Canadian Institute of Actuaries
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Educational Note Supplement

Calibration of Stochastic Risk-Free Interest Rate Models for Use in CALM Valuation

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Committee on Life Insurance Financial Reporting

May 2017

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Members should be familiar with educational note supplements. Educational note supplements expound or update the guidance provided in an educational note. They do not constitute standards of practice and are, therefore, not binding. They are, however, in conjunction with the source educational note, intended to illustrate the application (but not necessarily the only application) of the Standards of Practice, so there should be no conflict between them. They are intended to assist actuaries in applying standards of practice in respect of specific matters. Responsibility for the manner of application of standards of practice in specific circumstances remains that of the members.

Canadian Institute of Actuaries / Institut canadien des actuaires

MEMORANDUM

To: All life insurance practitioners

From: Pierre Dionne, Chair
Practice Council

Stéphanie Fadous, Chair
Committee on Life Insurance Financial Reporting

Date: May 17, 2017

Subject: **Educational Note Supplement: Calibration of Stochastic Risk-Free Interest Rate Models for Use in CALM Valuation**

The Committee on Life Insurance Financial Reporting (CLIFR), through its Calibration Working Group, has reviewed the development of calibration criteria for stochastic risk-free interest rate models since the last publication to reflect experience through the middle of 2016.

The results and recommendations of the previous working group were published in a research paper in December 2013.

These calibration criteria are directly applicable to Canadian risk-free interest rates or instruments denominated in Canadian dollars, but could be adapted for the U.S. and other developed countries.

The calibration criteria are based on historical interest rate data starting in the 1930s, which were considered sufficient to span a wide range of possible future risk-free interest rate outcomes. This educational note supplement has updated the stochastic risk-free interest rate calibration criteria that were based on historical experience of long-term risk-free interest rates through 2012 to include experience to June 2016. The updated distribution of rates used as the basis for the steady-state calibration criteria showed a decrease in historical experience and calibration criteria at the 2.5^{th}, 5^{th}, and 10^{th} percentile points. As a result, it was decided that it was appropriate to revise the calibration criteria.

As with all items promulgated by the Actuarial Standards Board (ASB), CLIFR intends to review updated experience from time to time, which could lead to revisions to the calibration criteria in the future.

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The focus of this educational note supplement is on the development of calibration criteria for calibrating stochastic risk-free interest rate models used in the production of risk-free interest rate scenarios for the Canadian Asset Liability Method (CALM) valuation of insurance contract liabilities. This may require that a large number of scenarios be generated. For valuation purposes a subset of scenarios or a reduced number of scenarios that are meant to represent the full set of stochastic scenarios may be used. Scenario reduction methodologies are beyond the scope of this paper. The actuary may refer to CIA guidance on the use of approximations, and other literature that is available[1] that deals with scenario reduction techniques.

Finally, CLIFR would like to acknowledge the contribution of the working group and thank the members—Jean-Yves Rioux, Jonathan Boivin, Salina Young, Brock McEwen, and John Campbell—for their efforts. The members have contributed based on their own skills and expertise. The thoughts in the educational note supplement reflect a general consensus view of the members of the working group. Nothing in this document should be construed as expressing the views of any of their employers, nor be considered a view or position regarding the policy of the regulators.

In accordance with the Institute's Policy on Due Process for the Approval of Guidance Material Other than Standards of Practice and Research Papers, this educational note supplement has been prepared by CLIFR, and has received approval for distribution by the Practice Council on May 16, 2017.

Questions or comments regarding this educational note supplement may be directed to Stéphanie Fadous at stephanie_fadous@manulife.com.

PD, SF

[1] The American Academy of Actuaries paper titled Modeling Efficiency Bibliography for Practicing Actuaries, published December of 2011, for example, includes a number of references related to scenario reduction techniques such as the paper by Chueh, Yvonne, Efficient Stochastic Modeling for Large and Consolidated Insurance Business: Interest Rate Sampling Algorithms, published in the *North American Actuarial Journal* in July 2002.

Table of Contents

1. Purpose/Summary

The purpose of this educational note supplement is the development of criteria for calibrating stochastic risk-free interest rate models used in the production of risk-free interest rate scenarios for the CALM valuation of insurance contract liabilities. Included are updates to the guidance for the long-term (term to maturity of 20 years and longer) risk-free interest rate and for the short-term (one-year maturity) risk-free interest rate, medium-term (five- to 10-year maturity) risk-free interest rates, and the slope[2] of the yield curve.

The CIA Standards of Practice include recommendations regarding the selection of stochastic risk-free interest rate scenarios. Different stochastic risk-free interest rate models, and parameterizations of the models, can produce significantly different sets of scenarios. Notwithstanding any definition for a plausible range on Canadian risk-free interest rates, the Standards of Practice provide little guidance on the selection, fitting, and use of a stochastic risk-free interest rate model. A goal of CLIFR is to narrow the range of practice, and this additional guidance supports this goal.

The calibration criteria presented in this educational note supplement are intended to be used for the validation of real-world scenario sets that project the evolution of the risk-free rates over long-term horizons for the valuation of insurance contract liabilities. Conversely, the calibration criteria presented in this educational note supplement would be inappropriate to validate a set of interest rate scenarios intended to reflect current market dynamics.

It would be considered best practice to model both general account and segregated fund account fixed-income assets consistently where risk-free real-world interest rate scenarios are utilized.

The normal approach to building a stochastic risk-free interest rate model and generating interest rate scenario sets would be to choose a model form and then to estimate an initial set of parameters for the model using statistical techniques. The scenario set resulting from the model would then be examined to determine if calibration criteria were satisfied. If necessary, the parameters would then be adjusted in order to produce a revised scenario set that satisfies the calibration criteria.

Strict adherence to the calibration criteria may not be necessary in order for the stochastic risk-free interest rate scenarios to be used, particularly where some of the short-term rates, long-term rates, or slopes do not have a material impact on the valuation. It may also be possible to satisfy left-tail calibration criteria, but not right-tail calibration criteria if it can be shown that this provides for a more conservative result. In these cases, refer to CIA guidance on materiality and the use of approximations.

Finally, there are many stochastic risk-free interest rate models that are available, ranging from fixed to stochastic volatility and single to multiple regimes. It is not possible to list all of the models. However, general comments are provided in appendix A.

For convenience, the calibration criteria for long-term and short-term risk-free rates and

[2] Defined as the long-term risk-free rate minus the short-term risk-free rate.

slopes are summarized below. Appendix C provides a comparison with the current criteria. For medium-term risk-free rates, qualitative guidance is presented in section 7. The calibration criteria are expressed as bond equivalent yields.

Calibration Criteria for the Long-Term Risk-Free Interest Rate (≥ 20-Year Maturity)

Horizon		**Two-Year**			**10-Year**			**60-Year**
Initial Rate		4.00%	6.25%	9.00%	4.00%	6.25%	9.00%	6.25%
Left-Tail Percentile	2.5th	2.70%	4.25%	6.40%	2.25%	2.85%	3.95%	2.30%
	5.0th	3.00%	4.55%	6.80%	2.45%	3.15%	4.50%	2.60%
	10.0th	3.20%	4.90%	7.20%	2.80%	3.70%	5.15%	2.90%
Right-Tail Percentile	90.0th	5.20%	7.65%	10.50%	6.90%	9.10%	11.50%	10.00%
	95.0th	5.55%	8.10%	11.00%	7.90%	10.10%	12.60%	11.90%
	97.5th	5.90%	8.50%	11.50%	8.70%	10.95%	13.60%	13.30%

A range of values around the historical median may be produced and would be acceptable, although a median at the 60-year horizon in the 4.00% to 6.75% range would generally be expected. A median outside of this range would need to be justified.

For all stochastic long-term risk-free interest rate models, the rate of mean reversion would not be stronger than 14.5 years (equivalent to a half-life of 10 years).

Calibration Criteria for the Short-Term Risk-Free Rate (One-Year Maturity)

Horizon		**Two-Year**			**60-Year**
Initial Rate		2.00%	4.50%	8.00%	4.50%
Left-Tail Percentile	2.5th	0.45%	1.25%	2.85%	0.60%
	5.0th	0.65%	1.55%	3.55%	0.80%
	10.0th	0.90%	2.00%	4.40%	0.85%
Right-Tail Percentile	90.0th	4.25%	7.50%	11.00%	10.00%
	95.0th	5.10%	8.35%	12.05%	12.00%
	97.5th	5.95%	9.15%	12.95%	13.65%

Calibration Criteria for Slope (the long-term rate less the short-term rate)

Horizon		**60-Year**
Left-Tail Percentile	5th	-1.00%
	10th	-0.10%
Right-Tail Percentile	90th	2.50%
	95th	3.00%

Further detail is provided in the rest of this educational note supplement.

2. Goals and Principles

To produce reasonable calibration criteria, the following principles were adopted. The calibration criteria would

- Be sufficiently robust to narrow the range of practice, but allow the actuary to apply reasonable judgment to specific circumstances;
- Be applied to the risk-free interest rate scenario sets produced;
- Be applied to the near term in addition to the steady-state portions of the risk-free interest rate scenarios produced;
- Promote the development of risk-free interest rate scenario sets that reflect yield curve shocks as well as long-term paths of declining and rising interest rates, consistent with history; and
- Encompass a wide distribution of risk-free interest rate scenarios as well as persisting environments over extended periods of time.

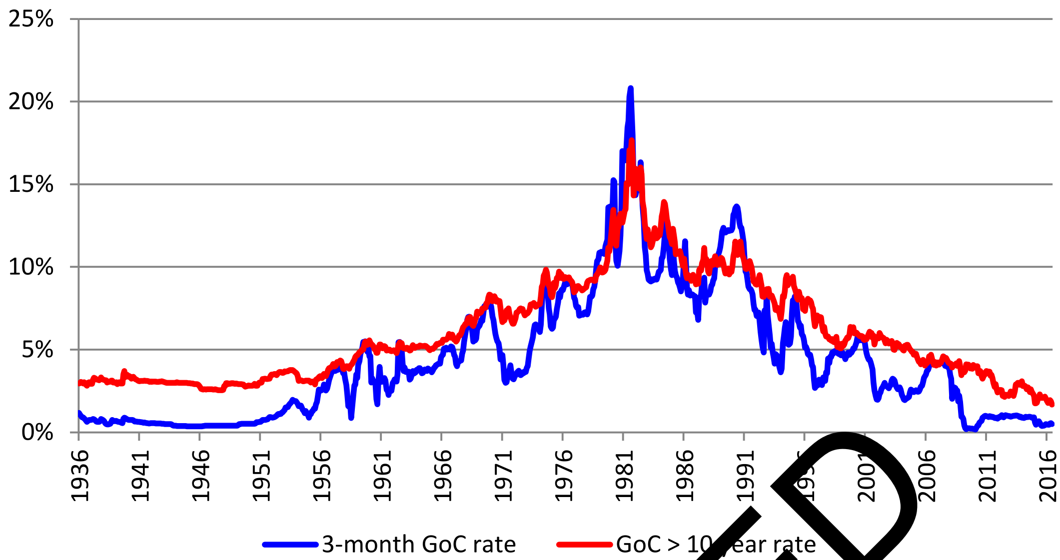
A combination of quantitative calibration criteria and qualitative guidance was developed. Quantitative criteria are provided for the short-term and long-term risk-free rates. A set of calibration criteria based solely on quantitative analysis may place too large a reliance on historical data, can be subjectively influenced by the choice of historical period, and does not take into consideration economic and monetary differences between the historical period selected and the current time. Qualitative guidance, such as that presented for medium-term risk-free rates in this educational note supplement, augments quantitative requirements and encourages the actuary to use judgment to assess the appropriateness of the stochastic risk-free interest rate model results.

Consideration was given as to whether to examine real rates (and inflation) or nominal rates. Nominal rates were chosen since modelling the complex relationship of real rates and inflation was impractical and the availability of historical nominal rates was better. The actuary would refer to the Standards of Practice if guidance is required to develop inflation assumptions that are consistent with nominal rates generated by the calibrated stochastic risk-free interest rate model.

3. Historical Interest Rates

Historical Canadian risk-free interest rates, starting in the 1930s, are illustrated in the graph below. There are three distinct patterns, beginning with the low interest rates of the 1930s depression through World War II, followed by steadily increasing interest rates through the 1970s and 1980s, and finally a period of steadily decreasing rates to June 2016. The working group decided to include historical experience to reflect these three periods, as it wanted to include data from a sufficiently long period of history to include changes in the monetary system, fiscal policy, etc., that may have influenced the level and volatility of interest rates.

Historical Short-Term and Long-Term Government of Canada Bond Rates
CAD – January 1936 to June 2016



Source: Bank of Canada, Series V122541 and V122487[3]

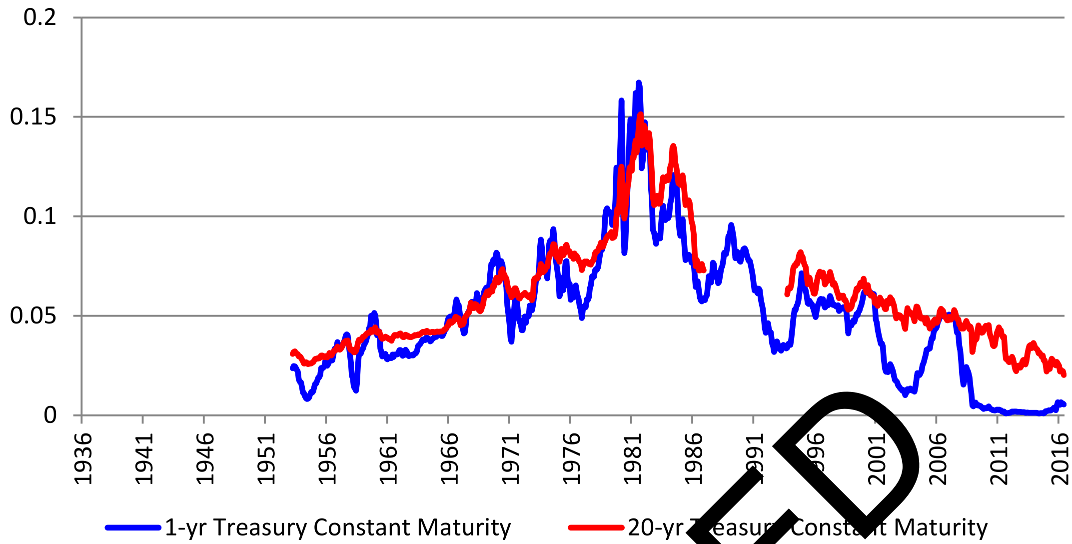
Although CANSIM series V122487 contains yields from 1919 to date, we have chosen to use only the rates since the founding of the Bank of Canada in 1935. The yields shown in the series for the period prior to 1936 are calculated on a different basis from those for the period from January 1, 1936, forward. We have chosen to use the date from January 1, 1936, rather than trying to adjust the older historical data to a consistent basis with the post-1936 data.

Historical U.S. interest rates are illustrated in the graph below and show similar patterns to those in Canada. These are provided for informational purposes only, and were not used to determine the calibration criteria for Canadian interest rates.

[3] The V122541 series is the Government of Canada Treasury bill – average yields – 3 month. The V122487 series is the Government of Canada marketable bonds—average yield—over 10 years.

Historical U.S. 20-Year Constant Maturities Treasuries and One-Year Treasury Constant Maturity Rates

USD - April 1953 to June 2016



Source: Federal Reserve Bank of St. Louis

The calibration criteria have been designed to support stochastic risk-free interest rate model development that would produce scenarios that have the following characteristics:

- Produce a wide range of interest rate scenarios, consistent with historical ranges;
- Produce periods of sustained low interest rates;
- Produce periods of sustained high interest rates (but with low probability of sustained extreme highs);
- Produce periods of trending low or trending high rates;
- Produce periods of inverted yield curves;
- Produce a reasonable slope between long-term and short-term rates; and
- Move between lows and highs over reasonable periods of time.

These characteristics can also be observed over the last 70 years in the graphs above.

4. Calibration Criteria for Long Term Interest Rate Models

This section provides the complete set of calibration criteria for the long-term risk-free interest rate, which is assumed to be a term of 20 years or greater.

Calibration criteria have been developed for the two-year, 10-year, and 60-year horizons. Interest rate scenarios at the two-year and 10-year horizons will be influenced by the initial starting interest rate, so calibration criteria at each of a 4.00%, 6.25%, and 9.00% starting long-term interest rate are provided. At the 60-year horizon, the impact of the starting rate is assumed to be minimal, so only calibration criteria at a single starting rate of 6.25% are provided. The calibration criteria are focused on the tails of the distribution (i.e., ≤10th percentile and ≥90th percentile).

Using fixed initial rates for calibration addresses the practical issue that, in most cases, stochastic risk-free interest rate models will be parameterized and tested, and scenarios generated, in advance of the valuation date, and it is to be expected that interest rates will change over this period.

The long-term rate calibration consists of the following three requirements: 1) satisfying 60-year calibration criteria; 2) satisfying near-term (two- and 10-year) calibration criteria; and 3) satisfying a mean reversion constraint.

The 60-year calibration criteria were established first, based on historical experience. The nearer horizon calibration criteria were then developed based on results from models that were parameterized to satisfy the 60-year calibration criteria.

The sections below describe the development of the calibration criteria in more detail.

4.1 Sixty-Year Calibration Criteria for the Long-Term Rate

The *steady state* is defined to be the point in time beyond which the distribution of model generated interest rates changes only negligibly, or the influence of the starting interest rate is minimal. Ideally, calibration criteria would be set at the steady state point. However, since this point can be very far in the future, and can vary by model type and parameterization, it is assumed for calibration purposes that a projection horizon of 60-years is sufficient to assume that steady state has been reached. The 60-year horizon criteria for the long-term rate are shown below.

The 60-Year Calibration Criteria

Initial Rate		6.25%
Left-Tail Percentile	2.5th	2.30%
	5.0th	2.60%
	10.0th	2.90%
Right-Tail Percentile	90.0th	10.00%
	95.0th	11.90%
	97.5th	13.30%

These calibration criteria will be satisfied if the stochastic risk-free interest rate model produces results that are less than or equal to each of the left-tail calibration criteria, and greater than or equal to each of the right-tail calibration criteria, with a long-term starting rate of 6.25%. The calibration criteria are expressed as bond equivalent yields.

Calibration criteria are provided for the left-tail and right-tail of the scenario distribution. From 1936 to June 2016, Canadian risk-free long bonds had mean and median returns of 6.00% and 5.21%, respectively[4]. The 35th to 65th percentiles are 3.94% and 6.78%, respectively. A range of values around the historical median may be produced and would

[4] Compared to 6.16% and 5.30% in the 2013 research paper reflecting experience through 2012.

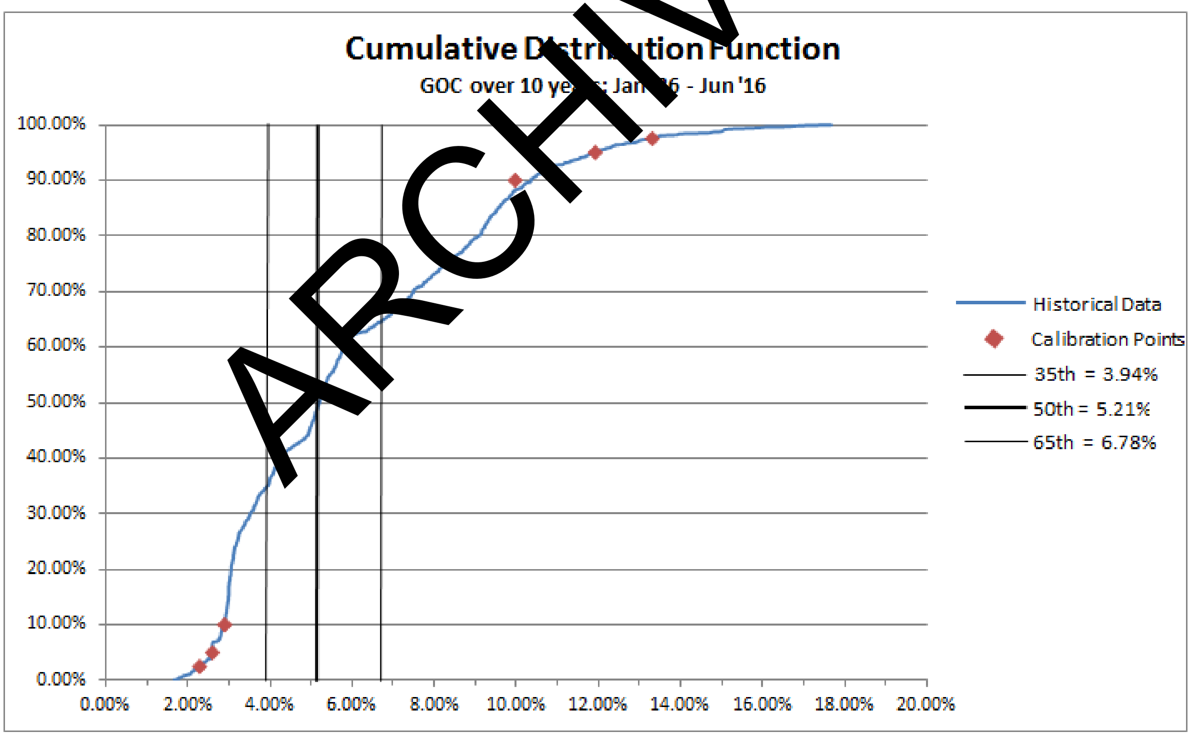
be acceptable, although a median in the 4.00% to 6.75%[5] range would generally be expected. A median outside of this range would need to be supported by justification.

4.1.1 Comparison to Historical

The following table and graph show that the updated calibration criteria are consistent with history through June 2016 at most calibration points.

		Calibration criteria	1936 –2016	Difference
Left-Tail Percentile	2.5th	2.30%	2.27%	0.03%
	5.0th	2.60%	2.59%	0.01%
	10.0th	2.90%	2.90%	0.00%
Right-Tail Percentile	90.0th	10.00%	10.36%	(0.36)%
	95.0th	11.90%	11.89%	0.01%
	97.5th	13.30%	13.30%	0.00%

The following graph also shows that the calibration criteria are a close fit to historical experience through June 2016



Source: Bank of Canada, Series V122487

[5] In the 2009 educational note, a range of 5.00% to 6.75% corresponded to the 40th and 60th percentiles of historical experience. The percentile range was expanded because of a significant difference between the 60th and 65th percentiles.

4.1.2 Comparison to Model Results

The 60-year calibration criteria were tested against two commonly used and publicly available model forms, with two different sets of parameters for each. The aim of the stochastic risk-free interest rate model testing was to determine whether common model forms with reasonable parameterizations could produce scenarios that satisfied the calibration criteria.

This was accomplished by testing different types of stochastic risk-free interest rate models, using three different parameterizations for the Cox-Ingersoll-Ross (CIR) model and two for the Brennan-Schwartz (BS) model. Testing results are shown in the table below. Details on the setup of the CIR and BS models are provided in Appendix B.

Sixty-Year Calibration Criteria—Model Testing Results

Percentile	Criteria	CIR Parameter Set 1	CIR Parameter Set 2	CIR Parameter Set 3	BS Parameter Set 1	BS Parameter Set 2
2.5th	2.30%	1.84%	1.83%	1.83%	2.23%	2.22%
5.0th	2.60%	2.28%	2.27%	2.26%	2.51%	2.51%
10.0th	2.90%	2.86%	2.85%	2.85%	2.89%	2.89%
Median		5.82%	5.81%	5.82%	5.15%	5.14%
90.0th	10.00%	10.31%	10.34%	10.35%	10.39%	10.39%
95.0th	11.90%	11.90%	11.93%	11.93%	13.03%	13.04%
97.5th	13.30%	13.43%	13.4[illegible]%	13.50%	16.16%	16.25%

4.2 Two-Year and 10-Year Calibration Criteria for the Long-Term Rate

For calibration criteria at shorter horizon points, the initial starting rate is important. For this reason, calibration criteria suitable for low, average, and high interest rates at the starting environment were developed. History has shown that interest rates can move significantly over short periods of time, and it is desirable to reflect the dynamics of lower and higher starting rate environments. Long-term starting rates of 4.00% and 9.00% were chosen as sample low and high rates to be used in developing the calibration criteria. This does not preclude the use of the calibrated model with long-term starting rates either below 4.00% or above 9.00%. Shorter horizon criteria for the long-term rate are shown below.

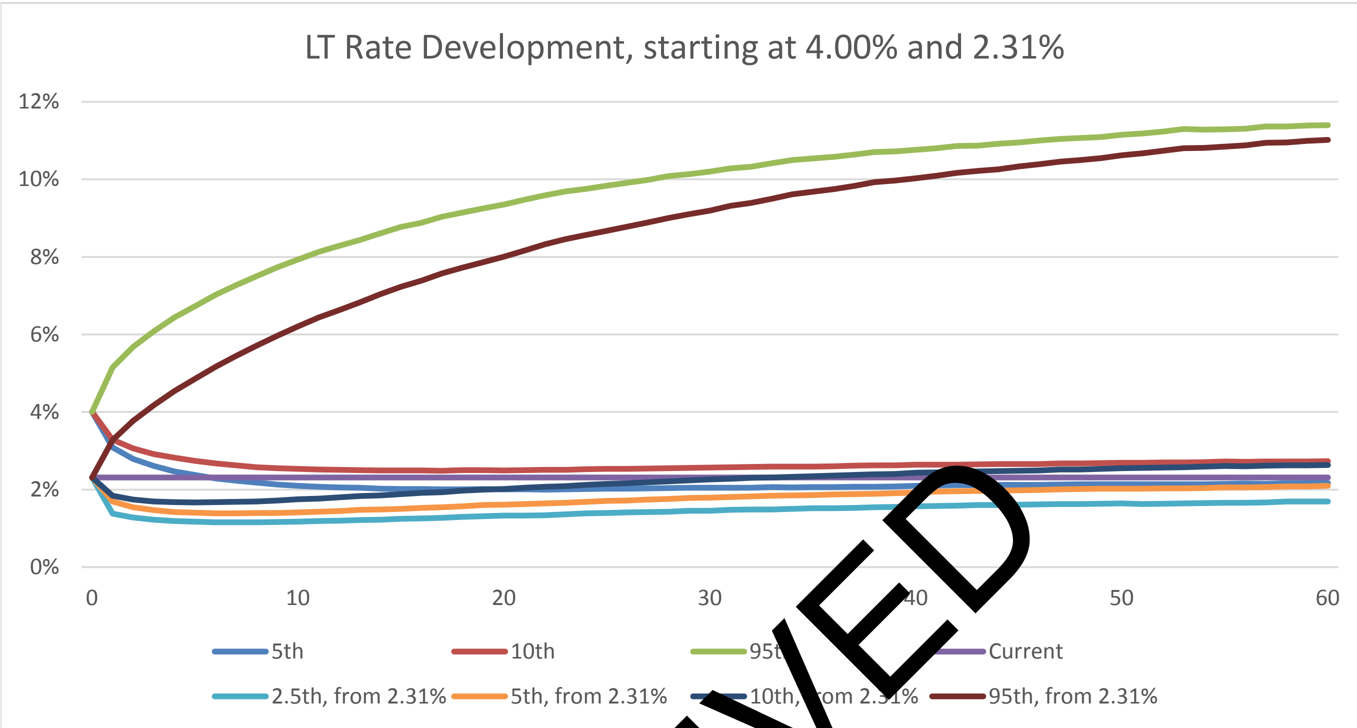
Two-Year and 10-Year Calibration Criteria

Horizon		**Two-Year**			**10-Year**		
Initial Rate		**4.00%**	**6.25%**	**9.00%**	**4.00%**	**6.25%**	**9.00%**
Left-Tail Percentile	2.5th	2.70%	4.25%	6.40%	2.25%	2.85%	3.95%
	5th	3.00%	4.55%	6.80%	2.45%	3.15%	4.50%
	10th	3.20%	4.90%	7.20%	2.80%	3.70%	5.15%
Right-Tail Percentile	90th	5.20%	7.65%	10.50%	6.90%	9.10%	11.50%
	95th	5.55%	8.10%	11.00%	7.90%	10.10%	12.60%
	97.5th	5.90%	8.50%	11.50%	8.70%	10.95%	13.60%

These calibration criteria will be satisfied if the stochastic risk-free interest rate model produces results that are less than or equal to each of the left-tail calibration criteria and greater than or equal to each of the right-tail calibration criteria, for each of the three long-term starting rates. The calibration criteria are expressed as bond equivalent yields.

To determine these calibration criteria, historical results were initially reviewed. However, since limited data are available to analyze the progression of rates from each of these starting rate environments, results from the CIR and BSM model forms that had been used to test calibration criteria at the 60-year horizon were used to develop the shorter horizon calibration criteria. The two-year and 10-year calibration criteria were set by choosing the least constraining value at each calibration point from among the results of the five stochastic risk-free interest rate models referenced in Appendix B. Models that satisfy these calibration criteria will produce a reasonable dispersion of interest rates at both the two-year and 10-year horizons.

If the actual long-term starting rate is less than 4.00%, or greater than 9.00%, then the models will produce distributions of scenarios that are shifted relative to the calibration criteria in the table above, as illustrated in the following graph in the case of a starting rate that is lower than 4.00%.



Appendix C provides a comparison of the long-term risk-free rate calibration criteria to the previous calibration criteria developed for the 2013 research paper.

4.3 Mean Reversion Calibration Criteria for the Long-Term Rate

Historical experience has shown that interest rates can stay at low levels for extended periods of time. The calibration criteria designed up to this point do not sufficiently constrain stochastic risk-free interest rate models to reflect economic environments where interest rates remain at low levels over an extended number of years.

For this reason, an additional constraint was thought necessary for all stochastic risk-free interest rate models so that the rate of mean reversion would not be stronger (i.e., not shorter or quicker) than 14.5 years (equivalent to a half-life of 10 years).

For simple stochastic risk-free interest rate models with an explicit mean reversion factor, this requirement can be satisfied by considering the value of the mean reversion parameter directly. For more complex models, this requirement can be satisfied by using a mathematical proof or using the procedure in Appendix D.

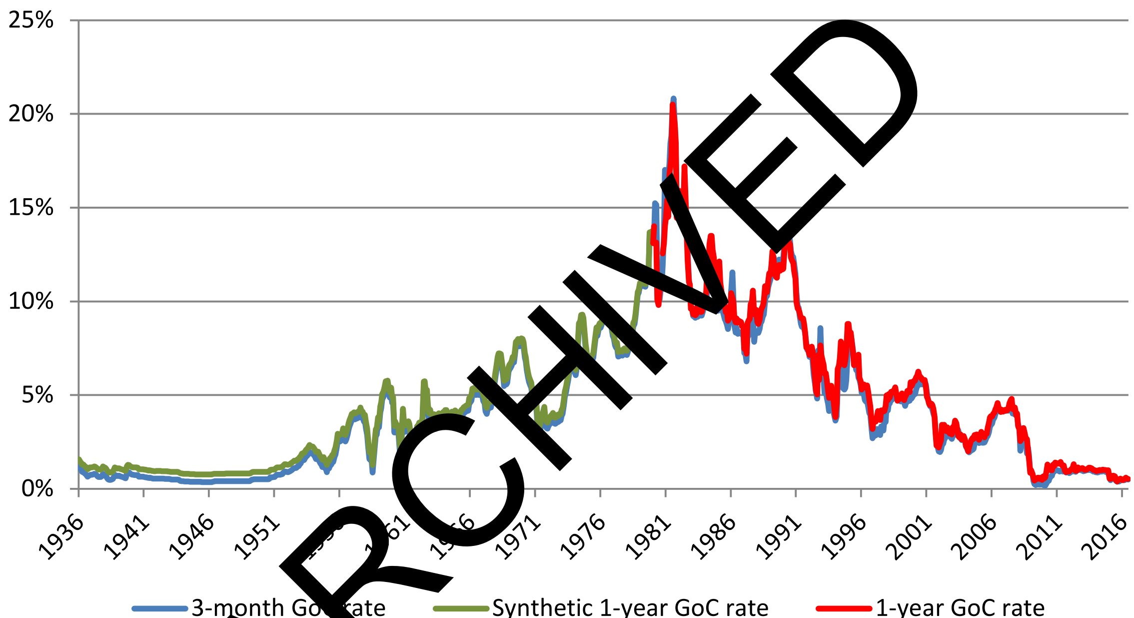
5. Short-Term Rate Calibration Criteria

This section provides the calibration criteria for the short-term risk free rate, which is assumed to be the one-year term.

The approach to determine calibration criteria for the short-term rate was consistent with the approach used for the long-term rate. That is, the 60-year calibration criteria were established first based on historical experience. The nearer horizon calibration criteria were then based on results from models parameterized to satisfy the 60-year calibration criteria. Where there is overlap in the methodology described for the long-term rates, it is not repeated here.

Historical experience for the one-year rate is available only from 1980 while historical experience for the three-month rate is available from the 1930s. Experience is highly correlated between the two sets of rates as shown in the graph below. In order to have a historical period for the short-term rate consistent with that for the long-term rate, a synthetic set of one-year rates was derived based on the three-month term for the full period and the relationship between the three-month and one-year rates over the period from 1980 to 2016. Details of the method are found in appendix E.

CAD - January 1936 to June 2016



5.1 Sixty-Year Calibration Criteria for the Short-Term Rate

The 60-year horizon criteria for the short-term rate are shown below.

Sixty-Year Calibration Criteria

Percentile		Initial Rate
		4.50%
Left-Tail	2.5th	0.60%
	5th	0.80%
	10th	0.85%
Right-Tail	90th	10.00%
	95th	12.00%
	97.5th	13.65%

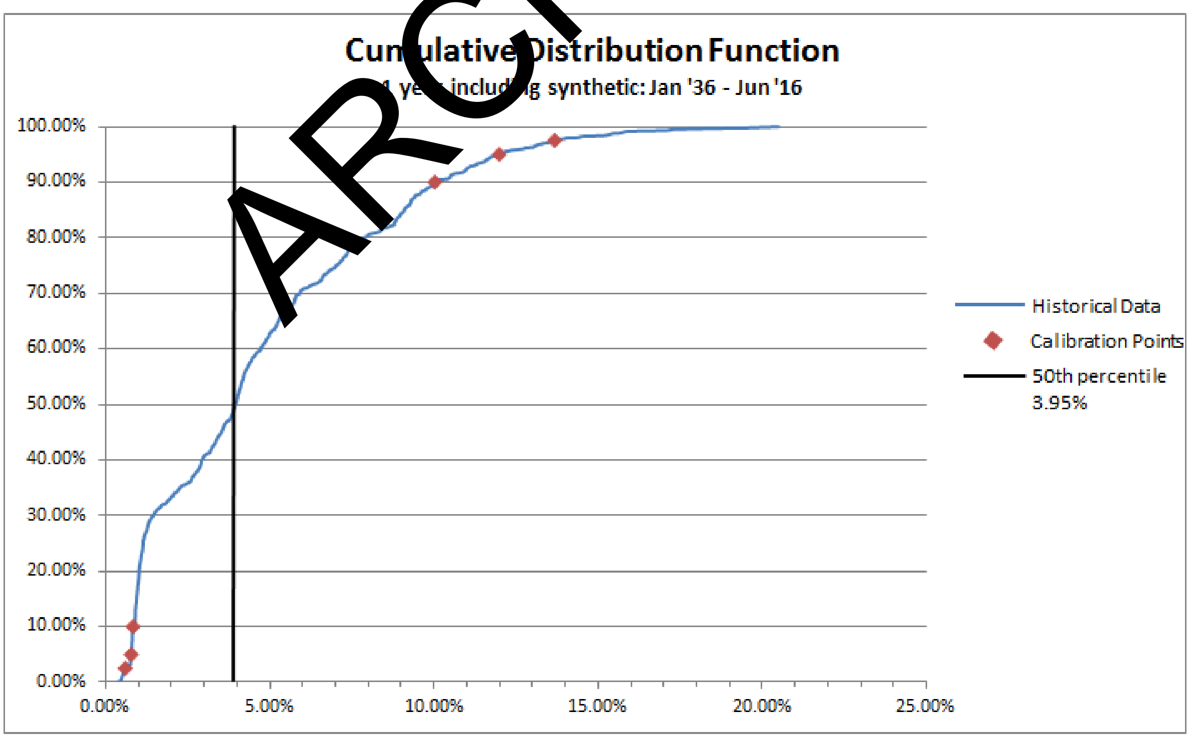
These calibration criteria will be satisfied if the distribution of one-year rates produced by the model at the 60-year point are less than or equal to each of the left-tail calibration criteria and are greater than or equal to each of the right-tail calibration criteria, with a short-term starting rate of 4.5%. The calibration criteria are expressed as bond equivalent yields.

5.1.1 Comparison to Historical

For reference, the following comparison to historical experience is provided:

Percentile		**Calibration criteria**	**Jan. 1936 – Jun. 2016**	**Difference**
Left-Tail	2.5th	0.60%	0.60%	0.00%
	5th	0.80%	0.78%	0.02%
	10th	0.85%	0.84%	0.01%
Right-Tail	90th	10.00%	10.03%	-0.03%
	95th	12.00%	12.04%	-0.04%
	97.5th	13.65%	13.69%	-0.04%

The historical interest rates are based on the actual one-year rates from 1980–2016 and on the synthetic one-year rates from 1936–1979. The calibration criteria are rounded from the historical distribution. The following graph also shows that the calibration criteria are a close fit to historical experience through June 2016.



5.2 Two-Year Calibration Criteria for the Short-Term Rate

Similar to the long-term risk-free interest rate, short-term starting rates of 2%, 4.5%, and 8% were chosen as representative of low, medium-, and high short-term risk-free rate environments, respectively. This does not preclude the use of the calibrated model with short-term starting rates less than 2%, or greater than 8%.

The two-year horizon criteria for the short-term rate are shown below.

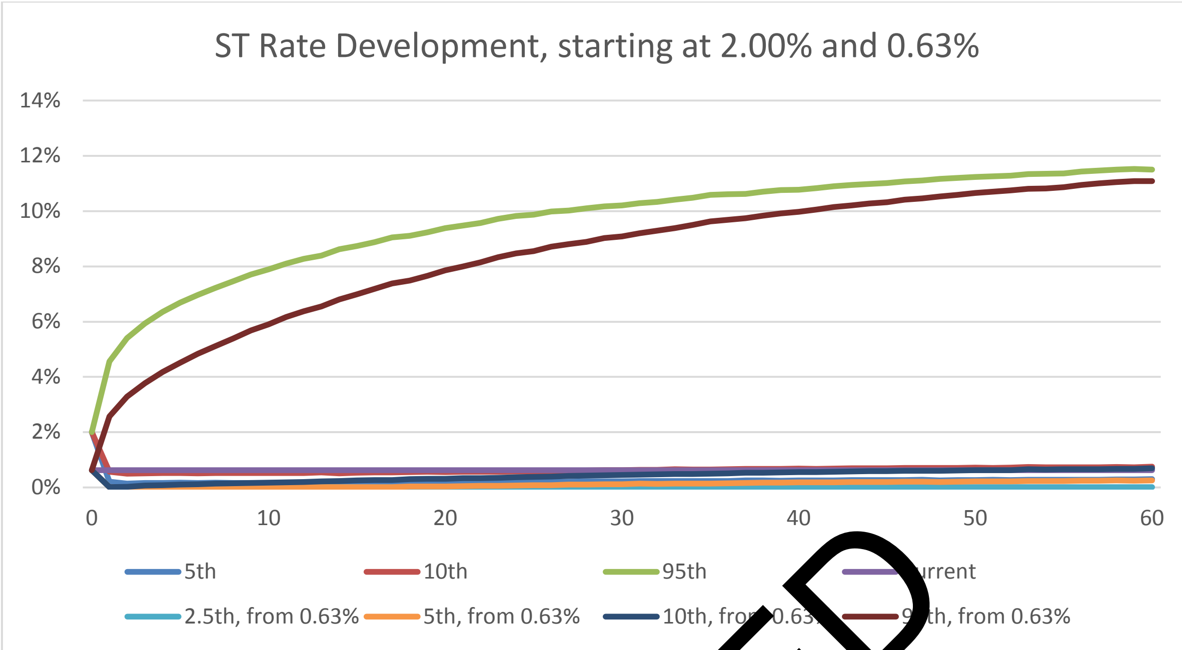
Two-Year Calibration Criteria

Percentile		**Initial Rate**		
		2.00%	**4.50%**	**8.00%**
Left-Tail	2.5th	0.45%	1.25%	2.85%
	5th	0.65%	1.55%	[illegible]
	10th	0.90%	2.00%	4.40%
Right-Tail	90th	4.25%	7.50%	11.0[illegible]%
	95th	5.10%	8.3[illegible]%	12.05%
	97.5th	5.9[illegible]%	9.15%	12.95%

These calibration criteria will be satisfied if the distribution of one-year rates produced by the model at the two-year horizon are less than or equal to each of the left-tail calibration criteria and are greater than or equal to each of the right-tail calibration criteria. The calibration criteria are expressed as bond equivalent yields.

If the actual long-term starting rate is less than 2.00%, or greater than 8.00%, then the models will produce distributions of scenarios that are shifted relative to the calibration criteria in the table above, as illustrated in the following graph in the case of a starting rate that is lower than 2.00%.

The changes to the two-year calibration criteria are larger than the changes to the 60-year calibration criteria. This has occurred because the 60-year calibration points are based on historical data, and the specific model parameterizations used influences the two-year calibration points. See appendix B for additional information on model parameterizations used.



6. Sixty-Year Slope Calibration Criteria

It is expected that the long-term and short-term rates will be correlated. As such, slope calibration criteria are provided. The calibration criteria also ensure that some scenarios produce inverted yield curves and that other scenarios produce steep yield curves.

The distribution of the slope of the yield curve (defined as the long-term rate less the short-term rate) would satisfy the following 60 years into the projection.

Sixty-Year Slope Calibration Criteria

Percentile	Calibration Criteria
5th	-1.00%
10th	-0.10%
90th	2.50%
95th	3.00%

These calibration criteria will be satisfied if the distribution of the slope values produced by the model 60 years into the projection are less than or equal to each of the left-tail calibration criteria and are greater than or equal to each of the right-tail calibration criteria.

6.1 Comparison to Historical

For reference, the following comparison to historical experience is provided.

Percentile		60-Year Criteria	Jan. 1936–Jun. 2016	Difference
Left tail	5th	-1.00%	-0.97%	-0.03%
	10th	-0.10%	-0.10%	0.00%
Right tail	90th	2.50%	2.52%	-0.02%
	95th	3.00%	2.98%	0.02%

The historical slopes are based on the difference between actual one-year rates and actual greater-than-10-year rates from 1980– June 2016 and on the difference between the synthetic one-year rates and actual greater-than-10-year rates from 1936–1979.

7. Medium-Term Rate Guidance

Medium-term rates are assumed to fall in the five- to 10-year maturity range. Qualitative guidance for medium-term risk-free rates is provided rather than quantitative calibration criteria.

The guiding principle for generating medium-term risk-free rates is that these rates would be generated using an appropriate methodology that logically connects the medium-term rates to the long-term and short-term rates. Depending on how the stochastic risk-free interest rate model is constructed, medium-term rates may be derived using one of following methods. That is, the medium-term rates may be either:

1. Modelled directly, with its own stochastic process (such as those outlined in Appendix B), along with other points on the yield curve where each has its own stochastic process with appropriate correlation between these processes; or
2. Modelled as a part of a principal component analysis, where changes in the yield curve characteristics (which can include, for example, one or more of yield curve level, slope, and curvature) are used to project the movements of the entire yield curve over time; or
3. Modelled where the entire yield curve is generated using term structure models of interest rates, with single or multiple factors; or
4. Estimated based on the modelled short-term and long-term rates, where the short- and long-term rates are modelled with their own stochastic processes.

Note that it is possible to directly calibrate the distributions of individual rates using methods 1 and 4, but not with methods 2 and 3.

If method 1 above is used, the stochastic process(es) for the medium-term rate(s) would be calibrated as consistently as practicable with both the short- and long-term rates' stochastic processes, so that the medium-term rate(s) will be consistent with both the short- and long-term rates. Consistency applies to both the calibration criteria methodology and to the final parameters selected. This is sufficient to meet the medium- term guidance requirements, provided that both the long- and short-term rates meet their respective calibration criteria.

If either of method 2 or 3 above is used, provided that the model is set up appropriately and that both the short-term rates and long-term rates meet their respective calibration criteria, the medium-term rates would naturally be consistent with both the short- and long-term rates. This is sufficient to meet the medium-term guidance requirements.

If the medium-term interest rates are not modelled and are instead estimated based on the modelled long-term and short-term rates (i.e., method 4), then the following are examples of the estimation techniques that can be used to derive the medium-term rates:

- Non-linear interpolation between short-term and long-term rates, or
- Regression with the short-term and long-term rates being the dependent variables.

The above estimation techniques would be sufficient to meet the medium-term guidance requirements, provided that both the long- and short-term rates meet their respective calibration criteria.

While the actuary is not constrained to using one of the estimation techniques above, some methodologies would be considered inappropriate. Unless evidence can be provided to the contrary, or if the impact of using these methodologies is not material, linear interpolation based on the short-term and long-term rates, or assuming medium-term rates are the same as the short-term or long-term rates, is not an appropriate methodology for the derivation of the medium-term rates and would not meet the medium-term guidance requirements.

8. Scenario Generation

The actuary would first demonstrate that the stochastic risk-free interest rate set satisfies all of the calibration criteria under the three sets of fixed starting rates:

- Short-term rate 2.00%, long-term rate 4.00%;
- Short-term rate 4.50%, long-term rate 6.25%; and
- Short-term rate 8.00%, long-term rate 9.00%.

This demonstration of calibration of the criteria would only need to be performed when the stochastic risk-free interest rate model and/or parameters are updated, or when the calibration criteria themselves are updated.

The initial conditions were left to be the same as the previous review because they remain reasonably lose to historical average rates.

	Historical Average	Initial Rate
Short Rate	4.70%	4.50%
Long Rate	6.00%	6.25%

Once it has been demonstrated that the stochastic risk-free interest rate model has been properly calibrated, the model may be used to generate interest rate scenarios for valuation using the same parameters and at least the number of scenarios[6] as was used for demonstrating calibration to the criteria, and by using actual starting risk-free interest rates that are appropriate for the valuation date.

[6] It may also be possible to run fewer scenarios than were used for calibration, which then becomes part of scenario reduction techniques and use of approximations.

It is possible for only a subset of the scenarios to be used in the actual CALM valuation. A discussion on scenario reduction techniques is beyond the scope of this educational note supplement, and the actuary would consult the literature that is available on this subject[7]. The actuary may also refer to subsection 1510 of the Standards of Practice on the use of approximations.

9. Calibration Criteria for Other Countries

The scenarios produced from stochastic risk-free interest rate models that satisfy the calibration criteria would be appropriate for valuations utilizing Canadian risk-free reinvestment assumptions. An actuary building a stochastic risk-free interest rate model for these U.S. government bonds and many (but not all) other developed economies would consider these calibration criteria as a starting point and make adjustments as he or she judges appropriate. In making such a judgment, rate history, market information, economic and political conditions may be considered. If calibration criteria relevant to the particular country or currency being modelled have been published, they could be used as an additional source of information and guide to aid the actuary in forming his or her opinion. It may be acceptable to use those calibration criteria if it can be demonstrated that they are broadly consistent with the calibration criteria in this educational note supplement (either the calibration criteria themselves are broadly consistent, or the approach taken to develop the calibration criteria is broadly consistent with this educational note supplement). In the absence of such a demonstration, it would not be appropriate to utilize the other country's calibration criteria without adjustment.

Countries with extended histories of either unusually low or high rates would be examples where the calibration criteria may not be appropriate. In some countries, history may be limited, and a wider distribution of rates relative to these limited observations may be needed in order to provide a margin for uncertainty.

Finally, the calibration criteria would not be appropriate for developing and emerging markets.

[7] The American Academy of Actuaries paper titled Modeling Efficiency Bibliography for Practicing Actuaries, published December of 2011, for example, includes a number of references related to scenario reduction techniques such as the paper by Chueh, Yvonne, Efficient Stochastic Modeling for Large and Consolidated Insurance Business: Interest Rate Sampling Algorithms, published in the *North American Actuarial Journal* in July 2002.

Appendix A

The CALM liability is determined by modelling the asset and liability cash flows over a defined set of scenarios, and comparing the resulting insurance contract liability balances. If the deterministic approach is taken, the set of scenarios are the ones prescribed in subsection 2330 of the Standards of Practice plus supplemental scenarios the actuary deems appropriate to the risk profile of the insurance contract liabilities. The insurance contract liability is set to be in the upper range of the results, and at least as great as the highest insurance contract liability resulting from the prescribed scenarios. If a stochastic approach is used, a large number of different interest rate scenarios are generated stochastically, with the insurance contract liability calculated under each scenario. The insurance contract liability is set to be consistent with the Standards of Practice, at the discretion of the actuary.

Stochastic Modelling of Interest Rates

The stochastic modelling of interest rates is similar to the stochastic modelling of equity returns (which is in general used to model variable annuity investment guarantees). It differs in that an important part of the modelling of interest rate movements is generally an assumption of non-negative rates, or a floor on the degree to which rates can become negative, and generally some form of reversion to a mean. The mean is usually chosen with regard to a relevant body of historical interest rates. The stochastic risk-free interest rate model used will define how rates move from one period to the next through a formula applied to values generated through a Monte Carlo simulation. The parameters in the stochastic risk-free interest rate model will typically represent mean-reversion level, volatility, and the strength (or speed) of the reversion to the long-run mean. This educational note supplement on calibration criteria does not prescribe the stochastic risk-free interest rate model form, or the setting of the parameters, but rather focuses on the scenarios resulting from an application of the scenario generator. This allows the actuary flexibility in the selection of a standard model formulation, or the modification of a standard formulation to create a new stochastic risk-free interest rate model that provides a better fit for the individual application under analysis.

Choice of Stochastic Modelling over Deterministic Modelling

Stochastic modelling of interest rates is not a radical departure from deterministic measures. It is an enhanced form of scenario testing whereby a wide range of random scenarios are developed using a model that is a representation of interest rate evolution in real life. In deciding whether stochastic modelling of interest rates would be utilized for the valuation, the actuary would consider the complexity of the interaction of interest rates with the asset and liability cash flows within the CALM model, as well as the materiality of the impact of the interest rate volatility on results. If the product design is such that most of the liability outflows will occur within a relatively narrow range around the mean of the distribution of outcomes, an approach of using the best estimate plus an explicit margin is appropriate. If, however, there are high benefit outflows that happen only in low-probability areas of the distribution (the tails) then a stochastic approach can give a more appropriate picture of the extent of interest rate exposures. Stochastic risk-free interest rate modelling may also be the preferred approach where there is no natural best estimate, such as when modelling interest rates that will be available for reinvestments 25 years or more into the future.

Practical Considerations

The stochastic CALM liability is set as the average of a subset of the highest resulting insurance contract liabilities. It is important to note that this can mean that the insurance contract liability is an average of scenarios that are neither the lowest interest rate scenarios nor the highest rate scenarios. For example, consider a product with high net positive cash flows from premiums in the next 10 years, and negative cash flows emerging over the subsequent 10 years, so that by year 20 the bulk of the cash flow is negative as benefits outweigh premiums and asset cash flows. An adverse scenario here will feature low interest rates in the first 10 years and higher rates in the years past year 20. This is a natural outcome of the stochastic modelling. If there is a need to develop a single average interest rate vector for the purpose of subdividing a block of business after the CALM run, then an odd pattern is possible.

Appendix B

This appendix presents the model parameters and model specifications for the stochastic risk-free interest rate model forms used in the development of the calibration criteria in this educational note supplement.

This information is provided to ensure transparency and to assist the actuary in understanding how the stochastic risk-free interest rate models are calibrated and used in determining the criteria. The actuary is cautioned against simply using these stochastic risk-free interest rate models in his or her work, but should instead develop sufficient expertise to apply actuarial judgment in selecting a particular stochastic risk-free interest rate model form and parameters, consistent with the calibration criteria.

The following forms of the **Brennan-Schwartz** model were used for developing and testing the criteria:

Long-term rates:

$$r_t^l = (1-\alpha_1)r_{t-1}^l + \alpha_1\tau_1 + \sigma_1 r_{t-1}^l\,\varepsilon_t$$

Short-term rates:

$$r_t^s = maximum\,((1-\alpha_2)r_{t-1}^s + \alpha_2\tau_2 + \sigma_2(r_{t-1}^s - d)\,\xi_t, floor)$$

where for $i = 1, 2$:

- τ_i is the mean-reversion level to which the process is reverting;
- α_i is the mean-reversion speed;
- σ_i is the volatility parameter;
- d is the displacement parameter;
- $\varepsilon_t, \xi_t \sim N(0,1)$ and
- $\rho = correl\left(\varepsilon_t, \xi_t\right)$
- $floor = -0.75\%$

The choice of floor of -0.75% is based on the lowest observed point in German historical 1-year data. Allowing for negative rates in the model parameterization was seen as appropriate given recent observed experience in some Organisation for Economic Co-operation and Development (OECD) countries, particularly Germany and Japan.

The continuous form of the Brennan-Schwartz model does not produce negative interest rates. The discretized form results in rare occurrences of negative rates. To allow for reasonable negative rate exposure for the short-term rate, a displacement term is added to the diffusion component for the short-term model. The volatility is scaled by rate – displacement. The displacement parameter was set to be -1.0% so that the higher volatility produces some negative rates (about 1.0% of the projected rates at year 60) and there is a buffer between the floor and the lowest generated rate.

In determining the criteria, two sets of parameters are considered and are shown in the following table. While the annualized parameters are shown below for illustrative purposes, the corresponding monthly parameters were used in the actual modelling.

- Two different parameter sets are illustrated to show that there are multiples ways to parameterize the model while satisfying the calibration criteria.
  - Mean-reversion speed is the linear regression coefficient of the relationship between the current rate (r_t) and its previous value (r_{t-1}).
  - Two values for the mean-reversion speed were determined using different historical periods.
  - The correlation parameter is estimated as the historical correlation between the long-term and short-term rate movements over the same periods.
- Mean reversion target and volatility: they are driven by statistical techniques to fit the historical distribution from January 1936 to June 2016. Models with faster mean reversion have higher volatility in order to meet the calibration criteria at year 60.

Annualized Parameters (*i* = 1, 2) Rate Model	**Parameter Set 1**		**Parameter Set 2**	
	Long-Term Rate Model	**Short-Term Rate Model**	**Long-Term Rate Model**	**Short-Term Rate Model**
α_i	3.50%	7.46%	4.25%	8.04%
$(1/\alpha_i)$[8]	(28.6 years)	(13.4 years)	(23.5 years)	(12.4 years)
τ_i	6.14%	4.88%	6.14%	4.88%
σ_i	14.38%	32.33%	15.84%	33.55%
ρ	0.6964		0.6998	

[8]In the table above, the rate of mean reversion in years is defined as *1/ the mean-reversion speed.*

The following form of the CIR model was used for developing and testing the criteria:

Long-term rates:

$$r_t^l = (1-\alpha)r_{t-1}^l + \alpha\tau + \sigma_1\sqrt{r_{t-1}^l}\,\varepsilon_t$$

Short-term rates:

$$r_t^s = maximum((1-\phi)r_{t-1}^s + \phi\left(r_{t-1}^l - \theta\right) + \beta\left(r_t^l - r_{t-1}^l\right) + \sigma_2\sqrt{r_{t-1}^l}\,\zeta_t, floor)$$

where

τ is the mean-reversion level to which the long-term rate is reverting;

α is the mean-reversion speed of the long-term rates;

σ_1 is the volatility parameter of the long-term rates;

θ is the steady-state spread between short-term rates and long-term rates;

ϕ is the mean-reversion speed of the spread between the long- and short-term rates;

β is a constant linked to the variation of long-term rates from one period to the next;

σ_2 is the volatility parameter of the short-term rates; and

$\varepsilon_t, \zeta_t \sim N(0,1)$

$\rho = correl\left(\varepsilon_t, \zeta_t\right)$

$floor = 0.01\%$

Three sets of parameters are used for developing the criteria and the parameters are estimated by fitting the model forms to their respective 60- year horizon calibration criteria. While the annualized parameters are shown below for illustrative purposes, the corresponding monthly parameters were used in the actual modelling.

Annualized Parameters (i = 1, 2)	Parameter Set 1		Parameter Set 2		Parameter Set 3	
	LT Rate Model	ST Rate Model	LT Rate Model	ST Rate Model	LT Rate Model	ST Rate Model
α ($1/\alpha$)	3.50% (28.6 years)	n/a	4.25% (23.5 years)	n/as	5.00% (20.0 years)	n/a
ϕ ($1/\phi$)	n/a	43.56% (2.3 years)	n/a	48.08% (2.1 years)	n/a	48.08% (2.1 years)
τ	6.30%	n/a	6.30%	n/a	6.30%	n/a
σ_i	3.19%	7.77%	3.52%	8.03%	3.82%	7.94%
θ	n/a	1.44%	n/a	1.47%	n/a	1.47%
β	n/a	9.50%	n/a	45.17%	n/a	54.47%
ρ	0.6017		0.4445		0.4151	

- Three different parameter sets are illustrated to show that there are multiples ways to parameterize the model while satisfying the calibration criteria.
    - Mean-reversion speed is the linear regression coefficient of the relationship between the current rate (r_t) and its previous value (r_{t-1}).
    - Three values for the mean-reversion speed were determined using different historical periods.
    - For the short term rate model: historical data show that the spread mean-reverts much faster than short or long rates, hence the high value for parameter ϕ.
    - The constant β and correlation ρ linking short and long term rates are determined using maximum likelihood estimation.
- Mean reversion target and volatility: they are driven by statistical techniques to

fit the historical distribution from January 1936 to June 2016. Models with faster mean reversion have higher volatility in order to meet the calibration criteria at year 60.

For both the Brennan-Schwartz and the CIR models, when used to derive the short-term rate calibration criteria at near terms, the long-term rate model parameters were paired only with the short-term rate model parameters within the same parameter set. Long-term rate calibration criteria were based solely on long-term rate model forms. The rates were projected at a monthly time step and at least 10,000 scenarios were run to ensure convergence.

Appendix C

This appendix provides a summary of how the risk-free interest rate calibration criteria in this educational note supplement compare to the previous calibration criteria presented in the 2013 research paper.

The revised and previous calibrations are shown in the following tables:

Long Term Rate															
		Published Criteria (in %)							Revised Criteria (in %)						
Horizon:		2-Year			10-Year			60-Year	2-Year			10-Year			60-Year
Initial Rate:		4%	6.25%	9%	4%	6.25%	9%	6.25%	4%	6.25%	9%	4%	6.25%	9%	6.25%
Left-tail percentile	2.5th	2.85	4.25	6.20	2.30	2.90	3.65	2.60	2.70	4.25	6.40	2.25	2.85	3.95	2.30
	5th	3.00	4.50	6.60	2.50	3.20	4.25	2.80	3.00	4.55	6.80	2.45	3.15	4.50	2.60
	10th	3.25	4.80	7.05	2.85	3.65	4.95	3.00	3.20	4.90	7.20	2.80	3.70	5.15	2.90
Right-tail Percentile	90th	5.15	7.80	10.60	6.85	9.35	11.60	10.00	5.20	7.65	10.50	6.90	9.10	11.50	10.00
	95th	5.55	8.30	11.20	7.85	10.40	12.80	12.00	5.55	8.10	11.00	7.90	10.10	12.60	11.90
	97.5th	5.85	8.70	11.70	8.85	11.40	13.90	13.50	5.90	8.50	11.50	8.70	10.95	13.60	13.30

		Short Term Rate (values in %)								
		Published Criteria (in%)					Revised Criteria (in%)			
Horizon:		2-Year			60-Year					60-Year
Initial Rate:		2%	4.5%	8%	4.5%		2%	4.5%	8%	4.5%
Left-tail percentile	2.5th	0.85	2.35	5.50	0.80		0.45	1.25	2.85	0.60
	5th	1.00	2.70	5.95	0.90		0.65	1.55	3.55	0.80
	10th	1.15	3.10	6.40	1.00		0.90	[illegible]	4.40	0.85
Right-tail Percentile	90th	3.00	5.90	9.75	10.00		4.25	7.50	11.00	10.00
	95th	3.35	6.30	10.25	12.00		5.10	8.35	12.05	12.00
	97.5th	3.60	6.65	10.65	13.50		5.95	9.15	12.95	13.65

For the long-term rates, the differences between the current calibration criteria and the previous calibration criteria in the 2013 research paper are shown in the following table:

Change in Calibration Criteria (Revised to Published, values in %)								
Horizon:		**2-Year**			**10-Year**			**60-Year**
Initial Rate:		**4%**	**6.25%**	**9%**	**4%**	**6.25%**	**9%**	**6.25%**
Left-tail percentile	2.5th	(0.15)	-	0.20	(0.05)	(0.05)	0.30	(0.30)
	5th	-	0.05	0.20	(0.05)	(0.05)	0.25	(0.20)
	10th	(0.05)	0.10	0.15	(0.05)	0.05	0.20	(0.10)
Right-tail Percentile	90th	0.05	(0.15)	(0.10)	0.05	(0.25)	(0.10)	[illegible]
	95th	-	(0.20)	(0.20)	0.05	(0.30)	(0.20)	(0.1[illegible])
	97.5th	0.05	(0.20)	(0.20)	(0.15)	(0.45)	(0.3[illegible])	(0.20)

For the short-term rates the differences between the current calibration criteria and the previous calibration criteria in the 2013 research paper are shown in the following table:

Change in Calibration Criteria (values in %)					
Horizon:		2-Year			60-Year
Initial Rate:		2%	4.5%	8%	4.5%
Left-tail percentile	2.5th	(0.40)	(1.10)	(2.65)	(0.20)
	5th	(0.35)	(1.15)	(2.40)	(0.10)
	10th	(0.25)	(1.10)	(2.00)	(0.15)
Right-tail Percentile	90th	1.25	1.60	1.25	-
	95th	1.75	2.05	1.80	-
	97.5th	2.35	2.50	2.30	0.15

Appendix D

One purpose of the calibration criteria is to ensure that scenarios robustly represent periods of sustained low rates, which limit investment income on reinvestments needed to support long-term guarantees. Although single-point-in-time tail calibration criteria go some way to ensuring this outcome, they do not exclude stochastic risk-free interest rate models that produce scenarios in which periods of low rates tend not to be sustained, so that few scenarios would display low interest rates averaged over a potentially extended period during which reinvestment could be financially important. Sustained periods of low rates can be statistically demonstrated if the scenarios that are relatively low in early years tend to stay relatively low in later years. As an example, although other approaches are possible, and as an alternative to a mathematical proof, satisfaction of the mean reversion criterion can be demonstrated with the following procedure:

1. Sort scenarios for lowest to highest long-term rate at projection year T0, where T0 is sufficiently long to accumulate substantial dispersion in rates, but not so long as to be beyond most expected reinvestments. For a typical long-term guaranteed block, T0 might be in the range of five to 10 years.
2. Group the scenarios by rate quartile at T0, from lowest (quartile 1) to highest (quartile 4). Calculate the magnitude of dispersion of low-rate scenarios from central scenarios dispersion (T0) = average rate (T0) within combined (quartile 2 and quartile 3) – average rate (T0) within quartile 1.
3. Using the same scenario grouping (ranked at T0, *not* re-ranked at T0+10) calculate 10-year-later dispersion (T0+10, ranked T0) = average rate (T0+10) within combined (quartile 2 and quartile 3) – average rate (T0) within quartile 1.
4. The mean reversion criterion over the projection period from T0 to T0 +10 is satisfied if dispersion (T0+10, ranked T0) > = 0.5 * dispersion (T0).
5. If the actuary can demonstrate that the model rate of mean reversion is similarly robust across other projection periods, this single test would be sufficient. If not, the test would be repeated across sufficient financially meaningful periods to demonstrate sustained periods of low rates.
6. Should periods of sustained high rates be financially stressful for a particular application in the opinion of the actuary, the demonstration would be repeated for these rates (quartile 4 relative to quartiles 2 and 3).

A model with a single regime and simple linear mean reversion (i.e., E(r(t+dt) =r(t) + (1/ reversion period)* dt* (long-term mean – r(t)) can be demonstrated to satisfy this calibration criteria (with sufficient numbers of scenarios) if the reversion period > 14.5 years[8]. If the projection period (dt) is greater than one month, the mean reversion period threshold may need to be slightly adjusted.

Models would generally not be used with characteristics that would invalidate the

[8] With this simple mean reversion, at the continuous limit, E(r(t+n))= long-term mean + exp(-n/reversion period) *(r(t) – long-term mean). For an elapsed period n of 10 years, the exponentially decaying weight on initial rate will be >= 0.5 when mean reversion period >= 10/ ln(2) =14.42.

statistical intent of this criterion (i.e., a cyclical component of rates with roughly 10-year periodicity). Should exceptional circumstances make such a model appropriate in the opinion of the actuary, the actuary would develop robust statistical methods appropriate to the model characteristics to demonstrate substantive sustained periods of low rates, consistent with this criterion.

Finally, it appears likely that stochastic risk-free interest rate models that satisfy both the long-term equilibrium tail calibration criteria, and reproduce close to historically representative volatility, will also satisfy this mean reversion criterion, although some models may possibly require modest parameter adjustment. Some mean reversion estimates based upon statistical fit to rate change history may estimate somewhat stronger (shorter period) or weaker (longer period) mean reversion than that of this calibration criteria. Statistical estimates of mean reversion tend to have large uncertainty, and may vary greatly depending upon the specific historical period used for estimation. Therefore, mean reversion that is stronger than that of this criterion, even if it is a statistical best estimate, may provide spurious comfort regarding the potential likelihood of sustained periods of extreme rates.

Appendix E

The historical one-year rates from 1936–1979 were estimated as follows:

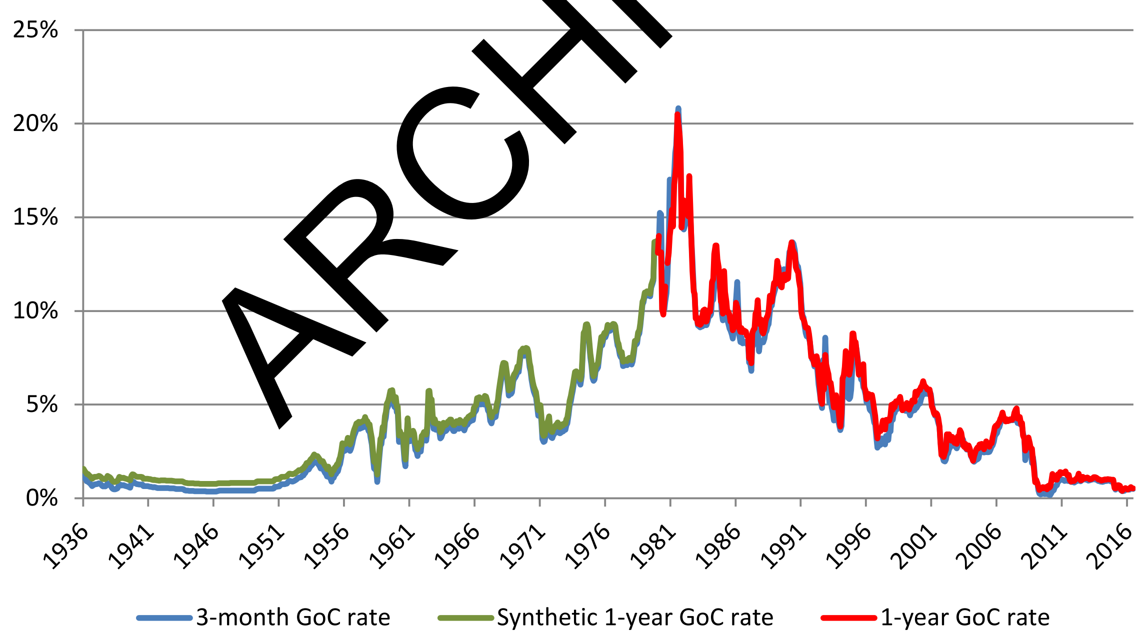
- Start with two monthly historical data series: three-month rates (Bank of Canada Series V122541) and one-year rates (Bank of Canada Series V122533). Pair the data according to dates.
- Perform a least squares linear regression using all available data pairs to estimate the relationship between the three-month and one-year rates.
    - For the analysis done for this educational note supplement, the available data pairs were from January 1980 to June 2016;
    - The estimated linear regression formula based on this pairs was

$$One\ year\ rate = 0.413289\% + 0.974584\ \times Three\ month\ rate$$

- Where the three-month rate is available, but the one-year rate is not, use the linear regression function estimated from the available data to calculate a synthetic one-year rate.

The final one-year time series is shown on the graph below, along with the three-month time series, for comparison.

CAD – January 1936 to June 2016



Source: Bank of Canada, Series V122541 and V122533