by amount of monthly income - Male non-registered									
	Exposed		Deaths		Actual/Expected		Standard Deviation			
Size	Policies	Income	Policies	Income	Policies	Income	Policies	Income		
0-1k	495,603	1,313,268,065	30,735	78,337,899	100.4%	98.1%	0.5%	0.8%		
1-2k	31,550	515,394,590	1,608	25,918,584	89.0%	88.0%	2.2%	2.3%		
2-3k	8,768	249,072,512	362	10,346,046	70.4%	70.4%	4.2%	4.2%		
3-4k	2,918	120,612,773	121	4,972,538	69.6%	69.4%	7.1%	7.1%		
4-5k	1,457	76,965,698	67	3,592,325	87.2%	87.7%	10.8%	10.8%		
5-6k	999	65,842,215	38	2,497,332	73.7%	73.3%	13.1%	13.2%		
6-7k	547	42,060,190	20	1,557,223	72.0%	71.9%	17.8%	17.8%		
7-8k	405	36,394,087	14	1,286,490	72.0%	71.2%	21.0%	21.0%		
8k +	1,992	408,760,450	48	7,954,195	52.7%	40.7%	10.0%	14.5%		
Total	544,240	2,828,370,581	33,014	136,462,633	98.9%	84.1%	0.5%	2.0%		

Table 22.	able 22. Experience by amount of monthly income - Female non-registered									
	Ex	posed	Deaths		Actual/Expected		Standard Deviation			
Size	Policies	Income	Policies	Income	Policies	Income	Policies	Income		
0-1k	579,352	1,749,939,014	30,399	84,078,796	103.2%	101.1%	0.6%	0.8%		
1-2k	39,560	640,603,856	1,635	25,936,704	90.2%	88.9%	2.2%	2.3%		
2-3k	9,743	273,774,856	372	10,482,993	82.7%	82.3%	4.5%	4.5%		
3-4k	2,882	118,050,090	90	3,592,547	78.6%	77.2%	8.9%	9.0%		
4-5k	1,571	83,477,496	49	2,618,057	79.8%	79.6%	12.2%	12.2%		
5-6k	871	57,121,721	33	2,125,133	79.6%	79.6%	14.8%	14.9%		
6-7k	523	40,130,413	10	773,372	48.8%	48.9%	21.1%	21.1%		
7-8k	409	37,359,526	7	639,183	55.6%	54.1%	26.6%	26.6%		
8k +	1,840	360,829,410	44	6,405,325	63.4%	45.6%	11.4%	15.1%		
Total	636,752	3,361,286,381	32,640	136,652,110	101.8%	89.6%	0.5%	1.7%		

As expected from the lesser variation in the RRSP data, the non-registered data show substantially more variation than RRSP.

15. Conclusion

This author believes that CIP2014 is an appropriate best estimate for payout annuities. It may be enhanced by adjusting for size of annuity. (Recall that annuities with annualized income of \$72,000 or more have been excluded.) Any who do further research in this area are encouraged to publish their findings.

CIP2014 is available in an Excel workbook <u>here</u>. It is also included in this document below in table 23.

16. References

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17. CIP2014 Mortality Rates

Table	23. Morta	lity rates of C	CIP2014	l.				
Age	Male	Female	Age	Male	Female	Age	Male	Female
18	0.00067	0.00015	51	0.00285	0.00141	84	0.06557	0.04377
19	0.00075	0.00017	52	0.00307	0.00153	85	0.07470	0.05050
20	0.00082	0.00018	53	0.00333	0.00168	86	0.08495	0.05832
21	0.00089	0.00019	54	0.00365	0.00186	87	0.09638	0.06734
22	0.00095	0.00020	55	0.00403	0.00207	88	0.10910	0.07765
23	0.00101	0.00022	56	0.00448	0.00231	89	0.12317	0.08932
24	0.00105	0.00023	57	0.00495	0.00258	90	0.13864	0.10238
25	0.00108	0.00024	58	0.00542	0.00287	91	0.15550	0.11683
26	0.00113	0.00025	59	0.00587	0.00318	92	0.17370	0.13261
27	0.00116	0.00027	60	0.00628	0.00350	93	0.19311	0.14961
28	0.00117	0.00027	61	0.00666	0.00384	94	0.21354	0.16771
29	0.00119	0.00028	62	0.00702	0.00421	95	0.23477	0.18671
30	0.00120	0.00030	63	0.00743	0.00464	96	0.25654	0.20644
31	0.00122	0.00031	64	0.00790	0.00511	97	0.27858	0.22670
32	0.00122	0.00034	65	0.00844	0.00562	98	0.30062	0.24728
33	0.00120	0.00036	66	0.00908	0.00620	99	0.32328	0.26871
34	0.00120	0.00039	67	0.00983	0.00686	100	0.34692	0.29128
35	0.00120	0.00042	68	0.01071	0.00761	101	0.37169	0.31508
36	0.00120	0.00045	69	0.01175	0.00844	102	0.39744	0.33995
37	0.00122	0.00048	70	0.01296	0.00934	103	0.42382	0.36552
38	0.00125	0.00053	71	0.01433	0.01031	104	0.45020	0.39120
39	0.00130	0.00057	72	0.01590	0.01134	105	0.47573	0.41616
40	0.00136	0.00061	73	0.01767	0.01244	106	0.49928	0.43937
41	0.00144	0.00065	74	0.01964	0.01366	107	0.51950	0.45956
42	0.00154	0.00069	75	0.02182	0.01502	108	0.53970	0.47973
43	0.00165	0.00075	76	0.02430	0.01656	109	0.55987	0.50988
44	0.00178	0.00080	77	0.02715	0.01834	110	0.58000	0.53000
45	0.00190	0.00086	78	0.03047	0.02041	111	0.60000	0.55000
46	0.00205	0.00092	79	0.03435	0.02284	112	0.62000	0.57000
47	0.00219	0.00101	80	0.03890	0.02571	113	0.64000	0.59000
48	0.00234	0.00109	81	0.04422	0.02913	114	0.66000	0.61000
49	0.00250	0.00119	82	0.05039	0.03320	115	1.00000	1.00000
50	0.00266	0.00129	83	0.05749	0.03805			