

Member's Paper

An Actuarial Balance Sheet Approach to Assessing Sustainability of Target Benefit Plans

By
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<p>Abstract</p> <p>A target benefit plan aims to provide a predefined retirement benefit (the "target benefit") with predefined contributions. Members' target benefits may be adjusted upwards or downwards depending on the financial status of the plan. In this paper, it is argued that the unit credit cost method (projected or unprojected) traditionally applied to the funding of defined benefit plans in Canada is not an appropriate method for assessing the financial sustainability of target benefit plans. That assessment can be done through the means of an actuarial balance sheet, which takes into account future contributions to be made for, and benefits to be earned by, current and future plan members. The actuarial balance sheet developed in this paper is adapted from the actuarial balance sheet methodology that has been applied to the Swedish social security system since 2001.</p>	<p>Résumé</p> <p>Un régime à prestations cibles a pour but de fournir des prestations de retraite prédéterminées (« prestations cibles ») à l'aide de cotisations prédéterminées. Les prestations cibles des participants au régime peuvent être ajustées à la hausse ou à la baisse, selon la situation financière du régime. Dans cet article, il est expliqué que la méthode de nivellement des prestations (projetée ou non projetée), traditionnellement appliquée au provisionnement des régimes à prestations déterminées au Canada, n'est pas une méthode appropriée pour évaluer la viabilité financière des régimes de retraite à prestations cibles. Cette évaluation peut être effectuée à l'aide d'un bilan actuariel, lequel tient compte des cotisations futures et des prestations qui seront acquises, par les participants actuels et futurs du régime. Le bilan actuariel élaboré dans le présent document est adapté de la méthodologie du bilan actuariel appliquée en Suède depuis 2001 dans le cadre du système de sécurité sociale.</p>
<p>Acknowledgements</p> <p>The author would like to acknowledge the valuable comments from Doug Andrews, Assia Billig, Malcolm Hamilton, Jean-Claude Ménard, and Barbara Sanders.</p>	<p>Remerciements</p> <p>L'auteur aimerait remercier Doug Andrews, Assia Billig, Malcolm Hamilton, Jean-Claude Ménard et Barbara Sanders pour leurs précieux commentaires.</p>

Table of Contents

1. Introduction 4

2. TBP Funding Regimes in Canada 5

3. Accrued Liability Based on Unit Credit Cost Method is an Ill-defined Funding Target for TBPs..... 6

4. How to Assess the Long-Term Sustainability of TBPs? 9

5. Numerical Illustration 18

6. Concluding Remarks..... 25

Bibliography 26

Appendix A: Summary of New Brunswick and Alberta Funding Regulations..... 28

Appendix B: Cost Patterns Under Unit Credit Cost Method vs. Entry Age Normal Cost Method 30

Appendix C: Parameters and Assumptions..... 33

1. Introduction

Historically, most workplace pension plans in Canada were of a defined benefit (DB) type that pays employees a guarantee pension at retirement. The financial health of such plans is greatly influenced by the performance of the pension fund assets accumulated to meet the benefit obligations and the interest rates on long term government bonds that are used to measure the liabilities for regulatory funding purposes. Due to the long-lasting collapse in government bond yields and the increased volatility of equity market returns since 2008, many DB plans have become significantly underfunded (at least until 2011) and their employer sponsors have been required to increase contributions substantially in order to meet their funding obligations under the pension legislation. Market volatility and low interest rates also have an adverse impact on the retirement accounts of defined contribution (DC) plan participants who face the challenge of accumulating sufficient assets to meet their retirement income needs. In the face of these problems, there has been a growing discussion in Canada about exploring innovative design solutions to address the financial issues affecting traditional DB and DC pension plans.

A plan design concept called the “target benefit plan (TBP)” has been widely discussed in Canada since it was endorsed by the *Alberta/British Columbia Joint Expert Panel on Pension Standards* (Alberta/British Columbia, 2008) and the *Ontario Expert Commission on Pensions* (OECP, 2008) in 2008. TBPs are plans that aim to provide a target “defined benefit” funded through fixed employer (and, if applicable, employee) contributions. If, based on some specified tests, the fixed contributions are determined to be insufficient to provide the target benefit, members share the shortfall risk through a reduction of their benefits. The risk is not only shared among existing plan members but could also be shared with future generations of plan members. Some industry participants (ACPM, 2012) (ACPM, 2014) (Aon Hewitt, 2012) have suggested that the TBP is a viable plan design to deliver DB-like benefits, as it provides for the pooling of investment and longevity risks while the employer is no longer bearing all of the funding risks associated with traditional DB plans.

To date, three Canadian jurisdictions have enacted legislation and detailed regulations governing the administration, investment and funding of TBPs: New Brunswick (where they are referred to as “Shared Risk Plans” or SRPs), Alberta and British Columbia¹ (New Brunswick, 2012) (Alberta, 2014) (British Columbia, 2015). Several other jurisdictions have passed enabling legislation but do not yet have regulations in place.

This paper discusses the actuarial methodologies used for assessing the funding level of TBPs, with specific references to the regulatory regimes in New Brunswick and Alberta. We contend that the *closed group* funding approach, which is traditionally applied to DB plans in Canada and is also adopted in Alberta/British Columbia’s TBP regulations, is not a proper methodology for assessing the financial status of TBPs. The *open group* funding approach adopted in the New Brunswick Shared Risk Plans Regulation is not a proper

¹ The TBP regulations implemented in British Columbia are virtually identical to those of Alberta’s.

one either, as it does not account for the contributions and liabilities for current and future plan members in an actuarially appropriate manner. It is demonstrated that the financial management of TBPs can be improved by using a modified version of an actuarial balance sheet that has been used by some countries to assess the financial sustainability of their social security systems.

2. TBP Funding Regimes in Canada

For the funding of DB plans in Canada, the normal cost of a plan is typically determined using the *unit credit* cost method. This is an actuarial cost method under which the benefits (projected or unprojected) of each plan member are allocated proportionally over the member's employment or membership. The present value of benefits allocated to the year following the valuation date is called the *normal cost*. The present value of benefits allocated to all periods prior to the valuation date is called the *actuarial or accrued liability*. The value (market or smoothed) of plan assets is compared with the accrued liability to determine if a surplus or unfunded liability exists. Unfunded liabilities are typically required to be amortized over a period of not more than 15 years. With the exception of negotiated cost multi-employer pension plans in some jurisdictions, DB plans are also subject to prescribed solvency funding requirements.

The TBP funding method adopted in the Alberta regulations (Alberta, 2014) is a closed group unit credit cost method with a prescribed discount rate and a provision for adverse deviations (PfAD), both being dependent on the plan's investment policy. If an actuarial valuation establishes an unfunded liability on a going concern basis, it is required to be amortized by special payments paid within a period that is the shorter of 15 years or the expected average remaining service life (EARSL) in respect of active members in the plan. No funding of solvency deficiencies is required. Use of going concern funding excesses is restricted. Risk management requirements include stress testing for factors that the actuary considers to pose a material risk to the plan's ability to meet its funding requirements.

The TBP funding regime in New Brunswick (New Brunswick, 2012) establishes an open group funded ratio which is calculated as: (i) the market value of plan assets plus the value of planned future contributions in excess of the normal costs on an "open group" basis over a period of not more than 15 years, divided by (ii) a liability equal to the present value of accrued benefits for members in the plan. The normal cost is calculated using the unit credit cost method and the discount rate reflects the plan's funding and investment policy. A funding deficit recovery plan must be implemented if the open group funded ratio falls below 1.0 in two successive actuarial valuations. Stress testing is mandated to provide a forewarning for administrators to take corrective actions so that target benefits can be delivered with a high degree of confidence.

Appendix A summarizes the key elements of the TBP funding regimes currently exist in New Brunswick and Alberta.

3. Accrued Liability Based on Unit Credit Cost Method is an Ill-defined Funding Target for TBPs

Broadly speaking, a TBP is a hybrid pension plan that blends the pooling of risks found in traditional DB plans with the cost predictability for employers of DC plans. Contributions may be required to be made by both of the employer and employees who participate in the plan, or by the employer alone, but the employer's contribution rate would be fixed by design (or the rate may vary within a predefined range). The contribution rate would be set according to the target benefit provided in the plan. As with DC plans, the employer would not be responsible for making any additional contributions should the plan assets fall below a specified funding target. Remedies of any funding shortfall would fall to the plan members, either through an increase in employee contributions or a reduction in benefits or both. On the other hand, any excess assets that are deemed to be not needed to keep the plan sustainable may be used to improve the benefits for plan members.

3.1 Misapplication of unit credit cost method in Alberta

Alberta's TBP regulations adopt a closed group unit credit cost method for determining the funded position and *minimum* funding requirements of a TBP. As analyzed below, we contend that this is not an appropriate actuarial method for assessing the funding level of TBPs.

We begin our analysis with a comparison of the cost patterns under the unit credit cost method and the *entry age normal* cost method. Entry age normal cost method is a level cost method that attempts to make the cost of benefits allocated to each future year in terms of level dollar amounts or as a level percentage of salary. Where the retirement benefit provided under a plan is related to members' salaries, the normal cost for a member is defined as a level percentage of salary which, if made every year from the member's entry age to the retirement age, will have a present value at the entry age equal to that of future benefits based on the actuarial assumptions used in the valuation². The accrued liability at a given age of a member is determined as the present value of future benefits less the present value of future normal costs as at that age. It can be shown that, for any active member in the plan,

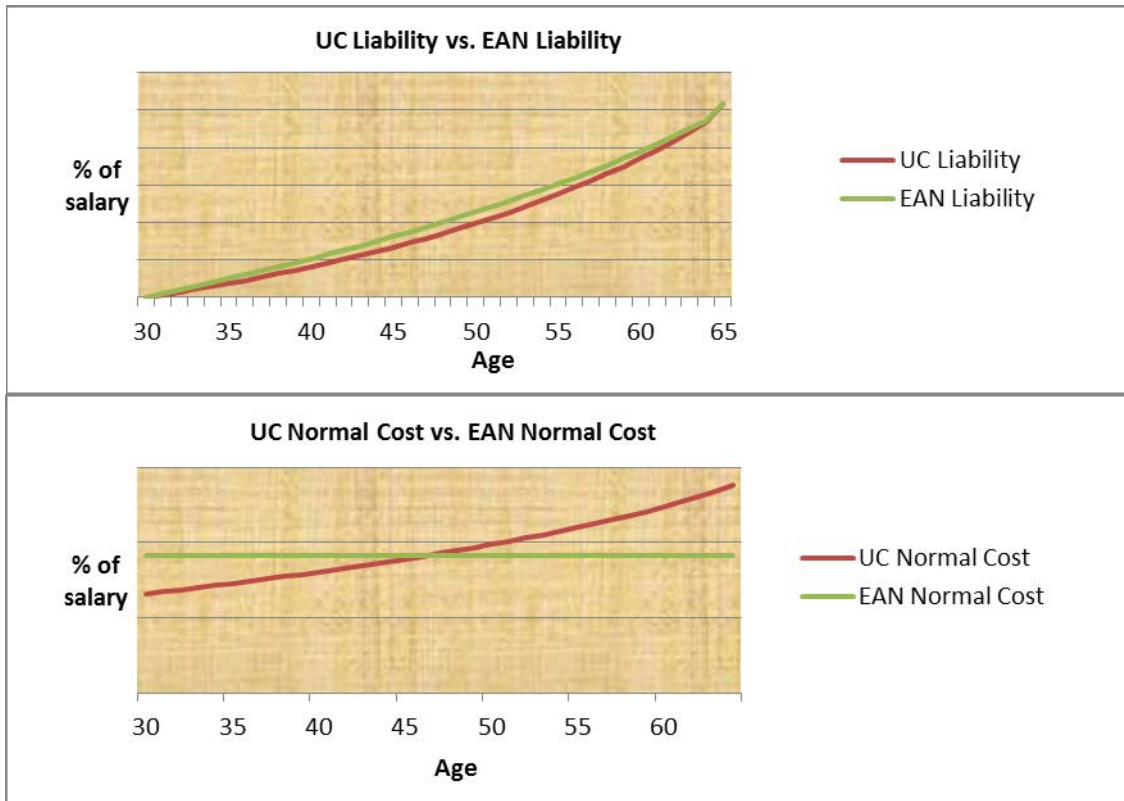
- The accrued liability determined under the unit credit cost method (the "UC liability") at any age of the member would be less than the accrued liability determined under the entry age normal cost method (the "EAN liability") at the corresponding age;
- The normal cost determined under the unit credit cost method (the "UC normal cost"), in terms of either dollar amount or percentage of salary, would increase with the age of the member, whereas the normal cost determined under the

² If *best-estimate* actuarial assumptions are used in the valuation, the normal cost for a member determined under the entry age normal cost method can be taken as an *actuarially fair* cost for the member. The normal costs paid over the member's career would accumulate to an amount sufficient to meet the benefits promised to the member, if all actuarial assumptions were exactly realized.

entry age normal cost method (the "EAN normal cost") would remain level throughout the member's career; and

- The UC normal cost would initially be less than the EAN normal cost, but would exceed the EAN normal cost after a certain age is reached.

This is illustrated graphically as follows. A mathematical proof is given in Appendix B.



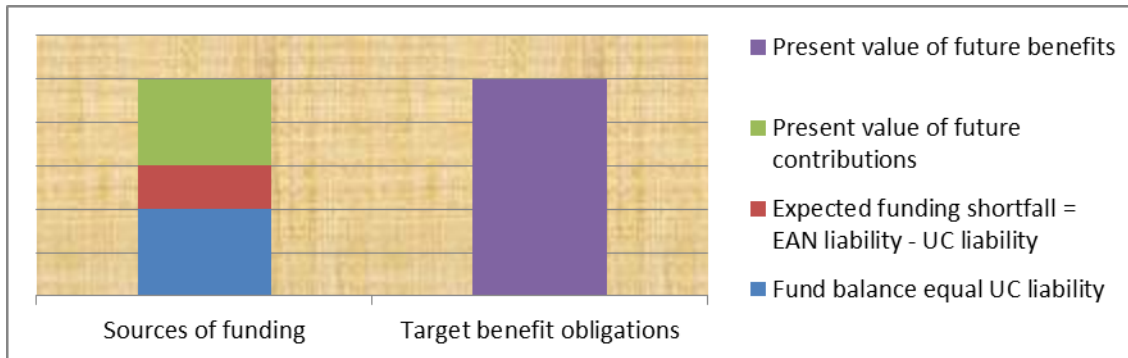
Now, consider a TBP with the following features:

- It provides a final salary pension payable at the normal retirement age;
- It does not provide any ancillary benefits;
- All employees join the plan at the same age; and
- The plan's fixed contribution rate is set as the normal cost rate determined according to the entry age normal cost method.

Theoretically, this particular plan should aim at funding towards the accrued liability determined under the entry age normal cost method (Anderson, 1992). In other words, the ideal fund balance, or desired amount of assets, to be held in the plan at any given point in time should be equal to the sum of the EAN liabilities for all members in the plan.

If the unit credit cost method is used to assess the funding of a plan that provides a defined benefit, adequacy of funding would be satisfied if the plan assets attain the level of UC liability for the plan and the contributions made are sufficient to meet the UC normal costs in respect of all members in the plan. However, this meaning of funding

adequacy is an illusion for a TBP that is designed to maintain a fixed contribution rate over the long term. For the example plan, assets at the level of UC liability plus the present value of future contributions for members in the plan will fall short of the present value of their future benefits, where the present values are calculated based on *best-estimate* actuarial assumptions applicable at the date of valuation. The shortfall is equal to the difference between the EAN liability and the UC liability, as demonstrated in the following graph.



If the actuarial assumptions used in the valuation were borne out by experience, the shortfall would eventually materialize and become manifest in situations where the plan's fixed contribution rate would not be high enough to meet the UC normal costs for all members in the plan (unless there is a continuing influx of new members joining the plan to prevent the UC normal cost rate from exceeding the plan's fixed contribution rate). At that point in time, members might be forced to make additional contributions to cover the funding shortfall or to have their anticipated benefits reduced.

While the above identified shortfall (which is not recognized in the funding assessment based on unit credit cost method) could potentially be reduced or extinguished by gains due to favorable plan experience, there is an equal likelihood that it could be passed onto future generations of members. Thus, use of the UC liability as a funding target could cause an implicit transfer of shortfall risk from current members to future members. This intergenerational risk transfer is not immediately evident to plan stakeholders and could potentially endanger the long-term sustainability of the plan.

3.2 Open group funding approach in New Brunswick

As discussed above, the closed group unit credit cost method adopted in Alberta is not an appropriate actuarial method for assessing the funding of TBPs. The open group approach adopted in New Brunswick is not a proper one either. First, the funding target prescribed therein is still based on the UC liabilities for members in the plan. Second, it includes any excess of future contributions over normal costs (again determined using the unit credit cost method) on an open group basis over a 15-year period as a part of the assets for determining the funded position of the plan (the so-called "open group funded ratio"). There is a risk transfer issue arising from such inclusion of excess contributions that is disputable.

The rising pattern of UC normal costs over a member's career implies that, beyond a certain age, the fixed contribution rate under the plan would not be sufficient to cover the member's UC normal costs (i.e., the cost of benefits expected to be accrued in a year by the member). All or part of the excess of contributions over the UC normal costs in the early career of a member should, arguably, be applied to fund the benefits accrued by the member in later years. Where the contributions made for future members over the next 15 years exceed their UC normal costs over the same period, a portion or all of that excess should be reserved for funding the benefits beyond the 15-year period earned by those same individuals and *not* be used to improve the funded ratio of the plan at the present time. In the opinion of this author, use of the New Brunswick's open group approach to assess the funded status of a TBP could cause a transfer of shortfall risk which might compromise the funding for future generations of members.

3.3 How should the funding target for a TBP be defined?

To avoid the obscure transfers of risk across different generations of members as exemplified above, we contend that the funding target for a TBP should be established using a level cost method that reflects the predefined contribution rate under the plan. In particular, the liability for active members should be defined as the difference between the present value of their future benefits and the present value of future contributions planned to be made for them. This prospective measure of liability will form part of the actuarial balance sheet developed in Section 4.2.

4. How to Assess the Long-Term Sustainability of TBPs?

“Sustainability” could be defined as an ability or capacity of something to be maintained or to sustain itself indefinitely. In the context of a TBP, it could mean the following: *“The assets already accumulated in the pension fund together with future contributions based on the predefined contribution rate and future investment earnings are expected to be able to support the retirement and other benefits targeted under the plan over the long term.”*

A distinctive feature of TBPs is that economic and demographic risks affecting a plan can be shared not only among current plan members (both active and inactive) but also with future plan members. This intergenerational risk sharing provides a valuable benefit to plan members when applied deliberately and transparently (CIA, 2015). Where the liabilities for current plan members (as defined in Section 3.3) are not fully met by the assets in the pension fund, the shortfall could be reduced or eliminated by lowering the target benefits for current plan members (including benefits that are currently payable). Additionally, contributions for future plan members that are expected not to be required to meet their target benefits could be used to cover a part or all of the shortfall. On the other hand, any excess of pension fund assets over the liabilities for current plan members could be used to restore or improve the target benefits for current and/or future plan members (including benefits that are currently payable), or be retained in the plan as a reserve for adverse contingencies. The nature and process

of intergenerational risk sharing employed by a plan should be clearly documented in the plan's risk management framework.

We seek to develop a financial statement for a TBP that presents a true and fair view of the plan's long-term sustainability, in the meaning described at the beginning of this section. In order to provide insights into the extent of intergenerational risk sharing, the statement should show clearly how the target benefit obligations are distributed among plan members and how they are being financed. To this end, we modify the actuarial balance sheet methodology used by the Swedish Inkomstpension system (Settergren, 2001) (Boado-Penas, Valdés-Prieto, & Vidal-Meliá, 2008) (Billig & Ménard, 2013) to suit the characteristics of TBPs.

4.1 The Swedish Inkomstpension system

The Swedish system is a notional defined contribution (NDC) system financed on a pay-as-you-go basis. The equivalent of 16% of each active participant's annual pensionable income is credited to the participant's notional account each year. The corresponding amounts received from active participants in a given year are used to pay pensions to pensioners for the same year. Differences between contributions received and pensions paid are transferred to a buffer fund which is used to cover periods when contributions are temporarily low due to an economic downturn. The interest credited to the notional accounts every year is either the increase in average income as measured by an *income index* or an approximation of the internal rate of return in the system as measured by a *balance index*. As a participant reaches a retirement age, the participant's notional account balance will be converted into a pension using an annuity factor that reflects the average life expectancy at retirement and an interest rate of 1.6%. A specific annuity factor is determined for each annual cohort of retired participants. The pension is subsequently indexed with the growth in average income or with the internal rate of return, minus 1.6%.

The Swedish government has published an actuarial balance sheet for its pension system (explained below) in its "Orange Report" (Swedish Pensions Agency, 2014) every year since 2001. An Automatic Balance Mechanism (ABM) has also been established and implemented to address any financial imbalances in the system. If the "balance ratio" that emerges from the actuarial balance sheet falls below one, the balancing mechanism is activated. When balancing is activated, pensions and notional account balances will be indexed by the change in a balance index³, instead of the change in the income index. If the balance ratio exceeds one during a period when balancing is activated, pensions and notional account balances will be indexed at a rate higher than

³ The balance index is the product of the income index and the balance ratio. The Orange Report provides the following example: If the balance ratio falls from 1.0000 to 0.9900 while the income index increases from 100.00 to 104.00, the balance index is calculated as the product of the balance ratio (0.9900) and the income index (104.00), for a balance index of 102.96. The indexation of pensions and notional account balances is then equal to 2.96% instead of 4%. The balance index for the next year is calculated by multiplying the balance index (102.96) by the ratio of the new to the old income index, multiplied in turn by the new balance ratio.

the increase in the income index. When the balance index reaches the level of the income index, balancing will be deactivated and the system returns to indexation solely by the change in the income index.

Actuarial balance sheet

The main entries on the actuarial balance sheet of a social security system are shown in Table 1. This structure is valid for social security systems with any level of funding (i.e., full, partial or *pure* pay-as-you-go).

Table 1: Main Entries on the Actuarial Balance Sheet of a Society Security System

Assets	Liabilities
Financial and real assets	Liability to pensioners
Contribution asset	Liability to contributors
Accumulated deficit (surplus)	
Total assets	Total liabilities

For a pay-as-you-go or partially funded system, the contribution asset (an implicit asset) on the above actuarial balance sheet is a call on the flow of future contributions to finance the liability already accrued in the system, where the liability is defined as the present value of future pension benefits to all persons to whom the system has a liability at the time of valuation, minus the present value of future contributions by the same individuals. For a fully funded system, the entry of contribution asset would be omitted.

Entries on the liabilities side

Under the Swedish Inkomstpension system, the liability to contributors who have not begun to draw a pension at time t (the time of measurement), AL_t^c , is valued as the sum of the notional account balances of all contributors at that time.

The liability to pensioners at time t , AL_t^p , is calculated as the sum of the pensions payable to each age group times the life expectancy of that age group at time t . The life expectancy is discounted to reflect the indexation of pensions by the increase in the income index or balance index with a reduction of 1.6%. The resulting life expectancy in number of years is termed as the "economic annuity divisor"; see Appendix B to the Orange Report.

Entries on the assets side

The novel entry on the actuarial balance sheet of the Swedish pension system is the "contribution asset" introduced by Settergren (Settergren, 2001) (Settergren & Mikula, 2005). The formula to calculate the contribution asset at time t , CA_t , is:

$$CA_t = TD \cdot C_t \quad (1)$$

where :

TD is the turnover duration, which is the time expected to pass from when a monetary unit enters the system as a contribution until it leaves in the form of a pension, and

C_t is the contribution revenue for the year beginning at time t .

This formula is derived on the premise of a pay-as-you-go system that is in a steady-state⁴. The turnover duration indicates the size of the pension liability that the present contribution flow can finance, given the present income and mortality patterns and the population growth rate⁵.

With the establishment of the above actuarial balance sheet, a social security system is said to be "sustainable" at time t as long as:

$$F_t + CA_t \geq AL_t^r + AL_t^a \quad (2)$$

where F_t is the fair value of the financial and real assets held within the system. This condition implies that the participants (both current and future) would have a reasonable expectation of receiving the benefits that have been committed to, without the sponsor of the system (i.e., the state) having to make non-statutory contributions.

By defining the *balance ratio* (BR_t) of the system as:

$$BR_t = \frac{F_t + CA_t}{AL_t^r + AL_t^a} \quad (3)$$

it can be said that the system is sustainable as long as the balance ratio is greater than or equal to 1.0. On the other hand, if the balance ratio is less than 1.0, the system is considered to be "unsustainable", which means that at some point in the future the sponsor might be forced to allocate additional funds to cover the deficit or that the promises made to the participants might be at least partially broken.

The accumulated deficit of the system at time t is defined as:

$$D_t \equiv (AL_t^r + AL_t^a) - (F_t + CA_t), \text{ if } > 0 \quad (4)$$

A negative value of D_t means that the system is in surplus.

⁴ In the context of the Swedish pension system, a steady state is defined as a situation where the average wage at each age, relative to the average wage of all ages, is constant over time and where the number of retirees at each age, relative to the total number of retirees, is constant over time (i.e., mortality rates are constant).

⁵ The turnover duration can be viewed as representing the period over which the liability of a steady-state system would be amortized by the present contribution flow. It has been shown to be equal to the difference between (i) the average weighted age for the pensioners (weighted by the amount of annual pensions considering the age-benefit profile), and (ii) the average weighted age for the contributors (weighted by the amount of real contributions considering the age-income profile). (Settergren & Mikula, 2005)

4.2 An actuarial balance sheet for target benefit plan

If the sponsor of a TBP has a fixed contribution commitment, any funding shortfall resulted from economic or demographic changes could be eliminated by either (i) reducing the benefits payable, or expected to be payable, to current plan members, or (ii) by relying on some forms of intergenerational risk sharing that provide less benefits for future plan members in relation to the contributions made for them, or a combination of both. Any reliance on intergenerational risk sharing must be made transparent to plan stakeholders including, in particular, current plan members as well as employees who are eligible to join the plan. If transparency is lacking in this regard, sustainability of the plan may potentially be jeopardized.

Suppose a TBP has adopted a policy which calls for risk-sharing between current members and future generations of members who join the plan over the next N years. We will develop an actuarial balance sheet for this plan based on the structure shown in Table 1.

Entries on the liabilities side

In the context of a TBP, the liability to pensioners at time t , AL_t^r , would include also the liability for other inactive members with immediate or deferred entitlements. Following the standard procedures of actuarial mathematics, this liability is calculated as the present value of the pension benefits payable to pensioners and other inactive members.

Liability to contributors at time t , AL_t^a , is the liability for all active members at that time. It is calculated as the present value of future benefits (i.e., projected benefits payable at termination, death, retirement, etc.) less the present value of future contributions based on the fixed contribution rate set out in the plan. The formula to calculate this liability is as follows:

$$AL_t^a = \sum_{j \in A_t} (PVFB_t^j - PVFC_t^j) \quad (5)$$

where A_t is the set of active members at time t , $PVFB_t^j$ is the present value of future benefits for member j at time t and $PVFC_t^j$ is the present value of future contributions for member j at time t .

The liability for active members can be decomposed into the following two components:

- The *past service liability*, PSL_t^a , being the present value of accrued benefits, and
- The *future service liability*, FSL_t^a , being the difference between the present value of benefits expected to accrue for service after time t and the present value of future contributions.

In order to present an actuarial balance sheet for the plan in an *actuarially* unbiased manner, the liabilities for plan members should be calculated using best-estimate

actuarial assumptions, with no margins included, that are applicable at the time of valuation. In particular, the discount rate used to calculate the present values should reflect the rate of return, net of investment expenses, that the pension fund is expected to earn over the long term.

Entries on the assets side

Apart from the financial and real assets held in the pension fund, contributions made for future members could be used to finance the liability for current members pursuant to the plan's risk-sharing policy. In this regard, we define a "contribution asset" at time t reflecting the plan's risk-sharing policy as follows:

$$CA_t = \sum_{n=1}^N \left[\sum_{j \in G_{t+n}} (PVFC_{t+n}^j - PVFB_{t+n}^j) \right] \cdot (1+i)^{-n}, \quad (6)$$

where:

$$n = 1, 2, \dots, N,$$

G_{t+n} is the generation of plan members⁶ who enter the plan at time $t+n$,

$PVFC_{t+n}^j$ is the present value at time $t+n$ of planned future contributions for member j in G_{t+n} ,

$PVFB_{t+n}^j$ is the present value at time $t+n$ of future projected benefits for member j in G_{t+n} , and

i is the discount rate used to calculate present values at time t

In applying Equation (6) to calculate the contribution asset, we need to make assumptions about future generations of members (e.g., number of new entrants, age distribution at plan entry, initial salaries, etc.) The selected assumptions should be consistent with the expected future experience of the plan.

Unlike the contribution asset for the Swedish pension system determined by Equation (1), the contribution asset for a TBP defined by Equation (6) does not rely on a steady-state assumption. A positive contribution asset obtained from Equation (6) is analogous to the hidden taxes that a pay-as-you-go pension system will apply to its participants in the future, whether in the form of excess contributions in relation to the pensions to be provided or in the form of lower pensions in relation to the contributions (Vidal-Meliá & Boado-Penas, 2013).

A positive contribution asset implies that the planned contributions for future plan members are projected to be more than sufficient to pay the benefits expected to be earned by them and that there are excess amounts for contributing to the financial

⁶ For ease of presentation, it is assumed that new members join the plan at the beginning of the year following the year of hire.

balance of the plan. A negative contribution asset implies that the contributions are insufficient.

The main entries on the actuarial balance sheet of a TBP are shown in Table 2 below:

Table 2: Main Entries on the Actuarial Balance Sheet of a TBP⁷

Assets	Liabilities
Fund assets (F_t)	Liability for pensioners and other inactive members (AL_t^a)
Contribution asset (CA_t)	Past service liability for active members (PSL_t^a)
Accumulated deficit (surplus) (D_t)	Future service liability for active members (FSL_t^a)
Total assets	Total liabilities

If a plan does not employ a policy which calls for risk-sharing between current members and future generations of members, the entry of contribution asset on the above actuarial balance sheet would be omitted. On the other hand, if a plan is assumed to continue on an indefinite basis and there is risk-sharing between current members and *all* future generations of members, the upper limit of the first summation in Equation (6) would be set equal to infinity, i.e., $N = \infty$. However, this type of risk-sharing is unlikely to be legally permitted in Canada (especially in the case of non-governmental pension plans.)

The above actuarial balance sheet provides valuable information about intergenerational risk sharing employed by the plan. It shows how the target benefit obligations are distributed among members, the sources of financing that comprise the pension fund assets and a contribution asset from future generations of members (if applicable), as well as the funding *shortfall* or *excess* at the time of valuation.

Balance ratio and balancing mechanisms

The *balance ratio* of the plan at time t is defined as follows:

$$BR_t \equiv \frac{F_t + CA_t}{AL_t^a + PSL_t^a + FSL_t^a} \quad (7)$$

If $BR_t \geq 1$, the plan is expected to be sustainable at least over the next N years without recourse to any increase in contributions or reduction in benefits.

Demographic and economic developments will change the balance ratio of a TBP from time to time. When the balance ratio falls significantly below 1.0, the benefits of current members and those of future members can be adjusted to restore the plan's financial balance. Benefit adjustment can be applied to future service accruals, indexation of

⁷ For simplicity, it is assumed that the employer pays all of the administrative costs other than investment expenses outside the plan fund.

pensions in pay if provided, updates of the earnings base in a career average earnings plan, or even the accrued benefits. Such adjustments will affect the value of one or more of the following entries on the actuarial balance sheet: PSL_t^a , FSL_t^a , AL_t^r and CA_t .

Benefit adjustment is not the only mechanism that could affect a plan's financial balance. Other mechanisms may include the following:

- Employer (and employee, if applicable) contributions could vary within a predefined and, presumably reasonably narrow, range. Such a change would affect the following values: FSL_t^a and CA_t .
- Change of investment policy - this could change the discount rate used to calculate present values, which in turn would affect the following values: AL_t^r , PSL_t^a , FSL_t^a and CA_t .
- Change of eligibility rules for benefits, e.g., an increase in the normal retirement age - this would affect the following values: PSL_t^a , FSL_t^a and CA_t .
- Use of a contingency fund, created and run in parallel to the pension fund, to meet any funding shortfall resulted from economic or demographic losses - this would affect the value of F_t .

The trigger for balancing mechanism may include a "no action" range of BR_t around 1.0, i.e., $1.0 - \mu < BR_t < 1.0 + \delta$, where μ and δ are positive amounts. If BR_t falls outside the range, the balancing mechanism may restore it to within the no action range; in particular, the ratio could be restored to 1.0 or to the nearest edge of the range. The range of balancing actions should be documented in the plan's risk management framework.

If the methodology used to assess the financial position of a TBP is based on a closed group unit credit cost method (projected or unprojected), the change in the funded ratio of the plan resulted from any benefit adjustment would only reflect the impact the adjustment has on AL_t^r and/or PSL_t^a . It does not provide any information about impacts on the future service liability for current active members (FSL_t^a), nor the contribution asset from future generations of members participating in risk-sharing (CA_t). On the other hand, the actuarial balance sheet developed above would show the full impact of a balancing mechanism on the plan's *net* worth. The full and transparent disclosure of cost impact will help stakeholders identify proper measures to address any financial imbalance in the plan.

The accumulated deficit of the plan at time t (D_t) is equal to:

$$(AL_t^r + PSL_t^a + FSL_t^a) - (F_t + CA_t) \quad (8)$$

Over any one-year period t to $t + 1$, the accumulated deficit at time $t + 1$ can be reconciled to that at time t in the manner described below:

- (a) First determine the expected accumulated deficit at time $t + 1$:

$$E(D_{t+1}) = D_t \cdot (1+i) - \sum_{j \in G_{t+N+1}} (PVFC_{t+N+1}^j - PVFB_{t+N+1}^j) \cdot (1+i)^{-N} \quad (9)$$

where i is the discount rate used to calculate liabilities at time t .

The last term in Equation (9) represents the expected contribution to the plan's financial balance at time $t+1$ from future members who enter the plan at time $t+N+1$. It will be omitted from the equation if members who join the plan after time $t+N$ are not required to share risk with their predecessors.

- (b) Then determine the actual accumulated deficit at time $t+1$:

$$D_{t+1} = (AL_{t+1}^r + PSL_{t+1}^a + FSL_{t+1}^a) - (F_{t+1} + CA_{t+1}) \quad (10)$$

- (c) Finally, calculate the gain or loss for the period:

$$\text{Gain (Loss)} \equiv E(D_{t+1}) - D_{t+1} \quad (11)$$

4.3 Other financial indicators

There are two other financial indicators which are useful for monitoring the financial health of target benefit plans. The first indicator, which we call the *current funded ratio*, is defined as follows:

$$CFR_t \equiv \frac{F_t}{AL_t^r + PSL_t^a}, \quad \text{for any time } t \quad (12)$$

This funded ratio is calculated on the premise that: (i) no new entrants to the plan are permitted, and (ii) current active members who are not receiving benefits at the valuation date are assumed to have no further contributions made for them beyond that date, and hence accrue no further benefits. It is the same as the *going concern funded ratio* typically reported in the actuarial valuation reports for DB plans in Canada, which measures the extent to which the liability for benefits already accrued by current active and inactive members is covered by the assets in the pension fund.

Another indicator, which we call the *termination funded ratio*, is also calculated using Equation (12) except that the past service liability is determined based on the members' actual salaries prior to the valuation date (if the benefits are salary-related), instead of projected salaries at retirement. This ratio measures the funded position of the plan if it were terminated on the valuation date. Contrary to the solvency ratio prescribed in Canadian pension legislation, the termination funded ratio is not calculated based on settlement assumptions. Rather, it is calculated on the assumption that the plan would continue in existence until the last benefit is paid out. The termination funded ratio may be used to determine the benefit entitlements for those members who terminate before retirement and elect to receive their benefits in a lump sum.

5. Numerical Illustration

In this section, we illustrate the use of the actuarial balance sheet developed in Section 4 for a stylized target benefit plan.

The predecessor of the TBP is a defined benefit pension plan with the following provisions:

- Benefit formula: 1.5% of final year pay per year of pensionable service
- Employee contributions: none
- Normal retirement date: attainment of age 65
- Earliest retirement date: age 55 with 2 years of service
- Pre-retirement death benefits: lump sum equal to the present value of the benefits to which the member would have been entitled had employment been terminated on the date of death
- Termination benefits: lump sum equal to the present value of accrued benefits based on the member's pay and years of service to the date of termination
- Normal form of pension: lifetime pension payable monthly
- Early retirement benefit: actuarial equivalent to normal retirement benefit
- Indexation: none

The demographic parameters and assumptions used to derive the numerical results presented in this section are set out in Appendix C.

Immediately prior to the conversion of the plan to a TBP, the plan's funded position and normal cost were as follows.

Table 3: Going Concern Financial Status

	<i>(\$ Million)</i>
<u>Assets</u>	
Market value of assets	4,379
<u>Liabilities</u>	
• Active members	2,930
• Pensioners	<u>1,449</u>
Total liabilities	4,379
Funding excess (shortfall)	Nil
Going concern funded ratio (assets ÷ liabilities)	1.0

Author's calculation

Normal cost: \$152 million or 11.6% of pay

The above valuation results were derived based on the unit credit cost method with projection of pay and a mortality assumption that does not allow for improvements in future mortality.

5.1 Design of target benefit plan

Assume that the TBP sponsor has adopted a policy that calls for risk-sharing between current members and future generations of members who join the plan over the next 15 years. With a contribution rate of 11.6% of pay (i.e., the UC normal cost rate) and using the approach described in Section 4, we obtain the following actuarial balance sheet for the plan:

Table 4: Actuarial Balance Sheet For Target Benefit Plan

Assets	\$ Million	Liabilities	\$ Million
Market value of fund assets	4,379	Liability for pensioners	1,449
Contribution asset	52	Past service liability for active members	2,930
Accumulated deficit (surplus)	120	Future service liability for active members	172
Total assets	4,551	Total liabilities	4,551

Author's calculation

The future service liability for current active members is estimated to be \$172 million. This means that, at a contribution rate of 11.6% of pay, the expected future contributions for current active members would fall short of their liability for future benefit accruals by \$172 million. This funding shortfall can be reduced or eliminated by either reducing future benefit accruals or raising the contribution rate, or both. In addition, through intergenerational risk sharing, all or part of that shortfall can be met by the contributions made for future generations of members.

The contribution asset on the actuarial balance sheet in Table 4 is that derived from new entrants who are expected to join the plan over the next 15 years. After taking account of this contribution asset, the plan still has an accumulated deficit of \$120 million and its balance ratio is 0.974. This implies that the existing plan design is not expected to be sustainable, and that some adjustments to the plan parameters would be necessary.

The following design options for the plan's contribution rate and future benefit accrual rate are considered:

- Option 1a - Decrease of contribution rate from 11.6% of pay to 11.0% of pay
- Option 1b - Decrease of contribution rate to 11.5% of pay
- Option 1c - Increase of contribution rate to 12.0% of pay
- Option 2a - Decrease of future benefit accrual rate from 1.5% to 1.45%
- Option 2b - Decrease of future benefit accrual rate to 1.40%
- Option 2c - Decrease of future benefit accrual rate to 1.35%

Table 5 shows the actuarial balance sheet of the original TBP and those emerged from the implementation of the above options.

Table 5: Actuarial Balance Sheets and Financial Indicators

All assets & liabilities in \$million	Original TBP	Design Option					
		1a	1b	1c	2a	2b	2c
<i>Assets</i>							
• Market value of fund assets	4,379	4,379	4,379	4,379	4,379	4,379	4,379
• Contribution asset	52	11	43	75	75	98	121
• Accumulated deficit (surplus)	<u>120</u>	<u>270</u>	<u>151</u>	<u>32</u>	<u>23</u>	<u>(73)</u>	<u>(169)</u>
Total assets	4,551	4,660	4,573	4,486	4,477	4,404	4,331
<i>Liabilities</i>							
• Liability for pensioners	1,449	1,449	1,449	1,449	1,449	1,449	1,449
• Past service liability for active members	2,930	2,930	2,930	2,930	2,930	2,930	2,930
• Future service liability for active members	<u>172</u>	<u>281</u>	<u>194</u>	<u>107</u>	<u>98</u>	<u>25</u>	<u>(48)</u>
Total liabilities	4,551	4,660	4,573	4,486	4,477	4,404	4,331
<i>Financial indicators</i>							
Balance ratio	0.974	0.942	0.967	0.993	0.995	1.017	1.039
Current funded ratio	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Contribution asset ÷ Total liabilities (%)	1.1%	0.2%	1.0%	1.7%	1.7%	2.2%	2.8%

Author's calculation

It can be seen that in order for the plan to achieve financial balance closely through an increase in the contribution rate, the rate would have to increase from 11.6% of pay to 12.0% of pay (Option 1c). The resulting balance ratio is 0.993. Financial balance could also be closely achieved through a decrease in future benefit accrual rate from 1.5% to 1.45% (Option 2a), in which case, the resulting balance ratio is 0.995.

Table 6 shows the effect on the plan's balance ratio of different combinations of design options taken two at a time.

Table 6: Balance Ratios for Different Combinations of Design Options

Design Option	2a	2b	2c
1a	0.962	0.983	1.004
1b	0.988	1.009	1.032
1c	1.015	1.037	1.060

Author's calculation

Two possibilities to achieve financial balance if two design options were taken simultaneously would be: (i) a decrease in the contribution rate to 11.0% of pay and a decrease in the future benefit accrual rate to 1.35% (Option 1a & Option 2c), with a resulting balance ratio of 1.004, or (ii) a decrease in the contribution rate to 11.5% of pay and a decrease in the future benefit accrual rate to 1.40% (Option 1b & Option 2b), with a resulting balance ratio of 1.009. Either combination would leave a small surplus margin in the plan.

For illustration purposes, it is assumed that the sponsor has chosen a combination of Option 1b and Option 2b for the design of TBP, i.e.,

- Target benefit formula:
 - Service prior to plan conversion - 1.50% of final year pay per year of pensionable service
 - Service subsequent to plan conversion - 1.40% of final year pay per year of pensionable service
- Fixed rate of employer contributions: 11.5% of member's pay

The plan's balance ratio is 1.009 and its actuarial balance sheet is as follows:

Table 7: Actuarial Balance Sheet For Target Benefit Plan

Assets	<i>\$ Million</i>	Liabilities	<i>\$ Million</i>
Market value of fund assets	4,379	Liability for pensioners	1,449
Contribution asset	90	Past service liability for active members	2,930
Accumulated deficit (surplus)	(42)	Future service liability for active members	48
Total assets	4,427	Total liabilities	4,427

Author's calculation

The contribution asset from future members equals approximately 2.0% (i.e., $90 \div 4,427$) of the liabilities for current pensioners and active members.

5.2 Application of balancing mechanisms

If it is decided that the mortality assumption underlying the actuarial balance sheet of the TBP should allow for improvements in future mortality, the inclusion of a mortality improvement scale would result in the following revised actuarial balance sheet:

Table 8: Revised Actuarial Balance Sheet For Target Benefit Plan

Assets	<i>\$ Million</i>	Liabilities	<i>\$ Million</i>
Market value of fund assets	4,379	Liability for pensioners	1,479
Contribution asset	13	Past service liability for active members	3,078
Accumulated deficit (surplus)	359	Future service liability for active members	194
Total assets	4,751	Total liabilities	4,751

Author's calculation

The balance ratio of the plan drops to 0.925 from 1.009. It will be necessary to invoke some benefit adjustments to restore the plan's financial balance, if the plan's fixed contribution rate of 11.5% of pay is to be maintained.

The plan's current funded ratio (i.e., the funded ratio based on the closed group unit credit cost method) is 0.961, and the UC normal cost rate is 11.497% of pay (which,

incidentally, is virtually identical to the plan's fixed contribution rate.) Had the unit credit cost method been used for determining funding adequacy, a reduction of the past benefit accrual rate from 1.5% to 1.44% (i.e., $1.5\% \times .961$) and a reduction of pensions in pay by 3.9% would be deemed to be sufficient to restore the plan to a fully funded status. No change to the future benefit accrual rate would be required. It will be shown that such a change (i.e., Option 1 below) is not an effective balancing measure.

Consider the following alternative balancing options:

Balancing Option	Description	Past benefit accrual rate	Future benefit accrual rate	Reduction of Pensioners' benefits (%)
1	<ul style="list-style-type: none"> Adjust accrued benefits only Restore current funded ratio to 1.0 	1.44%	1.40%	3.9%
2	<ul style="list-style-type: none"> Proportionate benefit adjustments reflecting impacts of mortality assumption change Restore balance ratio to 1.0 	1.43%	1.31%	2.0%
3	<ul style="list-style-type: none"> Preserve accrued benefits for current pensioners only Restore balance ratio to 1.0 	1.43%	1.30%	Nil
4	<ul style="list-style-type: none"> Preserve accrued benefits for both current pensioners and active members Restore balance ratio to 1.0 	1.50%	1.23%	Nil

Table 9 shows the actuarial balance sheets that emerged after implementation of the above balancing measures.

Table 9: Actuarial Balance Sheets and Financial Indicators

All assets & liabilities in \$million	Balancing Option			
	1	2	3	4
<i>Assets</i>				
• Market value of fund assets	4,379	4,379	4,379	4,379
• Contribution asset	13	58	65	101
• Accumulated deficit (surplus)	<u>181</u>	<u>0</u>	<u>0</u>	<u>0</u>
Total assets	4,573	4,437	4,444	4,481*
<i>Liabilities</i>				
• Liability for pensioners	1,421	1,449	1,479	1,479
• Past service liability for active members	2,958	2,930	2,930	3,078
• Future service liability for active members	<u>194</u>	<u>58</u>	<u>35</u>	<u>(76)</u>
Total liabilities	4,573	4,437	4,444	4,481
<i>Financial indicators</i>				
Balance ratio	0.960	1.000	1.000	1.000
Current funded ratio	1.000	1.000	0.993	0.961
Contribution asset ÷ Total liabilities (%)	0.3%	1.3%	1.5%	2.3%

Author's calculation

* rounding difference

The benefit changes under Option 1 would improve the plan's balance ratio to 0.960, which is still significantly below 1.0. Options 2, 3 and 4 are effective measures to bring the plan into financial balance. Option 2 requires proportionate adjustments to benefits for current pensioners and active members as well as future members, and may be perceived to be most equitable. Option 4 preserves the accrued benefits for both current pensioners and active members, but requires current active and future members to bear the entire cost of expected future mortality improvements, in the form of lower future benefit accruals.

5.3 A special case

For the target benefit plan considered in Section 5.1, it has been assumed that all future generations of members will have the same number of members and age distribution. Their initial pay at plan entry will grow at a constant rate γ (4% in the example) by generation. In addition, members are subject to constant mortality rates which do not improve over time.

Given the design of the TBP and the characteristics of future generations described above, the following relationship would hold:

$$CA(n) = CA(1) \cdot (1 + \gamma)^{n-1}, \quad n = 1, \dots, N \quad (13)$$

where $CA(n)$ is the contribution asset attributable to the n^{th} generation of members, i.e., $CA(n) = \sum_{j \in G_n} (PVFC_n^j - PVFB_n^j)$. The contribution asset for the plan, as defined by Equation (6), can then be expressed as follows:

$$CA = \sum_{n=1}^N [CA(1) \cdot (1 + \gamma)^{n-1}] \cdot (1 + i)^{-n} = CA(1) \cdot \frac{1 - \left(\frac{1 + \gamma}{1 + i}\right)^N}{i - \gamma} \quad (14)$$

If $\gamma < i$ and $N \rightarrow \infty$, the above formula reduces to:

$$CA = CA(1) \cdot \left(\frac{1}{i - \gamma}\right) \quad (15)$$

This means that the contribution asset can be defined as the present value of a perpetual annual fixed amount discounted by an interest rate equal to $i - \gamma$.

Before any plan design changes, the initial TBP had a benefit accrual rate of 1.5%, for both past and future service, and a fixed contribution rate of 11.6%. If the plan required current members and *all* future generations of members to participate in risk-sharing, the contribution asset from future members, calculated using Equation (15) with $i = 6\%$ and $\gamma = 4\%$, would amount to \$208 million. The actuarial balance sheet in Table 4 could then be restated as follows:

Table 10: Actuarial Balance Sheet For Target Benefit Plan

Assets	<i>\$ Million</i>	Liabilities	<i>\$ Million</i>
Market value of fund assets	4,379	Liability for pensioners	1,449
Contribution asset	208	Past service liability for active members	2,930
Accumulated deficit (surplus)	(37)	Future service liability for active members	172
Total assets	4,551	Total liabilities	4,551

Author's calculation

In this special case, excess contributions from future generations of members were expected to be sufficient to cover the future service liability for current members and the plan's balance ratio was 1.008. As the plan was already in financial balance, no change to the plan parameters (namely, the benefit accrual rate and contribution rate) would be considered necessary.

6. Concluding Remarks

We contend in this paper that the *closed group* unit credit cost method that has been traditionally applied to defined benefit pension plans does not produce a candid financial indicator for target benefit plans. As analyzed in Section 3 and demonstrated by the numerical example in Section 5, the fact that the funded ratio of a TBP calculated based on the unit credit cost method is 1.0 produces a "mirage effect" to plan stakeholders. This is because a funded ratio calculated as such would hide the presence of any potential deficit (or surplus) associated with future benefit accruals for current and/or future members under the plan's fixed contribution formula. In order to assess whether or not a TBP is sustainable over the long term, an actuarial balance sheet that takes into account future contributions and benefit accruals of current and/or future members should be compiled.

For the assessment of the financial sustainability of its pay-as-you-go pension system, Sweden has developed a "contribution asset" that links the assets and liabilities on the actuarial balance sheet of its system. That contribution asset is derived based on the assumption that the system is in a steady-state. We have adapted this actuarial balance sheet methodology to target benefit plans by defining a *contribution asset* that does not rely on a steady-state assumption and that reflects the policy of risk-sharing between current and future members employed by the plan. The ratio of assets to liabilities of the plan is called the *balance ratio*, where assets are comprised of financial and real assets held in the pension fund and a contribution asset pertaining to future generations of members, if applicable. This ratio provides an indication of the financial sustainability of the plan and serves as a trigger for the application of corrective measures to bring the plan into financial balance. The approach has been shown to provide a high level of transparency to plan stakeholders as regards intergenerational risk sharing.

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Appendix A: Summary of New Brunswick and Alberta Funding Regulations

Feature	New Brunswick (SRP)	Alberta (TBP)
Benefit structure	<p>Benefits are split into two parts:</p> <ul style="list-style-type: none"> • Base benefits which comprise pensions in pay and vested ancillary benefits • Ancillary benefits which may include escalated adjustments before and after retirement, early retirement subsidies and subsidized optional form of pension 	No specific requirements
Risk management goals	<ul style="list-style-type: none"> • Base benefits must be very strongly funded such that there is at least 97.5% probability that base benefits will not be reduced over a 20-year period, after taking into account: (a) the funding deficit recovery plan, and (b) the funding excess utilization plan • Ancillary benefits are required to be strongly funded such that, on average over a 20-year period, at least 75% of such benefits will be provided 	No explicit risk management goals other than stress testing satisfactory to the Superintendent (guidelines not yet released)
Contributions	Normal cost + admin expenses + an additional amount so that the risk management goals are met	<ul style="list-style-type: none"> • Normal cost + amortization of unfunded liabilities + prescribed PfAD⁸ + admin expenses • PfAD applies to normal cost but not to unfunded liability amortization payments
Funding method	<ul style="list-style-type: none"> • Unit credit cost method; projection of pay increases if benefits are in reference to final or final few years of pay before retirement • Discount rate not prescribed, but must be consistent with the plan's funding policy, investment 	<ul style="list-style-type: none"> • Unit credit cost method; projection of pay increases if benefits are in reference to final or final few years of pay before retirement • Prescribed benchmark discount rate (BDR) as baseline; higher PfAD if higher rate is used

⁸ The PfAD, provision for adverse deviations, is a percentage determined by reference to the plan's asset mix policy (higher PfAD for higher equity content) and discount rate that is in excess of a prescribed benchmark discount rate.

Feature	New Brunswick (SRP)	Alberta (TBP)
	policy and risk management goals and procedures <ul style="list-style-type: none"> • Generational mortality tables • Other assumptions must be consistent with plan experience 	
Funding deficit recovery plan	<ul style="list-style-type: none"> • A funding deficit recovery plan must be implemented if the open group funded ratio falls below 1.0 in two successive actuarial valuations • Open group funded ratio is the ratio of (i) over (ii), where (i) is the market value of plan assets plus the value of planned future contributions in excess of the normal costs on an "open group" basis over no more than 15 years, and (ii) is the plan's liabilities which is the present value of the past base and ancillary benefits. 	Unfunded liability is required to be amortized over the shorter of 15 years or EARSL
Use of funding excess	Permitted if open group funded ratio > 105%	Excess of fund assets over actuarial liability * (1 + prescribed PfAD)

Appendix B: Cost Patterns Under Unit Credit Cost Method vs. Entry Age Normal Cost Method

Suppose a plan provides a salary-related pension at the normal retirement age and there are no ancillary benefits payable under the plan. We will prove that, for any active member in the plan:

- (a) The UC liability for the member will always be less than the member's EAN liability; and
- (b) The UC normal cost for the member will be less than the member's EAN normal cost before a certain age, but will be greater than the EAN normal cost after that age.

The following notation will be used in the proof:

$B(y)$ is the projected annual pension (payable monthly for life) when a member reaches the normal retirement age y

$\ddot{a}_y^{(12)}$ is the life annuity factor at age y

i is the discount rate used in the valuation, and v is the inverse of $1+i$

s is the assumed salary increase rate, which is assumed to be less than i

θ is the normal cost rate determined in accordance with the entry age normal cost method. (Normal costs are assumed to be paid at the beginning of each year.)

e is the age at which a member joins the plan

x is the attained age of the member at the date of valuation and is less than y

S_x is the member's annual salary at age x

${}^s\ddot{a}_{x|y-x}$ is a temporary life annuity calculated using a service table and a net interest rate of $\left(\frac{1+i}{1+s}\right) - 1$

${}_{y-x}p_x$ is the probability that a member currently aged x will remain in the plan until age y , computed using a service table

(a) UC liability vs. EAN liability

Proof:

The UC liability for a member, AL^{UC} , can be written as follows :

$$AL^{UC} = \left[\left(\frac{x-e}{y-e} \right) B(y) \right] \cdot v^{y-x} \cdot {}_{y-x}p_x \cdot \ddot{a}_y^{(12)} \quad (16)$$

To determine the EAN liability for the member, AL^{EAN} , we need to first calculate the member's normal cost (Anderson, 1992):

$$NC^{EAN} = \theta \cdot S_x = \left(\frac{B(y) \cdot v^{y-e} \cdot {}_{y-e}p_e \cdot \ddot{a}_y^{(12)}}{{}^s\ddot{a}_{e:y-e}} \right) \cdot (1+s)^{x-e} \quad (17)$$

By the prospective definition of accrued liability (i.e., present value of future benefits minus present value of future normal costs), we obtain:

$$\begin{aligned} AL^{EAN} &= B(y) \cdot v^{y-x} \cdot {}_{y-x}p_x \cdot \ddot{a}_y^{(12)} - NC^{EAN} \cdot {}^s\ddot{a}_{x:y-x} \\ &= \left(B(y) \cdot v^{y-x} \cdot {}_{y-x}p_x \cdot \ddot{a}_y^{(12)} \right) \cdot \left(\frac{{}^s\ddot{a}_{e:x-e}}{{}^s\ddot{a}_{e:y-e}} \right) \end{aligned} \quad (18)$$

From Equations (16) and (18), it can be seen that:

$$AL^{EAN} > AL^{UC} \quad (19)$$

if and only if:

$$\left(\frac{{}^s\ddot{a}_{e:x-e}}{{}^s\ddot{a}_{e:y-e}} \right) > \left(\frac{x-e}{y-e} \right)$$

if and only if:

$$\left(\frac{{}^s\ddot{a}_{e:x-e}}{x-e} \right) > \left(\frac{{}^s\ddot{a}_{e:y-e}}{y-e} \right) \quad (20)$$

Since

$${}^s\ddot{a}_{e:x-e} = \sum_{z=e}^{x-1} \left(\frac{1+s}{1+i} \right)^{z-e} \cdot {}_{z-e}p_e$$

and the expression in the summation: $f(z) = \left(\frac{1+s}{1+i} \right)^{z-e} \cdot {}_{z-e}p_e$ is a decreasing function of $z \in [e, y)$, it is clear that the inequality in Equation (20) holds for $x < y$ and hence the inequality in Equation (19) is true.

The proof is complete.

(b) UC normal cost vs. EAN normal cost

Proof:

The UC normal cost (NC^{UC}) and the EAN normal cost (NC^{EAN}) for a member can be written, respectively, as follows:

$$NC^{UC} = \left[\left(\frac{1}{y-e} \right) B(y) \right] \cdot v^{y-x} \cdot {}_{y-x}p_x \cdot \ddot{a}_y^{(12)} \quad (21)$$

$$\begin{aligned} NC^{EAN} &= \theta \cdot S_x = \left(\frac{B(y) \cdot v^{y-e} \cdot {}_{y-e}p_e \cdot \ddot{a}_y^{(12)}}{{}_s\ddot{a}_{e|y-e|}} \right) \cdot (1+s)^{x-e} \\ &= \left[B(y) \cdot v^{y-x} \cdot {}_{y-x}p_x \cdot \ddot{a}_y^{(12)} \right] \cdot \left[\frac{\left(\frac{1+s}{1+i} \right)^{x-e} \cdot {}_{x-e}p_e}{{}_s\ddot{a}_{e|y-e|}} \right] \end{aligned} \quad (22)$$

Taking the ratio of NC^{UC} to NC^{EAN} , we obtain the following:

$$\frac{NC^{UC}}{NC^{EAN}} = \frac{\left[\frac{{}_s\ddot{a}_{e|y-e|}}{y-e} \right]}{\left[\left(\frac{1+s}{1+i} \right)^{x-e} \cdot {}_{x-e}p_e \right]} = \frac{(\sum_{z=e}^{y-1} f(z))/(y-e)}{f(x)}, \quad (23)$$

where $f(z) = \left(\frac{1+s}{1+i} \right)^{z-e} \cdot {}_{z-e}p_e$ is a decreasing function of z . There exists a value x^* between e and y such that $f(x^*)$ is equal to the average value of $f(z)$: $(\sum_{z=e}^{y-1} f(z))/(y-e)$. Since $f(x) > f(x^*)$ for $x < x^*$ and $f(x) < f(x^*)$ for $x > x^*$, it follows that $NC^{UC} < NC^{EAN}$ for $x < x^*$ and $NC^{UC} > NC^{EAN}$ for $x > x^*$.

The proof is complete.

Appendix C: Parameters and Assumptions

A. Current Membership Profile

Active Members			
Age	Number	Average service	Average annual salary
25 to 29	700	3.0	\$48,000
30 to 34	1,300	4.0	56,000
35 to 39	1,800	6.0	62,000
40 to 44	2,400	10.0	67,000
45 to 49	3,400	15.0	70,000
50 to 54	3,600	20.0	76,000
55 to 59	3,300	25.0	78,000
60 to 64	2,000	32.0	81,000
Total	18,500	16.8	\$70,800

Pensioners		
Age	Number	Average annual pension
65 to 69	3,000	\$26,100
70 to 74	2,000	21,000
75 to 79	1,350	18,000
80 to 84	850	15,000
85 to 89	450	13,000
90 to 94	120	10,000
Total	7,770	\$21,200

B. Actuarial Assumptions

Discount rate (<i>net of investment expenses</i>)	6.0% per annum
Salary increase rate	4.0% per annum
Pre-retirement decrements	None
Retirement age	Age 65
Administrative expenses other than investment expenses	Paid by employer directly

Mortality table with no improvement scale, generating the following life annuity factors:

Age	65	66	67	68	69	72	77	82	87	92
Annuity factor	10.77	10.51	10.25	9.99	9.71	8.86	7.30	5.77	4.41	3.23

Mortality table with an improvement scale, generating the following applicable life annuity factors:

1. For current pensioners

Current age	65	66	67	68	69	72	77	82	87	92
Annuity factor	11.03	10.76	10.49	10.21	9.93	9.03	7.41	5.84	4.44	3.24

2. For current active members

Current age	27	32	37	42	47	52	57	62
Age 65 annuity factor	11.96	11.85	11.74	11.62	11.50	11.37	11.24	11.11

3. For future active members

Generation	1	2	3	4	5	6	7	8
Age 65 annuity factor	11.91	11.94	11.96	11.98	12.00	12.02	12.04	12.06

Generation	9	10	11	12	13	14	15
Age 65 annuity factor	12.08	12.10	12.12	12.14	12.16	12.18	12.20

C. New Entrants Profile

New Entrants			
Generation	Number	Average entry age	Average annual salary
1	400	30	\$49,900
2	400	30	51,900
3	400	30	54,000
4	400	30	56,200
5	400	30	58,400
6	400	30	60,700
7	400	30	63,200
8	400	30	65,700
9	400	30	68,300
10	400	30	71,100
11	400	30	73,900
12	400	30	76,800
13	400	30	79,900
14	400	30	83,100
15	400	30	86,400

D. Calculation Formulae

Liability for pensioners

The formula to calculate the liability for pensioners is:

$$AL^r = \sum_{j \in R} P_y^j \ddot{a}_y^{(12)}, \quad 65 \leq y < \omega$$

where R is the set of pensioners, $\omega - 1$ is the age of the oldest pensioner, y is the attained age of pensioner j , P_y^j is the amount of pension payable to pensioner j , and $\ddot{a}_y^{(12)}$ is the life annuity factor at age y .

Liability for active members

The formula to calculate the past service liability for active members is:

$$PSL^a = \sum_{j \in A} \beta S_x^j (1+s)^{64-x} k \cdot \ddot{a}_{65}^{(12)} \cdot (1+i)^{-(65-x)}, \quad 20 \leq x \leq 64$$

where A is the set of current active members, β is the benefit rate which is 1.5% in the example, S_x^j is the annual salary for member j , x is the attained age of member j , k is the number of years of pensionable service for member j , s is the pay increase rate and i is the discount rate.

The formula to calculate the future service liability for active members is:

$$FSL^a = \sum_{j \in A} \left(\beta S_x^j (1+s)^{64-x} (65-x) \cdot \ddot{a}_{65}^{(12)} \cdot (1+i)^{-(65-x)} - \theta S_x^j \cdot {}^s \ddot{a}_{65-x} \right), \quad 20 \leq x \leq 64$$

where θ is the plan contribution rate, and ${}^s \ddot{a}_{65-x}$ is a term certain annuity calculated using a net interest rate of $\left(\frac{1+i}{1+s}\right) - 1$. Contributions are assumed to be made at the beginning of each year.

Contribution asset

The formula to calculate the contribution asset from future new members is:

$$CA = \sum_{n=1}^{15} \sum_{j \in G_n} \left[\theta S_e^j \cdot {}^s \ddot{a}_{65-e} - \beta S_e^j (1+s)^{64-e} (65-e) \cdot \ddot{a}_{65}^{(12)} \cdot (1+i)^{-(65-e)} \right] (1+i)^{-n},$$

where G_n is the n^{th} generation of new members, e is the member's plan entry age, $l \leq e \leq u$, and $[l, u]$ is the range of plan entry ages. It is assumed that future members all join the plan at the beginning of the year following the year of hire.