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Educational Note

Duration Considerations for P&C Insurers

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Committee on Property and Casualty Insurance Financial Reporting

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Members should be familiar with educational notes. Educational notes describe but do not recommend practice in illustrative situations. They do not constitute standards of practice and are, therefore, not binding. They are, however, intended to illustrate the application (but not necessarily the only application) of the Standards of Practice, so there should be no conflict between them. They are intended to assist actuaries in applying standards of practice in respect of specific matters. Responsibility for the manner of application of standards of practice in specific circumstances remains that of the members.

MEMORANDUM

To: Members in the property and casualty insurance area

From: Pierre Dionne, Chair
Practice Council

Raul Martin, Chair
Committee on Property and Casualty Insurance Financial Reporting

Date: March 7, 2017

Subject: **Educational Note: Duration Considerations for P&C Insurers**

This educational note has been prepared by the Committee on Property and Casualty Insurance Financial Reporting in accordance with the Institute's Policy on Due Process for the Approval of Guidance Material other than Standards of Practice and Research Documents, and received final approval for distribution from the Practice Council on February 28, 2017.

This guidance was published previously in the 2013-2015 Guidance to the Appointed Actuary for Property & Casualty Insurers, and going forward will be available as a stand-alone educational note.

As outlined in subsection 1220 of the Standards of Practice, "The actuary should be familiar with relevant Educational Notes and other designated educational material". That subsection explains further that a "practice that the Educational Notes describe for a situation is not necessarily the only acceptable practice for that situation and is not necessarily accepted actuarial practice for a different situation". As well, "Educational Notes are intended to illustrate the application (but not necessarily the only application) of the standards, so there should be no conflict between them".

Questions or comments regarding this educational note may be directed to Raul Martin at jscp@jscp.com.

PD, RM

Introduction and Scope

The Committee on Property and Casualty Insurance Financial Reporting (PCFRC) of the Canadian Institute of Actuaries (CIA) prepared this educational note to provide guidance to actuaries doing work for property and casualty (P&C) insurers related to duration of the insurer's interest rate sensitive claim liabilities, premium liabilities and assets.

In this document, the term "P&C returns" refers to the uniform returns approved by the Canadian Council of Insurance Regulators. The term "MCT Guideline" refers to the Minimum Capital Test (MCT) Guideline issued by the Office of the Superintendent of Financial Institutions (OSFI) or the version approved for use by provincial regulatory authorities.

Duration has become an increasingly relevant topic for a variety of reasons, including but not limited to the following:

- The MCT Guideline requires the calculation of estimated duration of insurer's interest-rate-sensitive assets, claim liabilities, and premium liabilities for purposes of the interest rate risk margin;
- Duration may be required for the estimation and selection of the margin for investment return rates in applying concepts from the educational note Margins for Adverse Deviations for Property and Casualty Insurance;
- Many insurers are employing the strategy to duration match liabilities to assets to help immunize the impact of relatively small shifts in the market yield curve on surplus; and
- Duration is a consideration in modelling market risk.

Furthermore, there are different interpretations on how duration is to be determined for certain asset classes (e.g., preferred shares).

Duration Defined

Duration is a concept or tool that is used to measure both the average maturity of a series of fixed future cash flows as well as to measure the sensitivity that interest rate changes have on the present value of a series of future cash flows. The calculation of the duration will depend on the duration measure chosen. The three most common types of duration measures are the following:

- *Macaulay duration* is computed as the weighted average of the time to each cash flow payment, using the present value of the future cash flow payment as weights. The Macaulay duration is calculated as follows:

- Macaulay Duration $= \frac{\sum_{t=0}^{n} t \times \text{PVCF}_t}{k \text{ x } \sum_{t=0}^{n} \text{PVCF}_t}$
- Where:

t	=	time to future cash flow payment
yield	=	market value yield to maturity of the cash flows consistent with *k* time period definition
k	=	number of periods, or payments, per year (e.g., k=2 for semi-annual periods)
n	=	number of periods until maturity (i.e., number of years to maturity times *k*)
PVCF_t	=	present value of the cash flow in period *t* discounted at the yield rate or market value of securities

- **Modified duration** measures the sensitivity of the present value of a series of fixed future cash flows to changes in interest rates. It is calculated as the following:
  - Modified Duration $= \frac{\text{Macaulay Duration}}{1+\text{Yield}}$
- **Effective duration** also measures the sensitivity of the present value of a series of fixed future cash flows and will give a similar estimate as the modified duration approach. In addition, the effective duration measures the fair value sensitivity of assets where interest rate changes would change future cash flows, such as in the case of interest rate derivatives, callable bonds, option embedded assets, etc. For example, bonds with embedded options may be called early, and therefore the yield to maturity would change on the bond and so the modified duration formula would no longer be an appropriate measure to use. The effective duration is calculated as the following:
  - Effective Duration $= \frac{\text{Fair value if yields decline} - \text{fair value if yields rise}}{2 \times \text{initial price} \times \text{change in yield in decimal}}$
  - or Effective Duration $= \frac{V_- - V_+}{2 \times V_0 \times \Delta y}$
  - Where:

Δy	=	change in yield in decimal
V_0	=	initial fair value
V_-	=	fair value if yields decline by Δy
V_+	=	fair value if yields increase by Δy

It is important to note that for the purpose of the MCT, the Macaulay duration is an intermediate step in the calculation of the interest rate sensitivity of an asset or liability and is *not* a measure of duration accepted by regulators. It is also necessary that the duration be

measured on an annual basis for the MCT interest rate margin calculation, as the application of the interest rate shock is measuring the impact of annual interest rate sensitivities. In other words, the definition of the duration needs to be consistent with the definition of the yield rate in terms of period of time, otherwise the results will be incorrect.

Also worth mentioning is that both the modified and effective durations provide only approximations of the sensitivity that changes in interest rates have on the present value of future cash flows. Both of these duration measures provide exact percentage changes for very small changes in interest rate (e.g., one basis point), but are generally less accurate for large changes, as the relationship between the change in interest rate and the change in present value of future cash flows is not linear. More accurate approximations of the impact of changes in interest rates on the present value of future cash flows can be achieved through considering the curvature (or convexity) of the price-yield relationship.

In an attempt to manage the effect that changes in interest rates have on their surplus position, insurers often endeavour to match the duration of their liabilities and assets. This approach is considered good practice. However, it can be demonstrated that there may be future cash flow shortfalls even in situations where the duration of liabilities and assets are perfectly matched. Accordingly, actuaries would consider future net cash flows as well as durations. The value of doing so is demonstrated in the educational note Discounting and Cash Flow Considerations for Property and Casualty Insurers (May 2016).

In the calculation of the interest rate risk margin, an interest rate shock factor is applied to the fair value of interest rate sensitive assets and liabilities and their duration. Actuaries are often involved in the calculation of the duration of liabilities and depending on the size of the insurer, may also be asked for support on the duration of assets.

Instructions on the calculation of the interest rate risk margin are provided in the MCT Guideline. The key points for the calculation of the duration are the following:

- Insurers may use either the modified duration or the effective duration to calculate the duration of assets and liabilities. However, the same duration methodology would apply to all assets and liabilities under consideration. Moreover, the same methodology is to be used consistently from year to year.
- Effective duration is the required measure when interest rate changes may change the expected cash flows.
- The portfolio duration can be obtained by calculating the weighted average of the duration for the assets or liabilities in the portfolio with the weights being proportional to the fair value of the cash flows or securities.

The following sections describe the theory and include some examples behind the calculations of duration of liabilities (both premium and claim) as well as assets.

Duration of Interest-Rate-Sensitive Liabilities

When evaluating the duration of the claim and premium liabilities, actuaries would consider the following:

- Assumptions underlying the duration calculation would be consistent with those underlying the discounting calculation (e.g., timing of payout) from the actuary's valuation work.
- The duration may be calculated by line of business using the payout patterns used for discounting. The line of business durations would then be weighted, using actuarial present value (APV) as weights, to derive the total premium or claim liabilities duration. This point is illustrated in appendix A, sheets 2 through 4.
- Alternatively, the duration may be evaluated for all lines of business on a combined basis, with the use of the effective duration approach. This point is illustrated for the duration of premium liabilities in appendix C.
- When the change in interest rate is small, the modified duration and effective duration are approximately the same, and the effective duration can be used to assess the reasonableness of the calculation of the modified duration, or even as a proxy for modified duration if appropriate.
- For premium liabilities, the following additional considerations apply:
  - The calculation would be adjusted for the future accident date; and
  - The future accident date would be adjusted to reflect policy terms of other than 12 months.
- For the purposes of input into the MCT calculation, the duration would be net of reinsurance and net of salvage and subrogation.

Interest-rate-sensitive liabilities include those for which the values are determined on a present value (PV) or actuarial present value basis. In accordance with the MCT Guideline, the interest-rate-sensitive liabilities to be included in the calculation of the interest rate risk margin are those for which their fair value will change with movements in interest rates. The following liabilities are considered sensitive to interest rates and are to be included:

- Net unpaid claims and adjustment expenses; and
- Net premium liabilities.

Other interest-rate-sensitive liabilities may include certain types of structured settlements. As per the OSFI guideline D5 Accounting for Structured Settlements, insurers may be required to recognize Type II structured settlement arrangements as an unpaid claim liability on the balance sheet (versus Type I structured settlements which have a disclosure-only requirement). The challenge to actuaries is that the value of the purchased annuities for Type II settlements will flow through the actuarial data as a single lump sum payment which could cause an understatement of the overall duration if not adjusted for. The additional challenge to actuaries is that embedded in the settlement structure value is the assumption of the prevailing interest rate (which is an input into the modified duration calculation). So, in the absence of the real future cash flows and the interest rate, the actuary may need to make a simplified yet reasonable assumption on the underlying payment pattern in order to reasonably approximate

the underlying future cash flows, and may want to consider using the valuation discount rate to complete the modified duration calculation.

P&C insurers may require supervisory approval in order to be able to consider other liabilities in the calculation of the interest rate risk margin.

Refer to appendix A (sheets 2–3) for an example of the duration calculations for unpaid claim liabilities, and to appendix A (sheet 4) for an example of the duration calculations for premium liabilities. Appendix A (sheet 5) shows how the durations calculated in sheets 3 and 4 may be carried into the calculation of the interest rate risk margin in P&C returns.

Refer to appendix B for an illustration of the cash flow matching model to derive the duration of the claim and premium liabilities.

Appendix C is similar to appendix A (sheet 4) except that it illustrates the duration calculation for premium liabilities on an all-lines-combined basis using the effective duration approach. The interest rate risk margin would be amended to reflect the appropriate fields from appendix C.

Duration of Interest-Rate-Sensitive Assets

Actuaries may be asked to calculate the duration of the interest-rate-sensitive assets in the insurer's portfolio, including for purposes of the calculation of the interest rate risk margin that is part of the MCT calculation. For most insurers, the main classes of interest-rate-sensitive assets are bonds and preferred shares. Refer to appendix A (sheet 1) for an illustrative duration calculation for fixed income securities.

Retractable preferred shares, and preferred shares with rate reset options, may lend themselves to the same duration calculation approach as bonds, particularly if a redemption date or rate reset date can be considered as equivalent to the maturity date of a bond.

As an alternative to the duration calculations referred to above, or to supplement the calculations for other classes of interest-rate-sensitive assets, actuaries may use estimates derived by the insurer's investment specialists. Before using the work of the investment specialist, the actuary would review the information for reasonableness, and identify which duration formula was used (i.e., Macaulay duration, modified duration, or effective duration) in order to ensure consistency between asset and liability durations.

Appendices

The examples in the appendices are provided to assist actuaries in calculating durations for the purpose of the interest rate risk margin in the P&C returns. They are intended to be illustrative, rather than prescriptive. Also included is an example of the use of those estimates in the calculation of the interest rate risk margin in accordance with the MCT Guideline (see appendix A, sheet 5).

Recognizing the link between concepts addressed in this educational note and those addressed in other recently issued educational notes, the appendices include exhibits taken from those other educational notes, as indicated below:

Exhibit	Description	Reference
Appendix A		
Sheet 1	Duration of bonds	2015 Year-end memo[1]
Sheets 2-3	Duration of unpaid claim liabilities	2015 Year-end memo[1]
Sheet 4	Duration of premium liabilities	N/A
Sheet 5	Interest Rate Risk Margin	2015 P&C return[2]
Appendix B	Net cash flow matching model	Discounting ed. note[3]
Appendix C	Duration of premium liabilities	Premium liabilities ed. note[4]

(1) Educational Note: 2015 Guidance to the Appointed Actuary for Property and Casualty Insurers (October 2015). Appendix B (Sheets 2-4)

(2) 2015 P&C Return – Page 30.66 – Capital (Margin) Required for Interest Rate Risk

(3) Revised Educational Note: Discounting and Cash Flow Considerations for Property and Casualty Insurers (May 2016). Appendix B (Sheet 4)

(4) Second Revision – Educational Note: Premium Liabilities (July 2016)

Appendix D, Sheet 1 is a deterministic approach to demonstrate that the duration of the net premium liabilities can be derived from the duration of a future accident year. Appendix D, Sheet 2 summarizes the results of testing performed by the PCFRC to assess the effect of various approximations of the Macaulay duration.

Duration of Bonds

Appendix A
Sheet 1

Year-end Information

Description	Bond #1	Bond #2	Bond #3
Valuation Date	2015/12/31	2015/12/31	2015/12/31
Maturity Date	2016/12/31	2017/06/30	2018/06/30
Coupon Rate	2.50%	6.60%	4.65%
Coupon # (k)	2	2	2
Par value	1,250.0	1,875.0	1,125.0
Market value	1,265.0	2,010.0	1,140.0
Semi-annual Coupon $	15.6	61.9	26.2
Yield (y) on a semi-annual basis	0.644%	0.859%	2.042%
Excel Yield (for comparison)	*0.644%*	*0.859%*	*2.042%*

Step 1: Future payment for assets

	Cash flows		
Year	Bond #1	Bond #2	Bond #3
2016.0	(1,265.0)	(2,010.0)	(1,140.0)
2016.5	15.6	61.9	26.2
2017.0	1,265.6	61.9	26.2
2017.5	-	1,936.9	26.2
2018.0	-	-	26.2
2018.5	-	-	1,151.2

Step 2: Calculation of duration for assets

Semi-annual (1 basis point) Δy = **0.01%**

	Annual (yrs) (1)	Semi-Annual Periods (2)	Cash Flows (3)	Present Value Factor (4)	Discounted Cash Flows (5)	PV Factor with -Δy (9)	PV Factor with +Δy (10)	Discounted Cash Flows with -Δy (11)	Discounted Cash Flows with +Δy (12)
Bond #1 yield:	0.64%								
	0.50	1	15.6	0.9936	15.5	0.9937	0.9935	15.5	15.5
	1.00	2	1,265.6	0.9872	1,249.7	0.9874	0.9870	1,249.7	1,249.2
	1.50	3	-	0.9809	-	0.9812	0.9806	-	-
	2.00	4	-	0.9746	-	0.9750	0.9743	-	-
	2.50	5	-	0.9683	-	0.9689	0.9679	-	-
	Total				1,265.0			1,265.2	1,264.8
(6) Macaulay duration	0.99386	1.98773				(13) Effective duration (semi-annual periods)			1.9750
(7) Modified duration	0.98750	1.97500				(14) Effective duration (annual basis)			0.98750
(8) Excel Duration (comparison):	*0.99386*								
Bond #2 yield:	0.86%								
	0.50	1	61.9	0.9915	61.3	0.9916	0.9914	61.4	61.3
	1.00	2	61.9	0.9830	60.8	0.9832	0.9829	60.8	60.8
	1.50	3	1,936.9	0.9747	1,887.8	0.9750	0.9744	1,888.4	1,887.3
	2.00	4	-	0.9664	-	0.9668	0.9660	-	-
	2.50	5	-	0.9582	-	0.9586	0.9577	-	-
	Total				2,010.0			2,010.6	2,009.4
(6) Macaulay duration	1.45435	2.9087				(13) Effective duration (semi-annual periods)			2.8839
(7) Modified duration	1.44197	2.8839				(14) Effective duration (annual basis)			1.44197
(8) Excel Duration (comparison):	*1.45435*								
Bond #3 yield:	2.04%								
	0.50	1	26.2	0.9800	25.6	0.9801	0.9799	25.6	25.6
	1.00	2	26.2	0.9604	25.1	0.9606	0.9602	25.1	25.1
	1.50	3	26.2	0.9412	24.6	0.9414	0.9409	24.6	24.6
	2.00	4	26.2	0.9223	24.1	0.9227	0.9220	24.1	24.1
	2.50	5	1,151.2	0.9039	1,040.5	0.9043	0.9034	1,041.0	1,040.0
	Total				1,140.0			1,140.5	1,139.5
(6) Macaulay duration	2.38980	4.7796				(13) Effective duration (semi-annual periods)			4.68397
(7) Modified duration	2.34198	4.6840				(14) Effective duration (annual basis)			2.34198
(8) Excel Duration (comparison):	*2.38980*								

Step 3: Market Value Weighted Duration of Assets

	Market Value	Modified Duration	Effective Duration
Bond #1	1,265.0	0.98750	0.98750
Bond #2	2,010.0	1.44197	1.44197
Bond #3	1,140.0	2.34198	2.34198
Total	**4,415.0**	**1.54415**	**1.54415**

(4) = 1 / (1 + y) ^ (2)
(5) = (3) x (4)
(6) Sumproduct of columns (2) and (5) divided by (5) Total; for annual basis divide by 2
(7) = (6) / (1 + y); for annual basis divide by 2
(8) DURATION (Valuation Date, Maturity Date, Coupon Rate, Annual Yield Rate, Coupon Frequency, basis)
(9) = 1 / (1 + y - Δy) ^ (2)
(10) = 1 / (1 + y + Δy) ^ (2)
(11) = (3) x (8)
(12) = (3) x (9)
(13) = [(11) total - (12) total] / [2 x Δy] / [(5) total]
(14) = (13) / 2

Duration of Unpaid Claim Liabilities

Appendix A
Sheet 2

Year-end Information

Unpaid as at December 31, 2015

Accident Year	Property	Liability
2011	-	32
2012	-	86
2013	-	127
2014	16	186
2015	137	258

Accident Year Payment Pattern

Age	Property	Liability
12	80%	35%
24	95%	68%
36	100%	80%
48	100%	85%
60	100%	90%
72	100%	95%
84	100%	99%
96	100%	100%

Yield (y)= 1.75%
Annual Δy = 0.10%

Unearned Premium Reserve for Property: 550
Unearned Premium Reserve for Liability: 380
Maintenance Expense Ratio (% UPR) = 3.50%
Maintenance Expenses should be paid during the time the UPR is being earned

Expected Loss Ratio for Property = 65.0%
Expected Loss Ratio for Liability = 80.0%

Step 1: Future payment for claims liabilities

Property

		Paid in						
Accident Year	Unpaid	2016	2017	2018	2019	2020	2021	2022
2011	-							
2012	-							
2013	-							
2014	16.0	16.0	-	-	-	-	-	
2015	137.0	102.8	34.3	-	-	-	-	-
Total	153.0	118.8	34.3	-	-	-	-	-

payout for AY 2015 @ 2016 = 137 / (1-80%) * (95% - 80%)
payout for AY 2015 @ 2017 = 137 / (1-80%) * (100% - 95%)
payout for AY 2014 @ 2016 = 16 / (1-95%) * (100% - 95%)

Liability

		Paid in						
Accident Year	Unpaid	2016	2017	2018	2019	2020	2021	2022
2011	32.0	16.0	12.8	3.2				
2012	86.0	28.7	28.7	22.9	5.7			
2013	127.0	31.8	31.8	31.8	25.4	6.4		
2014	186.0	69.8	29.1	29.1	29.1	23.3	5.8	
2015	258.0	137.2	47.6	19.8	19.8	19.8	15.9	4.0
Total	689.0	283.5	149.9	106.8	80.0	49.4	21.7	4.0

payout for AY 2015 @ 2016 = 258 / (1-35%) * (68% - 35%)
payout for AY 2015 @ 2017 = 258 / (1-35%) * (80% - 68%)
payout for AY 2014 @ 2016 = 186 / (1-68%) * (80% - 68%)
etc.

Duration of Unpaid Claim Liabilities

Appendix A
Sheet 3

Step 2: Calculation of duration for claims liabilities

Year (1)	Lag (yrs) (2)	Payment (3)	Present Value Factor (4)	Discounted Payment (5)	PV Factor with -Δy (8)	PV Factor with +Δy (9)	Discounted Cash Flows with -Δy (10)	Discounted Cash Flows with +Δy (11)
Property								
2016	0.5000	118.8	0.9914	117.7	0.9919	0.9909	117.8	117.7
2017	1.5000	34.3	0.9743	33.4	0.9758	0.9729	33.4	33.3
2018	2.5000	-	0.9576	-	0.9599	0.9552	-	-
2019	3.5000	-	0.9411	-	0.9443	0.9379	-	-
2020	4.5000	-	0.9249	-	0.9290	0.9208	-	-
2021	5.5000	-	0.9090	-	0.9139	0.9041	-	-
2022	6.5000	-	0.8934	-	0.8991	0.8877	-	-
Total				151.1			151.2	151.0
	0.7209	(6) Macaulay duration					(12) Effective duration	0.7085
	0.7085	(7) Modified duration						
Liability								
2016	0.5000	277.2	0.9914	274.8	0.9919	0.9909	274.9	274.6
2017	1.5000	149.9	0.9743	146.1	0.9758	0.9729	146.3	145.8
2018	2.5000	106.8	0.9576	102.3	0.9599	0.9552	102.5	102.0
2019	3.5000	80.0	0.9411	75.3	0.9443	0.9379	75.6	75.1
2020	4.5000	49.4	0.9249	45.7	0.9290	0.9208	45.9	45.5
2021	5.5000	21.7	0.9090	19.7	0.9139	0.9041	19.8	19.6
2022	6.5000	4.0	0.8934	3.5	0.8991	0.8877	3.6	3.5
Total				667.4			668.6	666.2
	1.8176	(6) Macaulay duration					(12) Effective duration	1.7863
	1.7863	(7) Modified duration						

Step 3: Weighted duration for claims liabilities

	PV of Unpaid Claims	PFAD	APV of Unpaid Claims	Modified Duration	Effective Duration
Property	151.1	5	156	0.7085	0.7085
Liability	667.4	115	782	1.7863	1.7863
Total	818.5	120	**938**	**1.6070**	**1.6070**

(3) From Appendix A, Sheet 2
(4) = 1 / (1 + y) ^ (2)
(5) = (3) x (4)
(6) Sumproduct of columns (2) and (5) divided by (5) Total
(7) = (6) / (1 + y)

(8) = 1 / (1 + y - Δy) ^ (2)
(9) = 1 / (1 + y + Δy) ^ (2)
(10) = (3) x (8)
(11) = (3) x (9)
(12) = [(10) total - (11) total] / [2 x Δy] / [(5) total]

Duration of Premium Liabilities

Appendix A
Sheet 4

Yield (y) = 1.75%
Annual Δy = 0.10%

Year (1)	Lag to Time Zero (yrs) (2)	AY Incremental Payment Pattern (3)	Present Value Factor (4)	Discounted to Time Zero (5)	PV Factor with -Δy (13)	PV Factor with +Δy (14)	Discounted with -Δy (15)	Discounted with +Δy (16)
Property								
2016	0.5000	80.0%	0.9914	79.31%	0.9919	0.9909	79.35%	79.27%
2017	1.5000	15.0%	0.9743	14.61%	0.9758	0.9729	14.64%	14.59%
2018	2.5000	5.0%	0.9576	4.79%	0.9599	0.9552	4.80%	4.78%
2019	3.5000	0.0%	0.9411	0.00%	0.9443	0.9379	0.00%	0.00%
2020	4.5000	0.0%	0.9249	0.00%	0.9290	0.9208	0.00%	0.00%
2021	5.5000	0.0%	0.9090	0.00%	0.9139	0.9041	0.00%	0.00%
2022	6.5000	0.0%	0.8934	0.00%	0.8991	0.8877	0.00%	0.00%
2023	7.5000	0.0%	0.8780	0.00%	0.8845	0.8715	0.00%	0.00%
Total				98.71%			98.78%	98.64%
	0.7451	(6) Macaulay Duration						
	0.7322	(7) Modified Duration						
	0.5000	(8) Mean Accident Date of an AY					0.5000	0.5000
	0.3333	(9) Mean Accident Date of UPR					0.3333	0.3333
	0.9900	(10) Discount Factor at Time Zero of Prem Liab					0.9905	0.9894
	0.5784	(11) Macaulay Duration					(17) Effective Duration:	0.5684
	0.5684	(12) Modified Duration						
Liability								
2016	0.5000	35.0%	0.9914	34.7%	0.9919	0.9909	34.71%	34.68%
2017	1.5000	33.0%	0.9743	32.2%	0.9758	0.9729	32.20%	32.10%
2018	2.5000	12.0%	0.9576	11.5%	0.9599	0.9552	11.52%	11.46%
2019	3.5000	5.0%	0.9411	4.7%	0.9443	0.9379	4.72%	4.69%
2020	4.5000	5.0%	0.9249	4.6%	0.9290	0.9208	4.65%	4.60%
2021	5.5000	5.0%	0.9090	4.5%	0.9139	0.9041	4.57%	4.52%
2022	6.5000	4.0%	0.8934	3.6%	0.8991	0.8877	3.60%	3.55%
2023	7.5000	1.0%	0.8780	0.9%	0.8845	0.8715	0.88%	0.87%
Total				96.6			96.85%	96.48%
	1.9282	(6) Macaulay Duration						
	1.8950	(7) Modified Duration						
	0.5000	(8) Mean Accident Date of an AY					0.5000	0.5000
	0.3333	(9) Mean Accident Date of UPR					0.3333	0.3333
	0.9695	(10) Discount Factor at Time Zero of Prem Liab					0.9712	0.9678
	1.7615	(11) Macaulay Duration					(17) Effective Duration:	1.7312
	1.7312	(12) Modified Duration						
Maintenance Expenses								
2016	0.5000	100%		99.1%	0.9919	0.9909	99.19%	99.09%
2017	1.5000	0%	0.9743	0.0%	0.9758	0.9729	0.00%	0.00%
Total				99.1%			99.19%	99.09%
	0.5000	(6) Macaulay Duration						
	0.4914	(7) Modified Duration						
	0.5000	(8) Mean Accident Date of an AY					0.5000	0.5000
	0.3333	(9) Mean Accident Date of UPR					0.3333	0.3333
	0.9942	(10) Discount Factor at Time Zero of Prem Liab					0.9946	0.9939
	0.3333	(11) Macaulay Duration					(17) Effective Duration:	0.3276
	0.3276	(12) Modified Duration						

	UPR	ELR	Undiscounted Prem Liabilities	Discount Factor	PV of Prem Liabilities	Total PFAD	APV of Prem Liabilities	Modified Duration	Effective Duration
Property	550	65.0%	357.5	0.9900	353.9	12	365.9	0.5684	0.5684
Liability	380	80.0%	304.0	0.9695	294.7	51	345.7	1.7312	1.7312
Maintenance		3.50%	32.6	0.9942	32.4	-	32.4	0.3276	0.3276
Total	930		694.1		681.0	63	**744.0**	**1.0983**	**1.0983**

(2) Assume that all policies have 12-month terms with equal earning
(3) From Appendix A, Sheet 2
(4) [1 + y]^-(2)
(5) = (3) x (4)
(6) = Sumproduct of columns (2) and (5) divided by (5) total
(7) = (6) / [1 + y]
(8) Average accident date of a future accident year (July 1st)
(9) Mean average accident date of premium liabilities (May 1st).
(10) = (5) total x (1 + y)^ [(8) - (9)]
(11) = (6) - (8) + (9)
(12) = (11) / [1 + y]
(13) = [1 + y - Δy]^-(2)
(14) = [1 + y + Δy]^-(2)
(15) = (3) x (13)
(16) = (3) x (14)
(17) [Discount Factor with +Δy - Discount Factor with -Δy] / [2 x Δy] / (10)

30.66

2015

Date

MCT (BAAT) MARKET RISK CAPITAL (MARGIN) REQUIREMENTS
($'000)

				Interest rate shock factor 0.01250	Interest rate shock factor (0.01250)
Capital (Margin) Required for Interest Rate Risk					
(55)		**Fair value** (01)	**Modified or effective duration** (02)	**Dollar fair value change (01)x(02)xΔy** (03)	**Dollar fair value change (01)x(02)x(-Δy)** (04)
Interest rate sensitive assets:					
Term deposits	01			0	0
Bonds and debentures	02	4,415.0	1.9141	85	-85
Commercial paper	03			0	0
Loans	04			0	0
Mortgages	05			0	0
MBS and ABS	06			0	0
Preferred shares	07			0	0
Other (specify)	08			0	0
Total interest rate sensitive assets	**09**	4,415.0		85	-85
Interest rate sensitive liabilities:					
Net unpaid claims and adjustment expenses	10	1,168.5	1.6070	19	-19
Net premium liabilities	11	744.0	1.0983	10	-10
Other as approved by OSFI				0	0
Total interest rate sensitive liabilities	**19**	1,682.5		29	-29
Allowable interest rate derivatives:		**Notional value** (05)		**Dollar fair value Δy** (06)	**Dollar fair value - Δy** (07)
Long positions	20				
Short positions	21				
Total allowable interest rate derivatives	**29**			0	0
Capital required for Δy shock increase	30			56	
Capital required for Δy shock decrease	31				0
Total interest rate risk margin	**39**				56

Note: Δy = 1.25%

Row 02 from Appendix A, Sheet 1
Row 10 from Appendix A, Sheet 3
Row 11 from Appendix A, Sheet 4

Appendix B

ABC INSURANCE COMPANY

31 DECEMBER 2015

CASH FLOW MATCHING MODEL

Cash Flow (in $000's) for Determination of Discount Rate

Reinvestment Rate		1.000%
Internal Rate of Return (IRR) on Cash Flows:	IRR per Col (4)	2.153%
Estimated investment expense ratio		0.250%
Indicated discount rate net of expenses		1.903%

	Cash In-flow from Assets					Cash Outflow			Net Inflow (Excess)				Reinvested Funds			
(1)	(2)	(3)	(4)	(4a)	(4b)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Year	Cash from Investment Sheet 3	(To)/ From Reinvestment See below	Total Inflow = (2) + (3)	Payment of Net Claim Liabilities	Payment of Net Prem Liabilities	Payment of Net Policy Liabilities Sheet 3	Cash Withdrawal = (4) - (5)	Total Outflow = (5) + (6)	Net inflow No Reinv/WD = (2) - (5)	Cumulative Excess Based on (8)	Net inflow With Reinv/WD = (4) - (7)	Cumulative Excess Based on (10)	Opening Balance = (15) prior year	Interest Earned on Reinv. =(12) * Reinv. Rate	Deposit / (Withdrawal) = -(3)	Closing Balance =(12) + (13)+ (14)
From Sheet 1	-349,985	0	-349,985	-275,865	-43,219			-349,985								
2016	140,960	-10,932	130,028	110,075	19,953	130,028	0	130,028	10,932	10,932	0	0	0	0	10,932	10,932
2017	87,733	-15,886	71,847	59,385	12,462	71,847	0	71,847	15,886	26,817	0	0	10,932	109	15,886	26,926
2018	54,773	-7,523	47,250	41,720	5,530	47,250	0	47,250	7,523	34,340	0	0	26,926	269	7,523	34,718
2019	2,648	27,826	30,473	27,400	3,073	30,473	0	30,473	-27,826	6,514	0	0	34,718	347	-27,826	7,240
2020	17,648	5,975	23,622	21,665	1,957	23,622	0	23,622	-5,975	539	0	0	7,240	72	-5,975	1,338
2021	32,033	-6,866	25,166	12,925	1,086	14,011	11,155	25,166	18,022	18,561	0	0	1,338	13	6,866	8,217
2022	893	8,299	9,191	8,715	476	9,191	0	9,191	-8,299	10,262	0	0	8,217	82	-8,299	1
2023	35,893	-3,391	32,502	4,875	273	5,148	27,354	32,502	30,745	41,007	0	0	1	0	3,391	3,392
2024	0	3,010	3,010	2,895	115	3,010	0	3,010	-3,010	37,997	0	0	3,392	34	-3,010	416
2025	0	400	400	345	55	400	0	400	-400	37,597	0	0	416	4	-400	20
2026	0	20	20	0	20	20	0	20	-20	37,577	0	0	20	0	-20	0
Total ex 2015	372,577	932	373,509	290,000	45,000	335,000	38,509	373,509	37,577		0					

Underlying Duration Calculation

IRR on Cash Flows (y):	2.257%	1.903%	1.903%
Payment Lag (EOP)	Disc Factor		
1	0.978	0.981	0.981
2	0.956	0.963	0.963
3	0.935	0.945	0.945
4	0.915	0.927	0.927
5	0.894	0.910	0.910
6	0.875	0.893	0.893
7	0.855	0.876	0.876
8	0.836	0.860	0.860
9	0.818	0.844	0.844
10	0.800	0.828	0.828
11	0.782	0.813	0.813
Macaulay Duration	2.747	2.617	2.122
Modified Duration	**2.687**	**2.568**	**2.082**

Notes

Cells in red are expansions to the educational note *Discounting and Cash Flow Considerations for P&C Insurers* .

(4a) See Revised Educational Note: Discounting and Cash Flow Considerations for P&C Insurers - Appendix B, S[illegible]

(4b) See Revised Educational Note: Discounting and Cash Flow Considerations for P&C Insurers - Appendix B, Sh[illegible]

(5) = (4a) + (4b)

ABC Insurance Company of Canada
Premium Liabilities Analysis
Net Basis
As of December 31, XXXX
(000s)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Class of Insurance	**Direct UPR**	**Assumed UPR**	**Gross UPR**	**Ceded UPR**	**Net UPR**	**Expected Reinsur. Premium**	**Selected Undisc. Loss Ratio (% Prem)**	**Losses + ALAE**	**Selected ULAE Ratio (% Loss + ALAE)**	**ULAE**	**Undisc. Losses + LAE**
Personal Property	10,000	0	10,000	500	9,500	500	80.0%	7,740	--	383	8,123
Commercial Property	0	0	0	0	0	0	0.0%	-	--	-	-
Aircraft	0	0	0	0	0	0	0.0%	-	--	-	-
Auto - Liability - Regular	50,000	0	50,000	1,000	49,000	3,000	90%	45,080	--	2,250	47,330
Auto - PA - Regular	25,000	0	25,000	3,000	22,000	1,500	115.0%	23,575	--	1,350	24,925
Auto - Other - Regular	30,000	0	30,000	500	29,500	1,000	67.0%	19,095	--	918	20,013
Auto - Liability - Facility	1,500	0	1,500	0	1,500	0	93.3%	1,400	--	-	1,400
Auto - PA - Facility	750	0	750	0	750	0	93.3%	700	--	-	700
Auto - Other - Facility	750	0	750	0	750	0	93.3%	700	--	-	700
Boiler & Machinery	0	0	0	0	0		0.0%	-	--	-	-
Credit	0	0	0	0		0	0.0%	-	--	-	-
Credit Protection	0	0	0	0	0	0	0.0%	-	--	-	-
Fidelity	0	0	0	0	(	0	0.0%	-	--	-	-
Hail	0	0	0	0	0	0	0.0%	-	--	-	-
Legal Expense	0	0	0	0	0	0	0.0%	-	--	-	-
Liability - Total	0	5,000	5,0	1,0	,000	250	73.0%	2,738	--	169	2,906
Other Approved Products	0	0	0	0	0	0	0.0%	-	--	-	-
Surety - Total	0	0	0		0	0	0.0%	-	--	-	-
Title	0	0		0	0	0	0.0%	-	--	-	-
Marine	0		0	0	0	0	0.0%	-	--	-	-
Accident & Sickness	0	0		0	0	0	0.0%	-	--	-	-
Total	**118,000**	**5,000**	**23,000**	**6,000**	**117,000**	**6,250**	**91.8%**	**101,028**	**--**	**5,069**	**106,097**

(1) From Prem Liab Ed Note, appendix B, sheet 1, column (1)
(2) From Prem Liab Ed Note, appendix B, sheet 1, column (2)
(3) = (1) + (2)
(4) From company accounting department or annual return
(5) = (3) - (4)
(6) From company
(7) Similar calculation as gross analysis (see Prem Liab Ed Note)
(8) = [(5) - (6)]x (7)

(9) n/a
(10) Prem Liab Ed Note, appendix B, sheet 1, column (10)
(11) = (8) + (10)

ABC Insurance Company of Canada
Premium Liabilities Analysis
Net Basis
As of December 31, XXXX
(000s)

	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)
Class of Insurance	**Discount Factor**	**Discounted Losses + LAE**	**Discount Factor (with MfAD)**	**Discounted Losses + LAE (with Int. PfAD)**	**Interest Rate PfAD**	**Claims Dev't. MfAD**	**Claims Dev't. PfAD**	**Ceded Discounted Losses +ALAE**	**Reinsur. MfAD**	**Reinsur. PfAD**	**Total PfAD**	**Discounted Losses with PfADs**
Personal Property	0.983	7,984	0.987	8,015	31	7.0%	559	74	1.0%	7	597	8,581
Commercial Property	--	0	--	0	0	0.0%	0	0	1.0%	0	0	0
Aircraft	--	0	--	0	0	0.0%	0	0	1.0%	0	0	0
Auto - Liability - Regular	0.922	43,647	0.943	44,642	994	11.0%	4,801	4,277	1.0%	43	5,838	49,485
Auto - PA - Regular	0.932	23,234	0.953	23,758	524	10.0%	2,323	5,833	1.0%	58	2,906	26,140
Auto - Other - Regular	0.977	19,553	0.988	19,773	220	7.0%	1,309	1,275	1.0%	13	1,601	21,154
Auto - Liability - Facility	0.929	1,300	0.929	1,300	0	15.4%	200	0	1.0%	0	200	1,500
Auto - PA - Facility	0.929	650	0.929	650	0	15.4%	100	0	1.0%	0	100	750
Auto - Other - Facility	0.929	650	0.929	650	0	15.4%	100	0	1.0%	0	100	750
Boiler & Machinery	--	0	--	0	0	0.0%	0	0	1.0%	0	0	0
Credit	--	0	--	0	0	0.0%	0	0	1.0%	0	0	0
Credit Protection	--	0	--	0	0	0.0%	0	0	1.0%	0	0	0
Fidelity	--	0	--	0	0	0.0%	0	0	1.0%	0	0	0
Hail	--	0	--	0	0	0.0%	0	0	1.0%	0	0	0
Legal Expense	--	0	--	0	0	0.0%	0	0	1.0%	0	0	0
Liability - Total	0.937	2,724	0.953	2,771	47	10.0%	272	890	1.0%	9	328	3,052
Other Approved Products	--	0	--	0	0	0.0%	0	0	1.0%	0	0	0
Surety - Total	--	0	--	0	0	0.0%	0	0	1.0%	0	0	0
Title	--	0	--	0	0	0.0%	0	0	1.0%	0	0	0
Marine	--	0	--	0	0	0.0%	0	0	1.0%	0	0	0
Accident & Sickness	--	0	--	0	0	0.0%	0	0	1.0%	0	0	0
Total	**0.940**	**99,742**	**0.957**	**101,558**	**1,816**	**9.7%**	**9,725**	**13,024**	**1.0%**	**130**	**11,671**	**111,413**

(12) Similar calculation as gross analysis (see Prem Liab Ed Note)
(13) = (11) x (12)
(14) Similar calculation as gross analysis (see Prem Liab Ed Note)
(15) = (11) x (14)
(16) = (15) - (13)
(17) Claims development MfAD used for the valuation of claims liabilities
(18) = (13) x (17)
(19) See Prem Liab Ed Note, Appendix C, Sheet 2
(20) Reinsurance MfAD used for the valuation of claims liabilities
(21) = (19) x (20)
(22) = (16) + (18) + (21) [input for P&C annual return Page 30.64, Col (14)]
(23) = (13) + (22)

ABC Insurance Company of Canada
Premium Liabilities Analysis
Net Basis
As of December 31, XXXX
(000s)

	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)
Class of Insurance	**Maint. Expense Ratio (% Gross Prem.)**	**Maint. Expenses**	**Contingent Comm. Rate (% Gross Prem.)**	**Contingent Comm.**	**Premium Liabilities**	**Unearned (Ceded) Comm.**	**Equ... UPR**	**Max. Allowable DPAE**	**Initial DPAE**	**Booked DPAE**	**Premium Deficiency**
Personal Property	3.00%	300	0.00%	0	9,381	429					
Commercial Property	3.00%	0	0.00%	0	0	0					
Aircraft	3.00%	0	0.00%	0	0	0					
Auto - Liability - Regular	3.00%	1,500	0.00%	0	53,985	258					
Auto - PA - Regular	3.00%	750	0.00%	0	28,390	774					
Auto - Other - Regular	3.00%	900	0.00%	0	33,054	129					
Auto - Liability - Facility	3.00%	45	0.00%	0	1,54	0					
Auto - PA - Facility	3.00%	23	0.00%	0	773	0					
Auto - Other - Facility	3.00%	23	0.00%	0	73	0					
Boiler & Machinery	3.00%	0	0.00%	0		0					
Credit	3.00%	0	0.00%	0	0	0					
Credit Protection	3.00%	0	0.00%	0	0	0					
Fidelity	3.00%	0	0.00%	0	0	0					
Hail	3.00%	0	0.00%	0	0	0					
Legal Expense	3.00%	0	0.00%	0	0	0					
Liability - Total	3.00%	150	0.00%	0	3,452	258					
Other Approved Products	3.00%	0	0.00%	0	0	0					
Surety - Total	3.00%	0	0.00%	0	0	0					
Title	3.00%	0	0.00%	0	0	0					
Marine	3.00%	0	0.00%	0	0	0					
Accident & Sickness	3.00%	0	0.00%	0	0	0					
Total	**3.00%**	**3,**	**0.00%**	**0**	**121,353**	**1,549**	**(2,804)**	**0**	**20,000**	**0**	**2,804**

(24) From Prem Liab Ed Note, appendix B, sheet 6, row (10)
(25) = (3) x (24)
(26) Based on company budget and projected loss ratios
(27) = (3) x (26)
(28) = (6) + (23) + (25) + (27)
(29) From company accounting department or annual return
(30) = (5) - (28) + (29)

(31) = max [(30) , 0]
(32) From company accounting department
(33) = min [(31) , (32)] [input for P&C return 20.10, row(43)]
(34) = - min [(30) , 0] [input for P&C return 20.20, row (15)]

ABC Insurance Company of Canada
Premium Liabilities Analysis
Net Basis
As of December 31, XXXX
(000s)

	(35)	(36)	(37)
Class of Insurance	**Premium Liabilities Δy= +0.1%**	**Premium Liabilities Δy= -0.1%**	**Premium Liabilities Effective Duration**
Personal Property			
Commercial Property			
Aircraft			
Auto - Liability - Regular			
Auto - PA - Regular			
Auto - Other - Regular			
Auto - Liability - Facility			
Auto - PA - Facility			
Auto - Other - Facility			
Boiler & Machinery			
Credit			
Credit Protection			
Fidelity			
Hail			
Legal Expense			
Liability - Total			
Other Approved Products			
Surety - Total			
Title			
Marine			
Accident & Sickness			
Total	**120,997**	**121,**	**3.803**

(35) = recalculation of (28) using discount rate + 0.1%
(36) = recalculation of (28) using discount rate - 0.1%
(37) = [(36)-(35)] / [2 x 0.1%] / (28)

Premium Liabilities Macaulay Duration

The following is a deterministic approach to demonstrate that the duration of the net premium liabilities can be derived from the duration of a future accident year.

Assume the following:

- i = yield-to-maturity discount rate.
- Assume losses are uniformly distributed and premiums are annual and evenly distributed.
- Let t = timing of payments of a future accident year (0.5/1.5/2.5/etc.) from the valuation or calculation date. For simplification, assume there is only one payment made each year and that the first payment is made at the average accident date.
- P_t is your cash flow payment at time t.
- Let x = difference between the mean accident date of a future accident year and the mean accident date underlying the unearned premium reserve = 1/6 (0.50 less 0.333).

$$Macaulay\ Duration_{AY} = \frac{\sum_t tP_t(1+i)^{-t}}{\sum_t P_t(1+i)^{-t}}$$

$$Duration_{NPL} \approx \frac{\sum_t (t-x)P_t(1+i)^{-(t-x)}}{\sum_t P_t(1+i)^{-(t-x)}} = \frac{(1+i)^x \sum_t tP_t(1+i)^{-t} - x(1+i)^x \sum_t P_t(1+i)^{-t}}{(1+i)^x \sum_t P_t(1+i)^{-t}}$$

$$\approx Macaulay\ Duration_{AY} - x$$

- Modified duration can then be calculated by dividing by (1+i).

The following table summarizes the results from the monthly testing of the duration of the premium liabilities performed by the Sub-committee on Premium Liabilities Ed Note Revisions of the Committee on Property and Casualty Insurance Financial Reporting (PCFRC) against the following:

1. Previous CIA interpolation approach with the median average accident date;
2. Previous CIA interpolation approach with the mean average accident date;
3. New approximation using the duration of a future accident year minus an adjustment for accident dates using the mean (.3333); and
4. New approximation using the duration of a future accident year minus an adjustment for accident dates using the median (.2929).

Summary of Results - Difference versus exact monthly calculation

	Macaulay Duration of the Premium Liabilities			
	YE 2014 CIA approx using interpolation w/ median AAD	YE 2014 CIA approx using interpolation w/ mean AAD	New approx assuming w/ mean AAD	New Approx w/ median AAD
yield rate 20.00%				
Stable Pattern	-10.3%	-7.1%	-0.8%	-2.5%
Decreasing Pattern	-14.0%	-8.7%	-1.1%	-3.5%
Long Tail	-11.1%	-7.8%	-0.8%	-2.4%
Short Tail	-15.4%	-5.2%	-0.8%	-9.8%
yield rate				
Stable Pattern	-9.2%	-6.2%	-0.4%	-2.0%
Decreasing Pattern	-11.8%	-7.8%	-0.6%	-2.8%
Long Tail	-10.0%	-6.8%	-0.4%	-2.0%
Short Tail	-15.2%	-5.1%	-0.5%	-9.4%
yield rate 3.50%				
Stable Pattern	-8.5%	-5.6%	-0.1%	-1.7%
Decreasing Pattern	-11.0%	-7.1%	-0.2%	-2.4%
Long Tail	-9.2%	-6.2%	-0.1%	-1.7%
Short Tail	-15.0%	-5.0%	-0.2%	-9.0%
yield rate 0.00%				
Stable Pattern	-8.1%	-5.3%	0.0%	-1.5%
Decreasing Pattern	-10.5%	-6.7%	0.0%	-2.1%
Long Tail	-8.8%	-5.8%	0.0%	-1.5%
Short Tail	-14.9%	-4.9%	0.0%	-8.7%