

Educational Note Supplement

Development of the Equilibrium Risk-Free Market Curve for the Base Scenario

Committee on Life Insurance Financial Reporting

December 2015

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Members should be familiar with educational note supplements. Educational note supplements expound or update the guidance provided in an educational note. They do not constitute standards of practice and are, therefore, not binding. They are, however, in conjunction with the source educational note, intended to illustrate the application (but not necessarily the only application) of the Standards of Practice, so there should be no conflict between them. They are intended to assist actuaries in applying standards of practice in respect of specific matters. Responsibility for the manner of application of standards of practice in specific circumstances remains that of the members.

MEMORANDUM

To: All life insurance practitioners

From: Pierre Dionne, Chair
Practice Council

Rebecca Rycroft, Chair
Committee on Life Insurance Financial Reporting

Date: December 17, 2015

Subject: **Educational Note Supplement – Development of the Equilibrium Risk-Free Market Curve for the Base Scenario**

The Committee on Life Insurance Financial Reporting (CLIFR) has prepared this educational note supplement to provide supplementary information on the method used to develop and apply the equilibrium risk-free market curve for the base scenario.

The information presented in this educational note supplement was reviewed by CLIFR in 2015 in support of CLIFR's review of the guidance provided related to the construction and application of an equilibrium risk-free market curve. As a result of this review, CLIFR prepared amendments to the educational note [Investment Assumptions Used in the Valuation of Life and Health Insurance Contract Liabilities](#) which were approved by the Practice Council in September 2015 and published on September 16, 2015. The amended educational note provided updated guidance on the recommended approach for determination of the equilibrium risk-free market curve for use in the base scenario.

CLIFR would like to acknowledge the contribution of its subcommittee that reviewed the potential approaches to developing the equilibrium risk-free market curve for the base scenario and provided a recommendation thereon to CLIFR. Members of the subcommittee were Saul Gercowsky, Edward Gibson, Brennan Kennedy, Josephine Marks (Chair), Jean-Philippe Morin, and May Zheng. CLIFR would also like to acknowledge the contribution of Simran Bhullar and Guillermo Szpigiel who were instrumental in developing and revising the spreadsheet used to determine the equilibrium risk-free market curve for the base scenario, which is included as appendix A to this educational note supplement and is also being made available electronically to CIA members.

In accordance with the Institute's Policy on Due Process for the Approval of Guidance Material other than Standards of Practice and Research Documents, this educational note supplement has been prepared by CLIFR and has received the approval for distribution from the Practice Council on December 16, 2015.

If you have any questions or comments regarding this educational note supplement, please contact Rebecca Rycroft, Chair of CLIFR, at her CIA Online Directory address, rebecca.rycroft@oliverwyman.com.

PD, RR

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1. Executive Summary

Life insurance practitioners had identified a number of concerns related to the derivation of forward rates and the resulting determination of insurance contract liabilities using the base scenario as specified in the 2014 educational note. As a result of this feedback, the Committee on Life Insurance Financial reporting (CLIFR) investigated approaches that could be used to derive forward rates from the equilibrium risk-free market curve.

The revised approach to deriving forward rates from the equilibrium risk-free market curve was intended to satisfy the following attributes:

- Achieves consistency in the construction of the initial yield curve;
- Produces yield curves that are appropriate from a capital market perspective;
- Uses an extrapolation method that is consistent with methods used in other jurisdictions;
- Uses a process that avoids mathematical inconsistencies;
- Produces reasonable insurance contract liabilities for a range of typical Canadian products;
- Produces insurance contract liabilities with a reasonable level of sensitivity to interest rate shifts; and
- Produces insurance contract liabilities that are consistent with stochastic Canadian Asset Liability Method (CALM) results at CTE(0).

CLIFR concluded that the use of linear interpolation in developing the initial par curve is a satisfactory approach for the development of the initial risk-free interest rate yield curve.

CLIFR also concluded that an extrapolation method that began using the 20-year spot rate, and then extended the spot curve beyond the yield curve horizon by grading the spot rates to the long-term ultimate rate of return (URR)-median by year 80, produced insurance contract liabilities for the base scenario that were aligned with the above attributes. This is further described as the “second approach”. The 2014 educational note was revised accordingly.

2. Background

In 2014, the Actuarial Standards Board (ASB) revised the Standards of Practice for Insurance Contract Valuations: Life and Health (Accident and Sickness) Insurance (Section 2300) with respect to the economic reinvestment assumptions and investment strategies utilized for long-tail liability cash flows under the CALM.

2.1 Construction of the Base Scenario

The specifications for the development of the base scenario, as set out in the 2014 revisions to the Standards of Practice, require the actuary to derive the forward interest rates implied by the equilibrium risk-free market curve at the balance sheet date. Paragraph 2330.09.1 specifies the following:

In the base scenario,

risk-free interest rates effective after the balance sheet date would be equal to the forward interest rates implied by the equilibrium risk-free market curve at that date, for the first 20 years after the balance sheet date, ...

Guidance was provided in the 2014 educational note [Investment Assumptions Used in the Valuation of Life and Health Insurance Contract Liabilities](#) on the approach to be taken to construct the equilibrium risk-free market curve and to derive the forward rates to be applied to cash flow reinvestments occurring in the first 20 years after the balance sheet date.

During the first year of implementation of the revised Standards of Practice, CLIFR and the ASB received feedback from life insurance practitioners identifying some challenges in the application of the base scenario when the market curve was extended. As a result of this feedback, CLIFR formed a subcommittee in 2015 to investigate other approaches that could be used in constructing the equilibrium risk-free market curve and in the determination of the rates to be used in the base scenario, and to make recommendations for revisions to the guidance, as appropriate.

As a result of these revisions to the guidance, the following approach is taken to construct the base scenario (the steps in brackets refer to those in appendix A):

1. Construct the initial risk-free interest rate par yield curve, based on the balance sheet date observable curve (steps 1 and 2)
2. Determine the rates for the first 20 years after the balance sheet date
 - a. Calculate the implied spot rates from the market data (step 3)
 - b. Extrapolate the spot rate beyond year 20 by grading to the URR (step 4)
 - c. Derive the implied forward par yields (steps 5 and 6)

The resulting par yields for the first 20 years are highlighted in yellow in appendix A.

3. Build the base scenario according to the Standards of Practice using the equilibrium risk-free market curve

This educational note supplement focuses on the construction of the initial risk-free interest rate par yield curve (item #1 above) and the derivation of the forward rates used in the determination of the rates for the first 20 years after the balance sheet date (item #2 above).

The purpose of this educational note supplement is to document the process and rationale that was followed in developing these recommended revisions and that supported the ultimate educational guidance provided by CLIFR. This educational note supplement also provides commentary on the challenges involved in developing an appropriate methodology for determining forward rates from the market curve (as used in the base scenario) and may be used as a source of guidance for developing an equilibrium risk-free market curve and deriving forward rates for economies other than the Canadian economy.

2.2 Feedback Received from Practitioners

Life insurance practitioners had identified a number of concerns related to the derivation of forward rates from the initial risk-free interest rate yield curve and the resulting determination of insurance contract liabilities under the base scenario as specified in the 2014 educational note. These included the following:

- Variability of implementation between practitioners due to inconsistencies in both the initial development of the risk-free interest rate yield curve and its extrapolation.
- Base scenario insurance contract liabilities which were higher than any of those of the prescribed scenarios, so that the base scenario would be selected as the valuation basis, resulting in a provision for adverse deviations (PfAD) of zero for interest rate risk.
- Unreasonable sensitivity in the insurance contract liabilities determined under the base scenario due to changes in interest rates, also resulting in PfADs that may be inappropriately volatile over time.
- Base scenario insurance contract liabilities which differed from results obtained under stochastic CALM at CTE(0). Although the Standards of Practice do not specify that the base scenario results need to correspond to CTE(0) results, the base scenario may be viewed as being intended to be broadly consistent with that achieved under a CTE(0) measure.

3. Mandate and Preferred Attributes

3.1 Mandate

An extrapolation of the risk-free interest rate yield curve is required in the base scenario to derive the implied forward rates to be used as the reinvestment rates for cash flows occurring in the first 20 years after the balance sheet date. It was expected that the development of the equilibrium risk-free market curve would be consistent with the reference point data (i.e., with current market conditions) for at least the first 20 years. Beyond the 20-year point on the Canadian yield curve however, interest rates are influenced by supply and demand considerations and the yield curve may be inverted as a result. Use of data beyond the 20-year point to derive the curve may not properly reflect the fundamental interest rate risk due to term structure alone.

The mandate of the CLIFR subcommittee was to examine the design and construction of the risk-free interest rate yield curve at the balance sheet date and to provide guidance to actuaries on the extension of the yield curve beyond the reference points traditionally available. The risk-free interest rate yield curve is based on current capital market conditions, with reference interest rates derived from benchmark bonds where the market is deep and liquid. The work of the subcommittee was focused on conditions prevalent in the Canadian market, although a similar approach could be used to extend yield curves in other jurisdictions.

3.2 Preferred Attributes

In reviewing the feedback received from life insurance practitioners, the subcommittee identified preferred attributes that would be taken into account in constructing and determining the equilibrium risk-free market curve used in the base scenario.

One issue identified was that slight deviations in the initial risk-free interest rate yield curve could lead to more material inconsistencies when using the initial curve as a starting point for extrapolation. These inconsistencies occur because slight differences in the par rates of the initial risk-free interest rate yield curve can give rise to more material deviations when these are used to determine spot rates or forward rates at longer terms. As a result, one of the preferred attributes would be achieving consistency in the construction of the initial yield curve.

The subcommittee also considered additional attributes when using the equilibrium risk-free market curve to derive forward rates. These attributes are the following:

- An extrapolation method that produces yield curves that are appropriate from a capital market perspective (i.e., consistent with known market characteristics);
- An extrapolation method that is consistent with methods used in other jurisdictions for extending interest rates beyond current market data;
- An extrapolation method that avoids mathematical inconsistencies (i.e., a method that does not give rise to mathematical contradictions such as discontinuities in interest rates or spurious interest rates);
- An extrapolation method that produces reasonable insurance contract liabilities for a range of typical Canadian products (i.e., not too liberal and not too conservative);
- An extrapolation method that produces insurance contract liabilities with a reasonable level of sensitivity to interest rate shifts; and
- An extrapolation method that produces insurance contract liabilities that are consistent with stochastic CALM results at CTE(0).

4. Methodology and Recommendations

4.1 Construction of the Initial Risk-Free Interest Rate Par Yield Curve

A common method used in the construction of a risk-free interest rate yield curve is to start with the discrete benchmark rates available in the market and then use linear interpolation to build an entire curve. Sources of data for the key par curve points include both Bloomberg (which is available by subscription only) and the Bank of Canada reference rates (which are publicly available on the government of Canada website). The reference rates from these two sources have historically been very consistent.

The subcommittee considered whether a method using non-linear interpolation would be materially different than linear interpolation.

Appendix B presents two methods for interpolating an interest rate yield curve, linear interpolation and non-linear interpolation. Appendix B shows the development of the par rates for the interest rate yield curve, the derivation of spot rates and of smoothed forward rates, and then the development of forward curves using each of these two interpolation methods. Appendix B also presents a sample initial risk-free interest rate yield curve for each method.

Recommendation

Linear interpolation is a satisfactory approach to be used in developing the basic par curve.

The use of a non-linear interpolation method does result in a more continuous interest rate yield curve, and the resulting interest rate yield curve exhibits a smoother transition before and after the points on the curve that are defined by the benchmark bonds. However, the subcommittee concluded that the additional value derived from a non-linear interpolation was insufficient to warrant the additional complexity.

The subcommittee noted that forward rates tend to be more sensitive than spot rates to minor differences in the initial values of the par interest rate yield curve. Therefore, the specification of a single method to construct the initial interest rate yield curve would probably be of greater importance in achieving consistency of practice if forward rates were being used to extrapolate the risk-free interest rate yield curve.

4.2 Determination of the Rates for the First 20 Years after the Balance Sheet Date

In the 2014 educational note, the guidance for determining reinvestment rates for the first 20 years after the balance sheet date had used the 20-year spot rate as the starting point for the extrapolation. The 20-year spot rate was used as the one-year implied forward rate from which to begin grading the one-year forward rates to the long-term URR-median over 20 years using a uniform transition.

The methodology used in developing this recommended approach had focused on extending the one-year forward rates beyond year 20 on the interest rate yield curve, with grading commencing at the 20th year so that the one-year forward rates would approach the long-term ultimate reinvestment rate (URR)-median by year 40, and then

constructing the interest rate yield curve for the base scenario from the resulting extrapolation of forward rates. The resulting interest rate yield curve would then be used to set risk-free reinvestment rates for cash flows occurring in the first 20 years after the balance sheet date.

In 2014, alternatives in methodology for extrapolating the interest rate yield curve that were considered included the following:

- Starting the extrapolation with the 19-year spot rate rather than the 20-year spot rate to avoid discontinuities that occur in that area of the curve;
- Transitioning to the long-term URR-median by year 60, with an inflection point at year 40, whereby 70 percent of the transition would occur by year 40 (i.e., using a 70/30 formula comparable to that used in the base scenario definition in the Standards of Practice);
- Using the short-term URR-median rather than the long-term URR-median as the target rate in year 40; and
- Greater calibration of the approach with results determined stochastically so that the mean URRs achieved under the two methods would be more closely comparable.

The subcommittee initially focused its 2015 review on alternatives that were modified from the current guidance as follows:

- Use of the one-year forward rate in year 20 (instead of the spot rate in year 20) as the starting point for extrapolation.
- Use of a shorter transition period for the one-year forward rates to reach the URR (i.e., reaching the URR earlier than year 40).
- Use of spot rates to extrapolate the curve rather than forward rates.

It was noted that it was unlikely that a single mathematical derivation could be devised to meet all the preferred attributes. In practice, the subcommittee was limited to testing various mathematical derivations and then assessing how they performed relative to the preferred attributes. While it was possible to identify a priori some of the mathematical features that might be beneficial, it was not possible to specify in advance what type of mathematical approach would achieve the best outcomes.

Alternative Approaches for Extrapolation

As a result of the review, two approaches emerged as the preferred candidates for consideration.

The first approach differed from the 2014 guidance in two respects.

1. It graded the forward rates linearly from the 20-year forward rate rather than from the 20-year spot rate.
2. It graded to the long-term URR-median by year 30 (i.e., the forward rates

converged with the long-term URR-median at year 30).

The second approach (which was selected) also differed from the 2014 guidance in two respects.

1. It started the extrapolation using the 20-year spot rate, but focused on projecting future spot rates and building smoother spot curves rather than building forward curves beyond the yield curve horizon.
2. It graded to the long-term URR-median by year 80 (i.e., the spot rates converged with the long-term URR-median at year 80).

In reviewing the two proposed approaches, the subcommittee developed three sample portfolios (insurance, guaranteed investment certificates (GICs), and annuities) for which each approach was tested. Results of these tests are provided in appendix C. The detailed results use year-end interest rate data from December 31, 2012, December 31, 2013, and December 31, 2014 for the analysis. For each approach, the subcommittee considered its impact on resulting insurance contract liabilities, its comparison with CTE(0) results using stochastic methods, and its sensitivity to interest rate shifts.

Assessment of the Extrapolation Approaches

Both of these extrapolation approaches satisfy many of the preferred attributes previously described in section 3.3. Both show less sensitivity to interest rate shifts than the prescribed scenarios, thus reducing the volatility of the PfADs. Both also give rise to insurance contract liabilities and interest rate sensitivities that are more closely comparable to those obtained under CTE(0) using a stochastic approach to CALM.

A disadvantage of the first approach is that it uses the 20-year forward rate as the starting point, which can result in variability in results. If this approach were selected, it would be advisable to ensure that all actuaries used the same initial risk-free interest rate yield curve, given that small deviations in the initial par curve can give rise to more material differences in the forward curve. Providing guidance on the development of the initial interest rate yield curve would help to address this concern.

Another disadvantage of the first approach is that such quick convergence of the forward rates to the long-term URR-median (i.e., by year 30) may be inappropriate. Market instruments do exist at the 30-year point, and even at the 40-year or 50-year point, and market pricing on these instruments may be inconsistent with the yield curve being determined by actuaries. However, these longer-term instruments are relatively few and may not constitute a liquid market due to supply/demand imbalances.

A technical objection to the first approach could also be raised in that its construction is inconsistent with the construction of the basic par yield curve used for the base scenario, which uses linear interpolation between years 20 and 40 and between years 40 and 60 with an inflection point at year 40. This concern may be countered by noting that extending forward rates is an intrinsically different exercise from extending par rates.

The primary disadvantage with the second approach is that it uses spot rate projections,

which is inconsistent with actuarial guidance emanating from Europe, where forward rates are being used for projections. Spot rates are more readily available in the market than forward rates, so it is easier to observe when spot rates are not market consistent than it is for forward rates. Once again, small deviations in the initial risk-free interest rate yield curve (par rates) may give rise to differences in the spot rates in the extrapolated yield curve, although this would not be as severe as for the extrapolation of a forward curve.

Extrapolated interest rates using the second approach were more aligned with the existing market curve for the periods tested, which may make this approach easier to justify in other jurisdictions (e.g., the U.S.) where there is a deeper, more liquid market at the long end of the curve. Conversely, using forward rates for the extrapolation, as is done in the first approach, makes it less obvious when the resulting interest rate yield curve is not consistent with market data at the long end of the curve.

Recommendation

The second approach to extrapolation of the interest rate yield curve was selected by CLIFR as being the preferred approach and the educational note [Investment Assumptions Used in the Valuation of Life and Health Insurance Contract Liabilities](#) has been revised accordingly.

After reviewing the results presented in appendix C, the overall conclusion was that each approach had some desirable features, and that neither was clearly superior to the other. Both approaches produced reasonable insurance contract liabilities for a selection of product types and both produced a reduced level of interest rate sensitivity when compared to the previous guidance.

Appendix A: Derivation of Forward Par Curve

Par Yields, Spot Rates, Forward Spots, and Forward Par Yields

Define a spot rate z_n as the yield on a zero-coupon bond maturing in n periods.
 Given an observed par yield curve p_n , the spot curve z_n is derived recursively:

Formula 1:

$$z_n = \left[\frac{(1 + p_n)}{(1 - p_n \sum_{k=1}^{n-1} (1 + z_k)^{-k})} \right]^{1/n} - 1$$

Define a forward spot $F(n,m)$ as the z_n on a zero purchased m periods from now.
 Given a spot curve z_n , the implied Forward spots $F(n,m)$ are derived via the relation:

Formula 2:

$$F(n,m) = \left[\frac{(1 + z_{m+n})^{m+n}}{(1 + z_m)^m} \right]^{1/n} - 1$$

The corresponding forward par yields $FP(n,m)$ are then derived via the formula

Formula 3:

$$FP(n,m) = \frac{1 - (1 + F(n,m))^{-n}}{\sum_{k=1}^n (1 + F(k,m))^{-k}}$$

A sample process is outlined below; sample 1- and 20-year rates are illustrated at right.

Construction of Implied Forward Par Yield Curves - Steps

- Step 1: Obtain current par yield curve from an appropriate source
- Step 2: Interpolate the par yield curve where yields are not directly available.
- Step 3: Derive the equivalent spot rate curve using Formula 1.
- Step 4: Beyond year 20, calculate an adjusted spot rate by using a uniform transition from the spot rate in year 20 to the median long-term ultimate risk-free reinvestment rate - median ($_{long}URR_{median}$) in year 80.
- Step 5: Derive the implied forward spots using Formula 2.
- Step 6: Determine the equivalent implied forward par yields using Formula 3.

Notes

1. Spot rate begins to grade to Median URR = **5.30%**
2. For each term, the time-0 forward spot equals the observed spot for that term.
3. For each term, only the first 20 forwards are used in the Base Scenario.

Illustration: 1- and 20-yr Terms

all rates annualized

	Observed Rates by Term				Implied Forwards by Year			
	Par	a=	Spots	Adj Spot	Spots		Par Yields	
					1-yr	20-yr	1-yr	20-yr
0					1.000%	2.399%	1.000%	2.300%
1	1.000%	0.000	1.000%	1.000%	1.000%	2.521%	1.000%	2.422%
2	1.000%	0.990	1.000%	1.000%	1.304%	2.647%	1.304%	2.552%
3	1.100%	1.970	1.101%	1.101%	1.508%	2.763%	1.508%	2.671%
4	1.200%	2.938	1.203%	1.203%	1.715%	2.873%	1.715%	2.784%
5	1.300%	3.891	1.305%	1.305%	1.925%	2.978%	1.925%	2.890%
6	1.400%	4.829	1.408%	1.408%	2.138%	3.077%	2.138%	2.990%
7	1.500%	5.748	1.512%	1.512%	2.356%	3.170%	2.356%	3.083%
8	1.600%	6.648	1.617%	1.617%	2.578%	3.258%	2.578%	3.167%
9	1.700%	7.528	1.724%	1.724%	2.805%	3.338%	2.805%	3.243%
10	1.800%	8.385	1.831%	1.831%	2.416%	3.413%	2.416%	3.309%
11	1.850%	9.219	1.884%	1.884%	2.532%	3.512%	2.532%	3.407%
12	1.900%	10.034	1.938%	1.938%	2.650%	3.610%	2.650%	3.504%
13	1.950%	10.828	1.993%	1.993%	2.770%	3.707%	2.770%	3.600%
14	2.000%	11.602	2.048%	2.048%	2.894%	3.802%	2.894%	3.694%
15	2.050%	12.355	2.104%	2.104%	3.021%	3.897%	3.021%	3.787%
16	2.100%	13.086	2.161%	2.161%	3.152%	3.990%	3.152%	3.877%
17	2.150%	13.797	2.219%	2.219%	3.286%	4.081%	3.286%	3.965%
18	2.200%	14.485	2.278%	2.278%	3.425%	4.170%	3.425%	4.050%
19	2.250%	15.152	2.338%	2.338%	3.569%	4.257%	3.569%	4.131%
20	2.300%	15.797	2.399%	2.399%	3.419%	4.342%	3.419%	4.208%
21	2.240%	16.419	2.315%	2.448%	3.517%	4.400%	3.517%	4.302%
22	2.180%	17.037	2.233%	2.496%	3.614%	4.537%	3.614%	4.397%
23	2.120%	17.652	2.152%	2.545%	3.711%	4.634%	3.711%	4.491%
24	2.060%	18.265	2.073%	2.593%	3.808%	4.732%	3.808%	4.586%
25	2.000%	18.876	1.995%	2.641%	3.906%	4.829%	3.906%	4.681%
26	2.000%	19.487	1.995%	2.690%	4.003%	4.927%	4.003%	4.775%
27	2.000%	20.085	1.995%	2.738%	4.100%	5.024%	4.100%	4.870%
28	2.000%	20.672	1.996%	2.786%	4.197%	5.122%	4.197%	4.965%
29	2.000%	21.247	1.996%	2.835%	4.295%	5.219%	4.295%	5.059%
30	2.000%	21.810	1.996%	2.883%	4.392%	5.317%	4.392%	5.154%
31	2.000%	22.363	1.996%	2.931%	4.490%	5.415%	4.490%	5.249%
32	2.000%	22.905	1.996%	2.980%	4.587%	5.512%	4.587%	5.343%
33	2.000%	23.436	1.996%	3.028%	4.684%	5.610%	4.684%	5.438%
34	2.000%	23.957	1.996%	3.076%	4.782%	5.708%	4.782%	5.533%
35	2.000%	24.468	1.996%	3.125%	4.879%	5.805%	4.879%	5.628%
36	2.000%	24.968	1.997%	3.173%	4.977%	5.903%	4.977%	5.723%
37	2.000%	25.459	1.997%	3.221%	5.074%	6.001%	5.074%	5.818%
38	2.000%	25.940	1.997%	3.270%	5.172%	6.098%	5.172%	5.913%
39	2.000%	26.412	1.997%	3.318%	5.269%	6.196%	5.269%	6.008%
40	2.000%	26.875	1.997%	3.366%	5.367%	6.294%	5.367%	6.103%
41	2.000%	27.328	1.997%	3.415%	5.465%	6.392%	5.465%	6.198%
42	2.000%	27.773	1.997%	3.463%	5.562%	6.490%	5.562%	6.293%
43	2.000%	28.209	1.997%	3.511%	5.660%	6.588%	5.660%	6.388%
44	2.000%	28.636	1.997%	3.560%	5.758%	6.685%	5.758%	6.483%
45	2.000%	29.055	1.997%	3.608%				

Appendix B: Constructing the initial risk-free interest rate par yield curve

Appendix B presents two methods for interpolating the interest rate par yield curve. The first method is the more traditional approach, involving linear interpolation. The second method is an alternative approach involving non-linear interpolation.

Initial Yield Curve Construction – Method 1 – Linear Interpolation

Step 1 – 10 point par curve

To obtain the 10 point par curve, use reference indices from Bloomberg or the Bank of Canada. For the purposes of this derivation, the following sources were used. These are illustrative and are not intended to be prescriptive. For this illustration, the rates being used are the daily rates. The Bloomberg References use the same benchmark bonds as are used by the Bank of Canada, although yields may vary slightly due to timing differences in recording the rates.

Point	Bloomberg Reference	Bank of Canada Reference
3-month T-bill	GCAN3M	V122531
6-month T-bill	GCAN6M	V122532
12-month T-bill	GCAN12M	V122533
2-year bond	GCAN2YR	V122538
3-year bond	GCAN3YR	V122539
4-year bond	GCAN4YR	GCAN4YR (Bloomberg)
5-year bond	GCAN5YR	V122540
7-year bond	GCAN7YR	V122542
10-year bond	GCAN10YR	V122543
20-year bond	GCAN20YR	GCAN20YR (Bloomberg)
Long-term bond	GCAN30YR	V122544

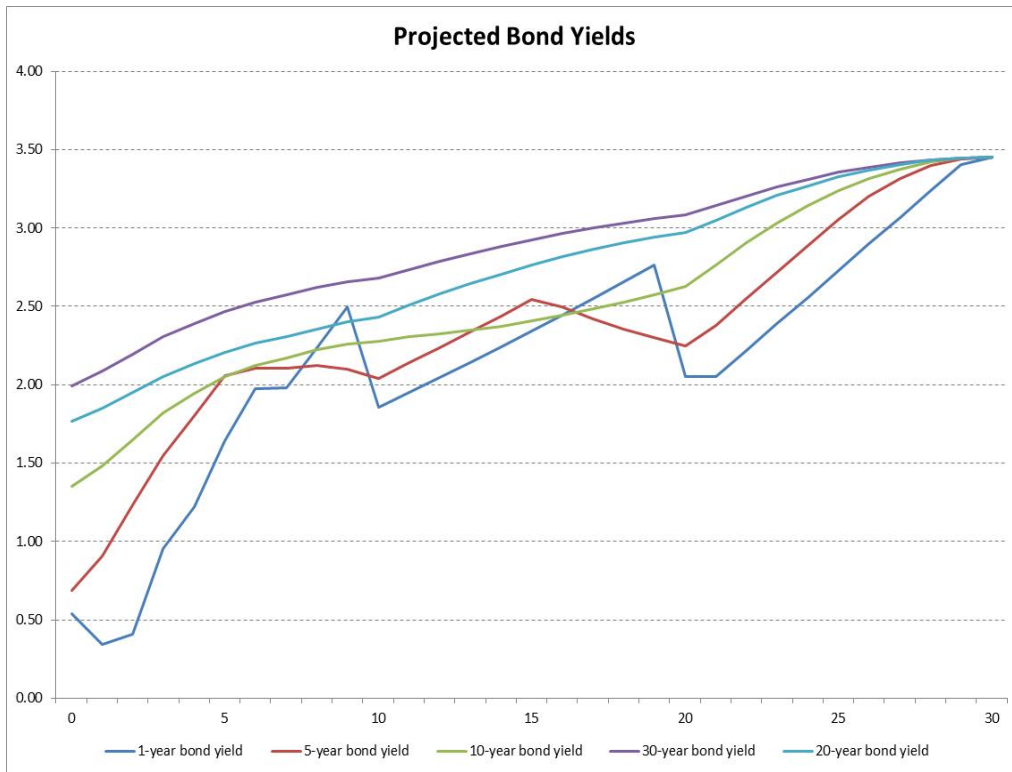
Step 2 – Interpolation to a 61 point par curve

To develop the starting yield curve with 61 points (i.e., a three month rate plus rates for semi-annual intervals over 30 years), linear interpolation is used between each point on the 10 point yield curve.

Steps 3-7 – Calculate spot rates.

Calculate smoothed forward discount rates. Derive forward monthly rates. Derive forward par curves.

Forward Rate Examples – Linear Interpolation Method



Initial Yield Curve Construction – Method 2 – Non-Linear Interpolation

Step 1 – 10 point par curve

To obtain the 10 point par curve, use the following reference indices from Bloomberg or the Bank of Canada.

Point	Bloomberg Reference	Bank of Canada Reference
3-month T-bill	GCAN3M	V122531
6-month T-bill	GCAN6M	V122532
12-month T-bill	GCAN12M	V122533
2-year bond	GCAN2YR	V122538
3-year bond	GCAN3YR	V122539
4-year bond	GCAN4YR	GCAN4YR (Bloomberg)
5-year bond	GCAN5YR	V122540
7-year bond	GCAN7YR	V122542
10-year bond	GCAN10YR	V122543
20-year bond	GCAN20YR	GCAN20YR (Bloomberg)
Long-term bond	GCAN30YR	V122544

Step 2 – Interpolation to a 61 point curve

The approach taken for the non-linear interpolation aims to smooth the curve across the known yield curve points by reflecting the slope between the known points in the interpolation calculations.

At a high level, an example of the method utilized to interpolate between the five-year and seven-year points is shown below.

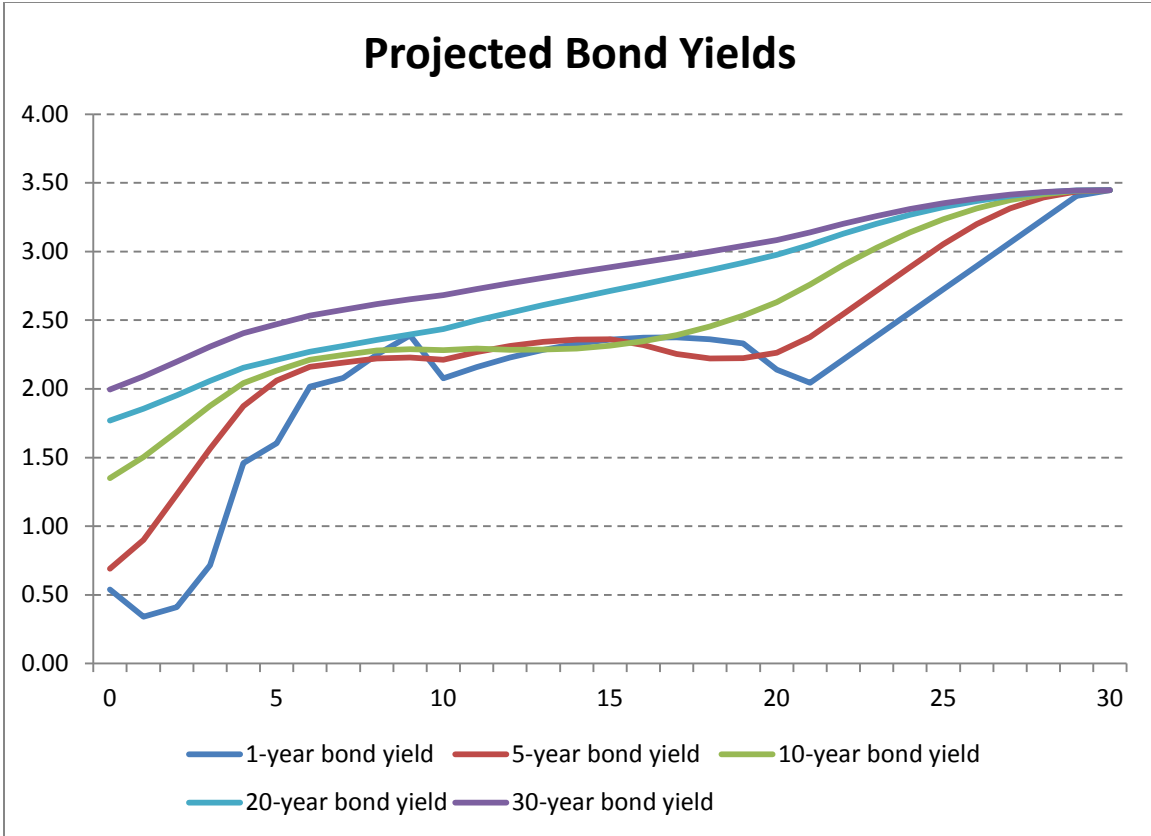
Par Yield Curve Key Point	Key Rate	Semi-Annual Slope between Key Rates	Linearly Interpolate Slope	Gross-up Interpolated Amount	Non-Linearly Interpolated Par Yield Curve
Formula	$P(x)$	$S(x) = [P(x) - P(x-2)]/4$	$L(x+0.5) = [S(7) - S(5)]/4 + L(x)$	$G(x) = L(x) * (P(7) - P(5)) / [L(5.5) + L(6) + L(6.5) + L(7)]$	$P(x+0.5) = P(x) + G(x+0.5)$
3.0 year	0.430%				
3.5 year					
4.0 year					
4.5 year					
5.0 year	0.690%	0.065%	0.065%		0.690%
5.5 year			0.068%	0.073%	0.763%
6.0 year			0.071%	0.076%	0.838%
6.5 year			0.074%	0.079%	0.918%
7.0 year	1.000%	0.078%	0.078%	0.082%	1.000%

Note that this methodology was stress tested under a number of various yield curve shapes to ensure that the results remained appropriate.

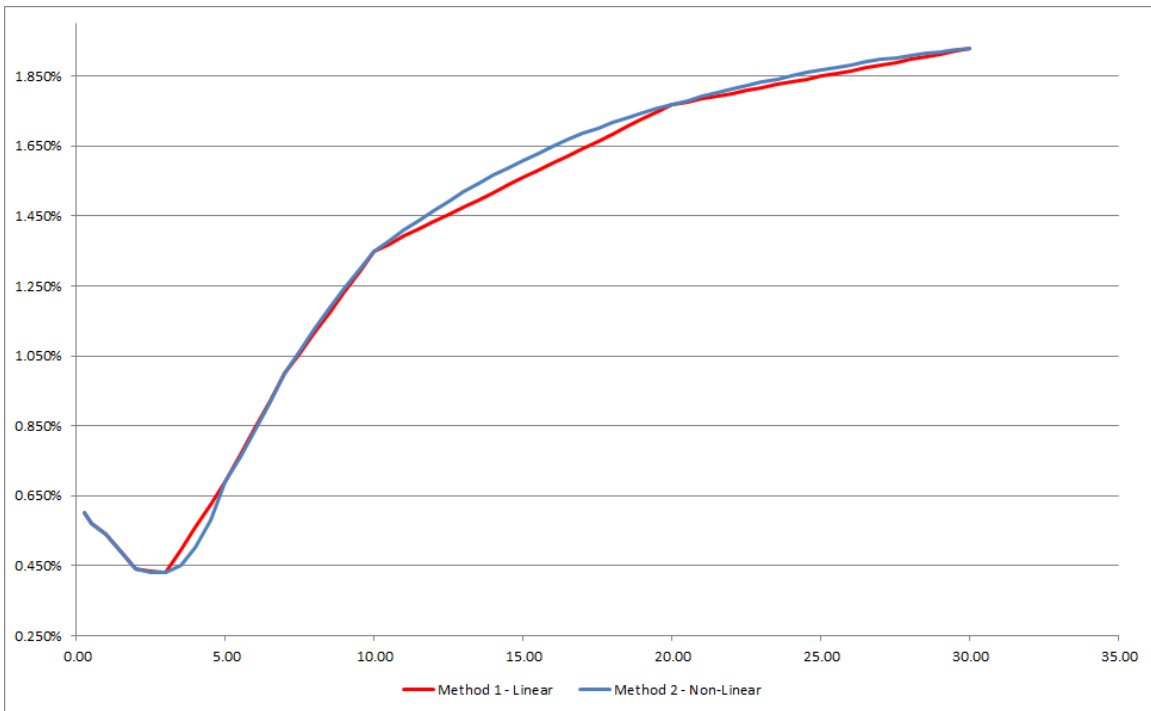
Steps 3-7– Calculate spot rates.

Calculate smoothed forward discount rates. Derive forward monthly rates. Derive forward par curves.

Forward Rate Examples – Non Linear Interpolation Method



Comparison of 61 point starting curves – Linear vs Non Linear Approach



Appendix C: Testing of Alternative Approaches for Determining the Rates for the First 20 Years after the Balance Sheet Date

Table 1

Insurance portfolio - starting yield curve analysis

Market value of the starting asset is the same for each test. It is a limitation of the model used in this analysis.

Curve tested: Approach tested:
 A) December 31st 2012 1) Deterministic
 B) December 31st 2013 2) Stochastic
 C) December 31st 2014

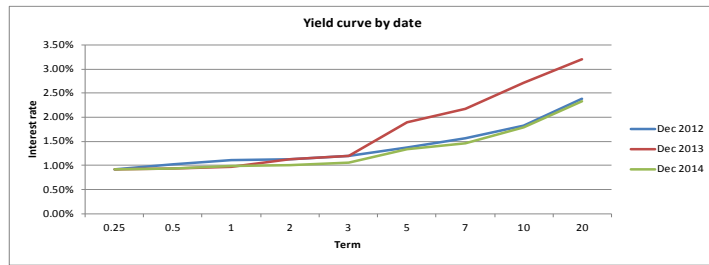


Table 1 shows the results obtained for the current approach, as well as the two alternative approaches described in section 4.2. The table shows the interest rate yield curve in effect at year-end 2012, 2013 and 2014, the resulting interest rate sensitivities for each approach and a comparison to stochastic results for a traditional life insurance product.

A) December 31st 2012

Current Approach	Base					+1%					-1%					Stochastic							
	Base	+1%	Impact	-1%	Impact	Base	+1%	Impact	-1%	Impact	Base	+1%	Impact	-1%	Impact		CTE	Base	+1%	Impact			
SBA	822.44	726.96	-95.48	915.75	93.31	SBA	739.91	661.44	-78.47	813.97	74.06	SBA	782.36	702.87	-79.49	858.19	75.83	0	777.86	724.59	-53.27		
SC1	950.09	899.34	-50.75	999.97	49.88	SC1	950.09	899.34	-50.75	999.97	49.88	SC1	950.09	899.34	-50.75	999.97	49.88	10	806.47	756.34	-50.13		
SC2	467.68	411.32	-56.37	520.10	52.41	SC2	467.68	411.32	-56.37	520.10	52.41	SC2	467.68	411.32	-56.37	520.10	52.41	20	828.62	781.62	-47.00		
SC3	638.21	618.99	-19.23	656.14	17.93	SC3	638.21	618.99	-19.23	656.14	17.93	SC3	638.21	618.99	-19.23	656.14	17.93	30	848.86	804.66	-44.21		
SC4	438.34	402.92	-35.42	470.41	32.07	SC4	438.34	402.92	-35.42	470.41	32.07	SC4	438.34	402.92	-35.42	470.41	32.07	40	865.04	826.87	-41.17		
SC5	638.21	618.99	-19.23	656.14	17.93	SC5	638.21	618.99	-19.23	656.14	17.93	SC5	638.21	618.99	-19.23	656.14	17.93	50	887.13	849.06	-38.08		
SC6	438.34	402.92	-35.42	470.41	32.07	SC6	438.34	402.92	-35.42	470.41	32.07	SC6	438.34	402.92	-35.42	470.41	32.07	60	907.14	872.13	-35.01		
SC7	841.64	785.24	-56.39	896.92	55.28	SC7	841.64	785.24	-56.39	896.92	55.28	SC7	841.64	785.24	-56.39	896.92	55.28	70	928.26	896.50	-31.76		
SC8	738.11	670.42	-67.69	803.48	65.37	SC8	738.11	670.42	-67.69	803.48	65.37	SC8	738.11	670.42	-67.69	803.48	65.37	80	952.20	924.59	-27.61		
C3 margin	127.65	172.38	44.73	84.21	-43.43	C3 margin	210.18	237.91	27.73	185.99	-24.19	C3 margin	167.73	196.47	28.74	141.78	-25.95	90	983.50	961.74	-21.76		
																				100	1090.85	1083.20	-7.65

B) December 31st 2013

Current Approach	Base					+1%					-1%					Stochastic							
	Base	+1%	Impact	-1%	Impact	Base	+1%	Impact	-1%	Impact	Base	+1%	Impact	-1%	Impact		CTE	Base	+1%	Impact			
SBA	743.11	644.20	-98.91	838.90	95.78	SBA	680.06	597.16	-82.90	757.77	77.71	SBA	716.37	632.60	-83.77	795.65	79.27	0	733.26	679.10	-54.16		
SC1	908.69	856.66	-52.03	959.33	50.64	SC1	908.69	856.66	-52.03	959.33	50.64	SC1	908.69	856.66	-52.03	959.33	50.64	10	764.48	713.12	-51.36		
SC2	421.98	361.64	-60.34	477.57	55.59	SC2	421.98	361.64	-60.34	477.57	55.59	SC2	421.98	361.64	-60.34	477.57	55.59	20	789.25	740.55	-48.70		
SC3	622.65	602.20	-20.45	641.61	18.96	SC3	622.65	602.20	-20.45	641.61	18.96	SC3	622.65	602.20	-20.45	641.61	18.96	30	811.82	765.72	-46.10		
SC4	409.66	371.04	-38.62	444.41	34.75	SC4	409.66	371.04	-38.62	444.41	34.75	SC4	409.66	371.04	-38.62	444.41	34.75	40	833.54	790.19	-43.34		
SC5	622.65	602.20	-20.45	641.61	18.96	SC5	622.65	602.20	-20.45	641.61	18.96	SC5	622.65	602.20	-20.45	641.61	18.96	50	855.20	814.81	-40.39		
SC6	409.66	371.04	-38.62	444.41	34.75	SC6	409.66	371.04	-38.62	444.41	34.75	SC6	409.66	371.04	-38.62	444.41	34.75	60	877.78	840.31	-37.47		
SC7	795.71	738.06	-57.65	851.83	56.11	SC7	795.71	738.06	-57.65	851.83	56.11	SC7	795.71	738.06	-57.65	851.83	56.11	70	901.56	867.58	-33.99		
SC8	683.04	612.84	-70.20	750.23	67.19	SC8	683.04	612.84	-70.20	750.23	67.19	SC8	683.04	612.84	-70.20	750.23	67.19	80	928.97	899.07	-29.90		
C3 margin	165.57	212.46	46.88	120.43	-45.14	C3 margin	228.63	259.50	30.87	201.56	-27.07	C3 margin	192.32	224.06	31.75	163.68	-28.64	90	965.16	941.19	-23.97		
																				100	1083.68	1071.81	-11.87

C) December 31st 2014

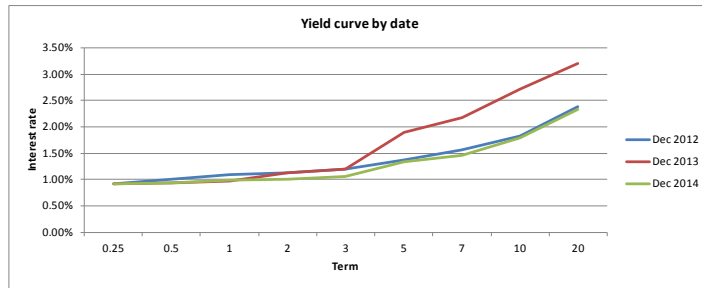
Current Approach	Base					+1%					-1%					Stochastic							
	Base	+1%	Impact	-1%	Impact	Base	+1%	Impact	-1%	Impact	Base	+1%	Impact	-1%	Impact		CTE	Base	+1%	Impact			
SBA	827.81	732.38	-95.43	921.01	93.20	SBA	745.22	667.12	-78.10	819.15	73.92	SBA	786.79	707.67	-79.12	862.47	75.68	0	775.96	721.86	-54.10		
SC1	952.86	901.96	-50.90	1002.48	49.61	SC1	952.86	901.96	-50.90	1002.48	49.61	SC1	952.86	901.96	-50.90	1002.48	49.61	10	804.74	753.87	-50.87		
SC2	470.61	414.44	-56.16	522.87	52.26	SC2	470.61	414.44	-56.16	522.87	52.26	SC2	470.61	414.44	-56.16	522.87	52.26	20	827.01	779.32	-47.68		
SC3	639.24	620.09	-19.14	657.10	17.86	SC3	639.24	620.09	-19.14	657.10	17.86	SC3	639.24	620.09	-19.14	657.10	17.86	30	847.32	802.48	-44.84		
SC4	440.12	404.91	-35.22	472.04	31.92	SC4	440.12	404.91	-35.22	472.04	31.92	SC4	440.12	404.91	-35.22	472.04	31.92	40	866.57	824.82	-41.75		
SC5	639.24	620.09	-19.14	657.10	17.86	SC5	639.24	620.09	-19.14	657.10	17.86	SC5	639.24	620.09	-19.14	657.10	17.86	50	885.73	847.11	-38.62		
SC6	440.12	404.91	-35.22	472.04	31.92	SC6	440.12	404.91	-35.22	472.04	31.92	SC6	440.12	404.91	-35.22	472.04	31.92	60	905.80	870.27	-35.53		
SC7	844.70	788.38	-56.32	899.82	55.12	SC7	844.70	788.38	-56.32	899.82	55.12	SC7	844.70	788.38	-56.32	899.82	55.12	70	926.96	894.74	-32.22		
SC8	741.82	674.09	-67.73	807.01	65.19	SC8	741.82	674.09	-67.73	807.01	65.19	SC8	741.82	674.09	-67.73	807.01	65.19	80	950.97	923.93	-28.04		
C3 margin	125.05	169.59	44.53	81.47	-43.59	C3 margin	207.64	234.84	27.20	183.33	-24.31	C3 margin	166.07	194.30	28.22	140.00	-26.07	90	982.37	960.25	-22.12		
																				100	1089.76	1081.89	-7.87

Insured Annuity portfolio - starting yield curve analysis

Table 3

Market value of the starting asset is the same for each test. It is a limitation of the model used in this analysis.

Curve tested: A) December 31st 2012 B) December 31st 2013 C) December 31st 2014
 Approach tested: 1) Deterministic 2) Stochastic



A) December 31st 2012

Current Approach					First Alternative Approach					Second Alternative Approach							
	Base	+1%	Impact	-1%	Impact		Base	+1%	Impact	-1%	Impact		Base	+1%	Impact	-1%	Impact
SBA	962.72	956.30	-6.42	970.82	8.10	SBA	957.40	952.42	-4.98	963.42	6.03	SBA	959.84	954.85	-4.99	965.93	6.09
SC1	973.32	969.85	-3.48	977.20	3.88	SC1	973.32	969.85	-3.48	977.20	3.88	SC1	973.32	969.85	-3.48	977.20	3.88
SC2	940.12	939.47	-0.65	940.92	0.80	SC2	940.12	939.47	-0.65	940.92	0.80	SC2	940.12	939.47	-0.65	940.92	0.80
SC3	946.80	946.75	-0.05	946.84	0.05	SC3	946.80	946.75	-0.05	946.84	0.05	SC3	946.80	946.75	-0.05	946.84	0.05
SC4	946.68	946.38	-0.30	947.06	0.37	SC4	946.68	946.38	-0.30	947.06	0.37	SC4	946.68	946.38	-0.30	947.06	0.37
SC5	945.20	945.19	-0.01	945.19	-0.01	SC5	945.20	945.19	-0.01	945.18	-0.01	SC5	945.20	945.19	-0.01	945.18	-0.01
SC6	945.47	945.25	-0.22	945.76	0.29	SC6	945.47	945.25	-0.22	945.76	0.29	SC6	945.47	945.25	-0.22	945.76	0.29
SC7	964.09	960.93	-3.16	967.66	3.58	SC7	964.09	960.93	-3.16	967.66	3.58	SC7	964.09	960.93	-3.16	967.66	3.58
SC8	957.73	954.62	-3.11	961.34	3.61	SC8	957.73	954.62	-3.11	961.34	3.61	SC8	957.73	954.62	-3.11	961.34	3.61
C3 margin	10.61	13.55	2.94	6.38	-4.23	C3 margin	15.92	17.43	1.50	13.77	-2.15	C3 margin	13.48	15.00	1.51	11.27	-2.21

Stochastic			
CTE	Base	+1%	Impact
0	954.84	952.76	-2.08
10	956.60	954.44	-2.16
20	958.04	955.87	-2.17
30	959.38	957.23	-2.15
40	960.71	958.59	-2.11
50	962.05	959.99	-2.06
60	963.46	961.48	-1.98
70	965.00	963.12	-1.88
80	966.77	965.02	-1.75
90	969.10	967.59	-1.52
100	976.29	975.26	-1.03

B) December 31st 2013

Current Approach					First Alternative Approach					Second Alternative Approach							
	Base	+1%	Impact	-1%	Impact		Base	+1%	Impact	-1%	Impact		Base	+1%	Impact	-1%	Impact
SBA	957.25	951.93	-5.31	963.95	6.70	SBA	953.56	949.38	-4.19	958.68	5.12	SBA	955.56	951.30	-4.26	960.78	5.23
SC1	970.21	967.03	-3.18	973.70	3.49	SC1	970.21	967.03	-3.18	973.70	3.49	SC1	970.21	967.03	-3.18	973.70	3.49
SC2	939.57	939.07	-0.50	940.20	0.63	SC2	939.57	939.07	-0.50	940.20	0.63	SC2	939.57	939.07	-0.50	940.20	0.63
SC3	946.83	946.82	-0.01	946.88	0.05	SC3	946.83	946.82	-0.01	946.88	0.05	SC3	946.83	946.82	-0.01	946.88	0.05
SC4	946.62	946.36	-0.26	946.95	0.33	SC4	946.62	946.36	-0.26	946.95	0.33	SC4	946.62	946.36	-0.26	946.95	0.33
SC5	945.21	945.27	0.06	945.20	-0.01	SC5	945.21	945.27	0.06	945.20	-0.01	SC5	945.21	945.27	0.06	945.20	-0.01
SC6	945.32	945.14	-0.18	945.55	0.23	SC6	945.32	945.14	-0.18	945.55	0.23	SC6	945.32	945.14	-0.18	945.55	0.23
SC7	961.19	958.39	-2.80	964.42	3.23	SC7	961.19	958.39	-2.80	964.42	3.23	SC7	961.19	958.39	-2.80	964.42	3.23
SC8	954.92	952.24	-2.69	958.07	3.15	SC8	954.92	952.24	-2.69	958.07	3.15	SC8	954.92	952.24	-2.69	958.07	3.15
C3 margin	12.97	15.10	2.13	9.76	-3.21	C3 margin	16.65	17.65	1.00	15.02	-1.63	C3 margin	14.65	15.73	1.08	12.92	-1.74

Stochastic			
CTE	Base	+1%	Impact
0	953.02	951.23	-1.79
10	954.73	952.84	-1.89
20	956.17	954.23	-1.94
30	957.54	955.58	-1.96
40	958.91	956.95	-1.96
50	960.31	958.37	-1.94
60	961.79	959.90	-1.90
70	963.43	961.61	-1.82
80	965.31	963.61	-1.70
90	967.86	966.34	-1.52
100	975.39	974.68	-0.72

C) December 31st 2014

Current Approach					First Alternative Approach					Second Alternative Approach							
	Base	+1%	Impact	-1%	Impact		Base	+1%	Impact	-1%	Impact		Base	+1%	Impact	-1%	Impact
SBA	963.17	956.63	-6.54	971.29	8.12	SBA	957.79	952.77	-5.02	963.91	6.12	SBA	960.16	955.09	-5.07	966.28	6.12
SC1	973.45	969.96	-3.49	977.32	3.88	SC1	973.45	969.96	-3.49	977.32	3.88	SC1	973.45	969.96	-3.49	977.32	3.88
SC2	940.16	939.51	-0.65	940.95	0.80	SC2	940.16	939.51	-0.65	940.95	0.80	SC2	940.16	939.51	-0.65	940.95	0.80
SC3	946.80	946.76	-0.04	946.86	0.06	SC3	946.80	946.76	-0.04	946.86	0.06	SC3	946.80	946.76	-0.04	946.86	0.06
SC4	946.72	946.41	-0.31	947.09	0.37	SC4	946.72	946.41	-0.31	947.09	0.37	SC4	946.72	946.41	-0.31	947.09	0.37
SC5	945.18	945.20	0.03	945.19	0.02	SC5	945.18	945.20	0.03	945.19	0.02	SC5	945.18	945.20	0.03	945.19	0.02
SC6	945.48	945.28	-0.21	945.78	0.30	SC6	945.48	945.28	-0.21	945.78	0.30	SC6	945.48	945.28	-0.21	945.78	0.30
SC7	964.22	960.98	-3.24	967.81	3.59	SC7	964.22	960.98	-3.24	967.81	3.59	SC7	964.22	960.98	-3.24	967.81	3.59
SC8	957.86	954.72	-3.14	961.46	3.60	SC8	957.86	954.72	-3.14	961.46	3.60	SC8	957.86	954.72	-3.14	961.46	3.60
C3 margin	10.27	13.33	3.05	6.03	-4.24	C3 margin	15.66	17.19	1.54	13.42	-2.24	C3 margin	13.29	14.87	1.58	11.04	-2.25

Stochastic			
CTE	Base	+1%	Impact
0	954.80	952.72	-2.08
10	956.57	954.41	-2.16
20	958.00	955.83	-2.17
30	959.35	957.20	-2.16
40	960.68	958.56	-2.12
50	962.02	959.96	-2.06
60	963.44	961.45	-1.99
70	964.98	963.09	-1.88
80	966.74	964.99	-1.75
90	969.08	967.56	-1.52
100	976.27	975.26	-1.01