

# Study

# Canadian Group Annuitant Mortality Experience – Calendar Years 2007–2016

# Research Council – Experience Research Committee

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# Canadian Group Annuitant Mortality Experience Calendar Years 2007–2016

# 1 Executive Summary

This is the second report of a study of experience under the group annuities of life insurance companies. There are 10 years of data included, and the amount of experience is more than that of any other CIA annuitant experience study.

This report includes the development of a new mortality table, <u>GAC2012</u>, based on the experience under study. There are some significant differences from CPM2014priv, which was used for expected mortality in the prior study.

This study shows that the new table, projected on MI-2017, fits the experience well, particularly by age and by year of experience. As observed in the prior report, the variation in experience by size is significant, and the size adjustment factors developed for CPM2014 seem to fit the data well.

# 2 Table of Contents

1	Exe	cutive Summary	1
2	Tab	le of Contents	1
3	Intr	oduction	2
4	Ove	erview	3
5	Ana	ılyses	9
	5.1	Single/Joint	9
	5.2	Defined Benefit or not	10
	5.3	Size of income	10
	5.4	Duration from retirement	14
6	Sup	plementary Information	15
7	Pro	ject Oversight Group	15
8	App	pendix: Construction of GAC2012	16
	8.1	Subset of data	16
	8.2	Graduation for main ages	22
	8.3	Extension to younger ages	24
	8.4	Extension to older ages	26
	8.5	Final table	29
	8.6	Excluding secondary annuitants	30
9	Ref	erences	32

# 3 Introduction

In 2014, the CIA published a mortality table based on data collected from a number of pension plans. Some insurance companies wondered if the experience under group annuitant policies would be similar or distinct from the data set underlying the new table. Accordingly, the CIA started an intercompany study of group annuitant experience of life insurance companies in Canada, referred to as the Group Annuitant Mortality Study (GAMS). The first report on experience, for calendar years 2007–2012 was published in 2016. The study has now been expanded to include four additional years of data, to 2016. This report examines the experience similarly to the prior report. (Some of the observations of the prior report are repeated here; the prior report is not a prerequisite for understanding this one.) Since the prior report, the CIA has published a new mortality improvement scale, MI-2017; that scale is used in this report in most cases for mortality improvement. (CPM-B is used in a few cases for comparison.)

Six life insurance companies currently participate in the intercompany study. Table 1 shows the companies and the proportion of exposure, measured by income, contributed by each.

Table 1. Proportion of exposure				
by income from cont	ributing			
companies.				
Company	Share			
Canada Life	18%			
Desjardins	7%			
Industrial-Alliance	15%			
Manulife	12%			
Standard Life 19%				
Sun Life	28%			

Unless stated otherwise in this report, expected mortality is calculated on GAC2012 with projection on MI-2017 to the appropriate calendar year, without size adjustments. (GAC2012 is a new table, a Group Annuitant (Canada) mortality table with experience centred on 2012. The table is based on the same experience as presented in this report. The construction of the table is described in the <a href="mailto:appendix">appendix</a> of this report.) This mortality basis is called "qGA" in this report for convenience. GAC2012 is used rather than CPM2014priv, as was used in the prior report, because GAC2012 was developed on the data under study. MI-2017 is used rather than CPM-B because it was more recently developed by the CIA and intended as a general-purpose improvement scale. The use of qGA should not be inferred as an indication of it being recommended for this block of business. This study will observe how closely qGA fits to group annuitant mortality experience.

Several tables in this report show standard deviations in the actual-to-expected (A/E) ratios. These ratios are calculated on the assumption that the exposure of each life to death in the next year is independent of the exposure for all other lives, that the number of deaths for any group of lives with the same sex-age-year is binomially distributed, and that the mean of the distribution is given by qGA. The formula for standard deviation is shown below, by income, where  $K_i$  is the annualized income and  $n_i$  is the number of annuitants with that income and that sex-age-year. The sum is over all annuitants under consideration. The same formula may be used by count except that  $K_i$  is 1 in all cases.

Standard Deviation of A/E by Income = 
$$\frac{\left(\sum_{i} K_{i}^{2} n_{i} p_{i} q_{i}\right)^{0.5}}{\sum_{i} K_{i} n_{i} q_{i}}$$

#### 4 Overview

All six companies contributed data for the calendar years 2007 to 2016. The data were subjected to checks to ensure consistency from one year to the next, and corrections were made as needed. All companies have signed off on their data as sufficiently accurate for the purpose of this study.

The data submitted and study method are very similar to that used by the individual annuitant mortality study. The exception is that this study has also requested industry codes for each group and postal codes for each annuitant. Not all companies can provide industry codes or postal codes at this time, and accordingly reporting on these has been deferred until enough companies can include these data.

This study distinguishes experience based on sex, year of birth, year of experience, amount of annualized income, whether the pension arises from a DB plan, and whether the annuity is single life or joint.

Each record is taken as distinct. Although companies are encouraged to submit only one record per life, not all are able to do so. Thus, the heading "count" in tables refers to a record count and may be a little higher than a life count. Studies by amount of annualized income consider the income of each record; in some cases records may appear in a band lower than is appropriate for the life. Of course, in some cases records for a life may appear in multiple companies; there is no feasible way to combine such records.

Each contributing company provided factors to be applied to reported deaths to make an estimate of incurred but not reported deaths (IBNR). All deaths and the annualized income of deaths in this report have been adjusted for IBNR as of the date of the data extract for 2016. The same factor is used for both count and income of deaths. Exposures are not adjusted.

Practices in verifying the continued existence of annuitants vary significantly between companies, particularly for secondary annuitants. The IBNR factors vary correspondingly.

The IBNR factors of the contributing companies are considered confidential. However, table 2 is included to give readers an idea of the magnitude of the factors. The table shows the simple average of the IBNR factors for the six contributing companies, combining male and female. For example, deaths on single life annuities submitted three years earlier are increased by 0.8% to allow for IBNR. Please note that the average is not necessarily meaningful because of the variability by company, particularly in the earlier years.

Table 2. Simple average of IBNR factors.					
Delay	Single	Primary	Secondary		
1	6.4%	5.6%	50%		
2	1.2%	1.1%	18%		
3	0.8%	0.6%	9.5%		
4	0.3%	0.1%	6.3%		
5	0.1%	0.0%	3.4%		
6	0.0%	0.0%	2.2%		

Table 3 compares the exposure and deaths included in this report (and used to construct GAC2012) to the totals from the studies that underlie the construction of the individual mortality table CIP2014 and the private sector pension table CPM2014priv. (Not all ages were actually used in table construction.) The table shows that the group annuitant data set is sizable, larger than the others, except for the count of deaths.

Table 3. Data in recent studies. (annualized income in thousands)							
		Exp	osure	De	eaths		
Source	Table	Count	Income	Count	Income		
Group Annuitants	GAC2012	3,773,070	21,844,766	180,609	888,721		
Individual Annuitants	CIP2014	3,649,413	14,845,090	225,438	809,949		
DB Pensioners (priv)	CPM2014priv	962,899	10,519,535	46,838	372,876		

Chart 1 shows the distribution of exposure, by income, into mainly quinquennial age groups. The median age is 75.9 for males and 74.7 for females. There is relatively little exposure other than in the range 60–89. Note that the age distribution for females is slightly flatter than for males.

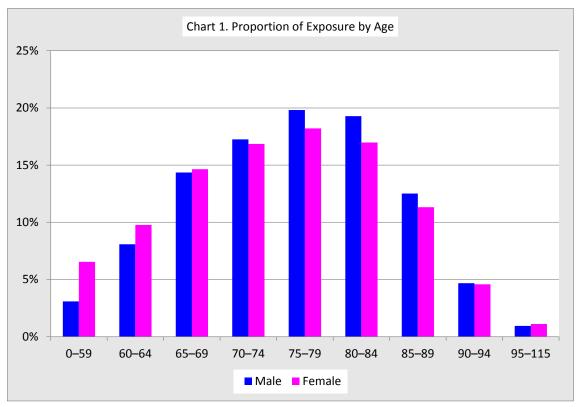


Chart 2 shows the distribution of exposure, by income, over each of the 10 calendar years included in the study. The distribution is fairly flat, increasing slightly toward more recent years. The increase is greater for females than for males.

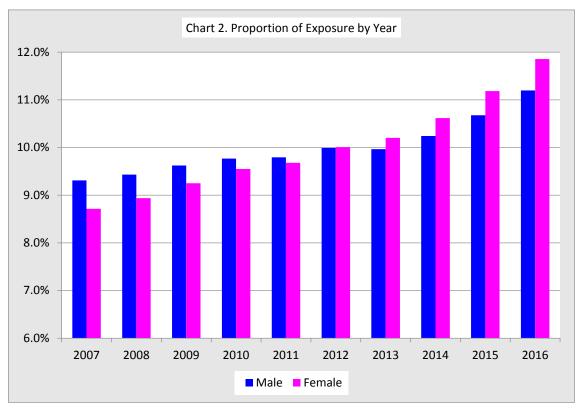


Chart 3 shows the distribution of the annuities in the study by amount of monthly income, weighted by income. As the chart suggests, the average size is in the first income band. The average size for males is \$518 per month, and for females, \$446. (However, there may be some small annuities that represent adjustments to the income for a pensioner rather than the full amount of the pension; 13% of male annuities by count are for less than \$50 per month, 14% for females. If these small annuities are ignored, the averages increase to \$590 and \$513 for males and females, respectively.) Although there are some large annuities included, the chart shows that small annuities predominate.

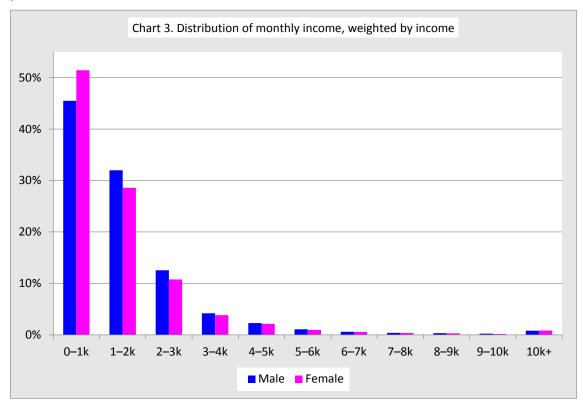


Table 4 shows the summarized experience by mainly quinquennial age groups, and table 5 by years of experience. Both show data by income only. If qGA were a good fit for the mortality experience of this data set, the A/E ratios would be mostly within one or two standard deviations of 100%. In fact, the fit shown by table 4 is very good except for ages under 60 and over 94, neither of which has sufficient data to support table construction. The fit shown by table 5 is almost as good. The A/E ratios are within two standard deviations of 100% except for males in 2007 and 2013 and females in 2016.

Table 4. Summarized experience by age. Expected on GAC2012 with MI-2017. (annualized income in thousands)

meetine in thousands,										
	Expo	sure	Deaths		sure Deaths Actual/Expecte		xpected	Standard Deviation		
Ages	Male	Female	Male	Female	Male	Female	Male	Female		
0–59	366,403	651,504	2,181	2,123	119.2%	117.0%	10.2%	10.6%		
60–64	959,499	974,043	7,354	4,796	94.9%	102.4%	5.0%	6.2%		
65–69	1,706,242	1,458,575	22,329	11,505	101.5%	101.7%	2.7%	3.7%		
70–74	2,049,217	1,679,568	43,720	21,776	99.3%	98.6%	1.9%	2.6%		
75–79	2,354,065	1,814,940	83,363	42,293	99.8%	100.8%	1.4%	1.9%		
80–84	2,290,408	1,690,909	136,378	67,135	99.9%	99.5%	1.2%	1.7%		
85–89	1,487,608	1,128,002	153,379	82,864	99.0%	99.5%	1.2%	1.6%		
90–94	555,340	456,149	96,968	58,565	100.8%	98.8%	1.6%	1.7%		
95–115	112,051	110,243	27,944	24,047	90.8%	97.4%	2.3%	2.3%		
All	11,880,835	9,963,931	573,616	315,104	99.3%	99.6%	0.6%	0.8%		

Table 5. Summarized experience by year. Expected on GAC2012 with MI-2017. (annualized income in thousands)

	Expos	sure	Deaths		Actual/Expected		Standard Deviation	
Year	Male	Female	Male	Female	Male	Female	Male	Female
2007	1,106,055	868,278	49,822	24,791	94.5%	99.1%	1.9%	2.6%
2008	1,120,731	890,585	52,616	26,184	97.4%	99.6%	1.9%	2.6%
2009	1,143,225	921,608	53,976	26,946	98.1%	97.3%	1.9%	2.5%
2010	1,160,638	951,711	56,433	29,585	99.9%	101.0%	1.9%	2.5%
2011	1,163,559	964,355	55,125	30,754	96.4%	100.6%	1.9%	2.4%
2012	1,187,036	997,268	57,955	31,852	99.3%	99.2%	1.9%	2.4%
2013	1,183,927	1,016,563	61,138	34,004	104.4%	101.6%	1.9%	2.4%
2014	1,216,879	1,057,921	59,599	35,446	99.7%	100.5%	1.7%	2.3%
2015	1,268,383	1,114,262	63,324	38,179	102.6%	102.3%	1.7%	2.3%
2016	1,330,400	1,181,381	63,629	37,365	99.8%	94.6%	1.7%	2.2%
All	11,880,835	9,963,931	573,616	315,104	99.3%	99.6%	0.6%	0.8%

Table 6 shows A/E ratios by age group with expected on qGA and on CPM-2014priv with CPM-B (the basis used in the prior report). The difference between the two bases appears in the last two columns. The difference can be rather large, even in the middle age groups. Not surprisingly, we conclude that the mortality basis developed from the data fits the data much better.

Table 6. Actual-to-expected ratios on two mortality bases, calculated by							
annualize	d income.						
	GAC2012,	/MI-2017	CPM2014p	riv/CPM-B	Differenc	es in A/E	
Ages	Male	Female	Male	Female	Male	Female	
0–59	119.2%	117.0%	103.0%	123.8%	16.2%	-6.7%	
60–64	94.9%	102.4%	82.9%	99.1%	11.9%	3.3%	
65–69	101.5%	101.7%	102.5%	100.4%	-1.1%	1.4%	
70–74	99.3%	98.6%	106.9%	103.4%	-7.6%	-4.8%	
75–79	99.8%	100.8%	101.3%	106.5%	-1.5%	-5.7%	
80–84	99.9%	99.5%	96.6%	96.1%	3.3%	3.3%	
85–89	99.0%	99.5%	97.7%	95.9%	1.3%	3.6%	
90–94 100.8% 98.8% 99.7% 95.8% 1.2% 3							
95–115	90.8%	97.4%	84.5%	90.8%	6.3%	6.6%	
All	99.3%	99.6%	98.1%	97.7%	1.2%	1.9%	

# 5 Analyses

# 5.1 Single/Joint

The data distinguish three types of annuitant: single life, joint life for the primary annuitant, and joint life for the secondary annuitant. (After the death of the primary annuitant, the secondary annuitant, if then living, is thereafter considered the primary annuitant.) Not all companies were able to include data for a secondary annuitant. It was not always possible to distinguish a single surviving annuitant of a joint and last survivor annuity from a single life annuitant, particularly when the insurance company took on the risk after the first death.

Table 7 summarizes the experience by annuity type.

Table 7. Summa	Table 7. Summarized experience by annuitant type. Expected on GAC2012 with MI-2017. (annualized							
income in thous	ands)							
	Expo	sure	Dea	ths	Actual/E	xpected	Standard	Deviation
Type	Male	Female	Male	Female	Male	Female	Male	Female
Single	4,528,449	2,742,235	265,707	137,068	103.3%	106.8%	0.7%	0.9%
Joint primary	6,893,580	2,015,036	291,101	80,710	95.8%	97.9%	0.8%	1.8%
Joint secondary	458,806	5,206,661	16,808	97,326	102.6%	92.0%	6.6%	1.5%
All	11,880,835	9,963,931	573,616	315,104	99.3%	99.6%	0.6%	0.8%

The A/E ratios are lower in the case of females for secondary annuitants than for primary, and significantly so. It is generally observed that mortality rates for married are markedly lower than for widows or other singles. A female would be secondary annuitant only if the primary is still alive. The secondary annuitant is often, but not always, living with the primary. Therefore, it is not surprising that the A/E ratio for secondary is noticeably lower than for primary. The fact that the A/E ratio for male secondary is higher than for male primary is more surprising, but there is very little exposure for male secondary annuitants.

#### 5.2 Defined benefit or not

The data distinguish three types of pension plan: defined benefit (DB), other types (not DB) such as defined contribution and group RRSP, and type unknown. One company used "unknown" when its administrative system did not have the pension plan type.

Table 8 summarizes mortality experience by pension plan type. Unfortunately, there is so much "unknown" that little can be inferred. For those that are distinguished, the difference is that experience between DB and not DB does not appear to be statistically significant.

Table 8. Summarized experience by pension plan type. Expected on GAC2012 with MI-2017.								
(annualized	income in th	ousands)						
	Expos	sure	Dea	ths	Actual/E	xpected	Standard I	Deviation
Plan	Male	Female	Male	Female	Male	Female	Male	Female
DB	5,641,739	4,963,704	252,269	131,264	100.7%	96.1%	0.9%	1.1%
Not DB	1,441,313	1,297,650	78,758	45,584	98.2%	98.9%	1.4%	1.7%
Unknown	4,797,783	3,702,578	242,589	138,256	98.3%	103.3%	0.9%	1.2%
All	11,880,835	9,963,931	573,616	315,104	99.3%	99.6%	0.6%	0.8%

# 5.3 Size of income

The data underlying CPM2014 showed a strong correlation between the level of mortality and the size of the pension, so much so that size adjustment factors were published along with the CPM2014 tables. It is important to discern if the group annuity data show a similar relationship.

Table 9 shows the experience by income band. Each band is shown as \$6,000 wide in annualized income. Like the CPM report, the bands are adjusted approximately based on the average weekly earnings (AWE). The CPM report suggests that AWE was about 95% during 2007–2016 of what it was in 2014. Accordingly, the first income band is actually of annualized amounts of 0–\$5,699, the second \$5,700–11,399, etc. This adjustment to the bands was used for all years of experience. It was not practicable to reflect the variation in AWE year by year as was done in CPM. The expected mortality in table 9 is on qGA, and like all prior tables and charts, has no size adjustment.

As indicated by the standard deviations, little can be inferred for the higher bands, but the lower bands show a strong decreasing trend with increasing income. The observations from the higher bands should be used with caution.

Т	Table 9. Summarized exp	erience by annual	income range.	Expected of	on GAC2012 with I	VII-2017
l۷	without size adjustments	(annualized incor	me in thousand	15)		

without size adjustments. (annualized meome in thousands)								
Annual	Expo	sure	Deaths		Actual/Expected		Standard Deviation	
Income	Male	Female	Male	Female	Male	Female	Male	Female
0–5,999	2,510,104	2,600,554	148,427	107,206	108.0%	107.1%	0.5%	0.5%
6,000–11,999	2,657,655	2,329,001	143,861	78,191	105.3%	103.1%	0.7%	1.0%
12,000–17,999	2,321,169	1,758,022	100,960	45,945	101.7%	97.9%	1.1%	1.7%
18,000–23,999	1,485,704	1,114,943	58,707	27,418	94.4%	92.9%	1.7%	2.5%
24,000–29,999	1,163,482	817,346	41,589	19,001	85.3%	92.0%	2.2%	3.4%
30,000–35,999	450,590	348,270	18,701	8,870	94.7%	90.7%	3.8%	5.4%
36,000–41,999	316,292	244,159	12,448	5,774	84.9%	85.6%	4.8%	7.1%
42,000–47,999	222,812	162,939	9,243	4,268	79.3%	94.3%	5.7%	9.4%
48,000–53,999	175,734	140,644	7,149	3,699	82.6%	85.0%	7.1%	10.2%
54,000-59,999	123,377	93,602	6,485	3,124	94.2%	87.0%	8.4%	11.7%
60,000–65,999	92,148	71,076	5,630	2,474	111.5%	97.5%	10.4%	14.8%
66,000–71,999	46,995	31,931	1,399	1,053	53.1%	104.8%	14.9%	24.5%
72,000+	314,772	251,444	19,015	8,081	79.5%	73.6%	8.1%	13.2%
All	11,880,835	9,963,931	573,616	315,104	99.3%	99.6%	0.6%	0.8%

Chart 4 shows the A/E ratios graphically. The downward slope is quite evident. The slope for males appears to be slightly steeper than for females, but the difference is not statistically significant.

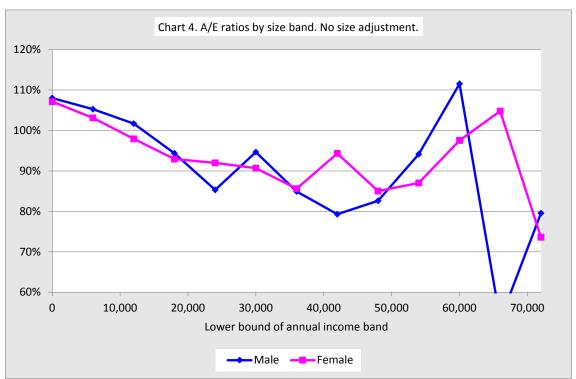


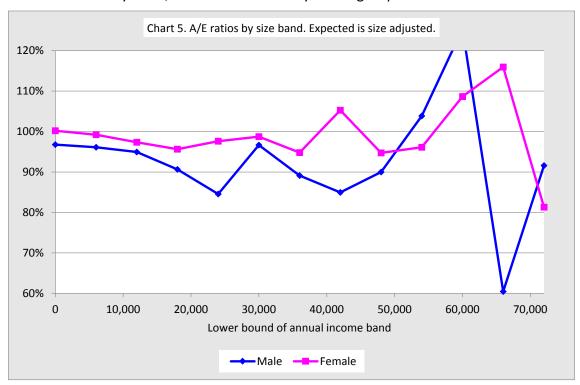
Table 10 is similar to table 9 except that the expected mortality rates have been size adjusted using the factors published with CPM2014priv. If the size adjustment factors were appropriate for the group annuitant data, the A/E ratios would be very flat in table 10.

Table 10. Summarized experience by annual income range. Expected on GAC2012 with MI-2017
with size adjustments. (annualized income in thousands)

with size adjustments. (annualized income in thousands)								
Annual	Expo	sure	Dea	ths	Actual/E	xpected	Standard Deviation	
Income	Male	Female	Male	Female	Male	Female	Male	Female
0–5,999	2,510,104	2,600,554	148,427	107,206	96.8%	100.2%	0.4%	0.5%
6,000–11,999	2,657,655	2,329,001	143,861	78,191	96.1%	99.2%	0.7%	1.0%
12,000–17,999	2,321,169	1,758,022	100,960	45,945	95.0%	97.4%	1.1%	1.7%
18,000–23,999	1,485,704	1,114,943	58,707	27,418	90.6%	95.6%	1.7%	2.5%
24,000–29,999	1,163,482	817,346	41,589	19,001	84.6%	97.6%	2.2%	3.5%
30,000–35,999	450,590	348,270	18,701	8,870	96.7%	98.7%	3.8%	5.7%
36,000–41,999	316,292	244,159	12,448	5,774	89.1%	94.8%	4.9%	7.5%
42,000–47,999	222,812	162,939	9,243	4,268	84.9%	105.3%	6.0%	10.0%
48,000–53,999	175,734	140,644	7,149	3,699	90.0%	94.7%	7.5%	10.7%
54,000-59,999	123,377	93,602	6,485	3,124	103.8%	96.1%	8.9%	12.4%
60,000–65,999	92,148	71,076	5,630	2,474	125.4%	108.6%	11.1%	15.6%
66,000–71,999	46,995	31,931	1,399	1,053	60.4%	116.0%	16.0%	25.8%
72,000+	314,772	251,444	19,015	8,081	91.6%	81.3%	8.8%	13.9%
All	11,880,835	9,963,931	573,616	315,104	94.1%	98.3%	0.5%	0.7%

The overall A/E ratios in table 10 are down from those in table 9, from 99.3% for males to 94.1%, and from 99.6% for females to 98.3%. The issue is that the CPM size adjustment factors are not tuned for the GAMS data. If the male adjustment factors were all decreased by 0.065 for males and 0.016 for females, the A/E ratios would be the same overall as in table 9.

Chart 5 shows the A/E ratios of table 10 graphically. It is clear that the lines are fairly flat for under the \$48,000–53,999 band. The trend being flat suggests that the variation by size is similar to that observed in the CPM study. That is a startling fact! Size adjustment factors were developed from a completely different data set and in conjunction with a different mortality table, but nonetheless they fit the group annuitant data well.



It is important to state that the relationship between the level of mortality and the amount of pension income is one of correlation not causality. There are many objections that can be raised to size adjustments. For example, one would not expect different mortality for two men who are otherwise very similar but one worked for 40 years under one pension plan and the other spent 10 years under each of four different plans, but size adjustment suggests heavier mortality for the latter.

This study does not allow us any insight into the individual circumstances of pensioners beyond the demographic data submitted. Nonetheless, it is clearly in the data that mortality rates go down dramatically as income goes up, at least over the range for which there is an abundance of data. Actuaries may be advised to be alert to possible applications of the size relationship, but whether a size adjustment is appropriate in a particular case is beyond the scope of this study.

# 5.4 Duration from retirement

In most cases, the contributing companies are not able to determine the date of retirement. The recorded date of issue relates to when the company took on the risk; in the case of a de-risking strategy by the plan trustees, that date may be many years after

retirement. Accordingly, duration from the issue of the annuity is not relevant and is not studied in this report.

# 6 Supplementary Information

For those who wish to study the data further on their own, a tool used in developing this report is available for download. That tool is an Excel workbook and a binary file containing the summarized intercompany data. These two files and a text file with installation instructions are combined into a <u>zip file</u>.

# 7 Project Oversight Group

The Group Annuitant Experience Project Oversight Group (POG) is responsible for the content of this report. The members of the POG who participated in the review of this report are listed below. Bob Howard was engaged by the CIA to compile the data for the report on behalf of POG.

Caroline Archambault
Jonathan Boivin
Paul Burnell
Anna Doudina
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Olivier Guilbaud (Chair)
Jennie Leung
Edward O'Malley
Sylvain Veilleux
Benny Wan

# 8 Appendix: Construction of GAC2012

Because there is now a large enough aggregate of data to warrant constructing a table, the CIA requested that table construction be part of this project. The details of construction are presented as an appendix because, although actuaries need to know that the table fits the underlying data well, knowledge of construction is not essential. However, many will want to understand the method of construction, and a few may wish to verify that the method was applied accurately or attempt a variation of the table from first principles.

# 8.1 Subset of data

The table should be based on the data, but that does not mean that all data must be used. There are segments of the data where exposure is too thin to be usable.

#### 8.1.1 Ceiling on income included

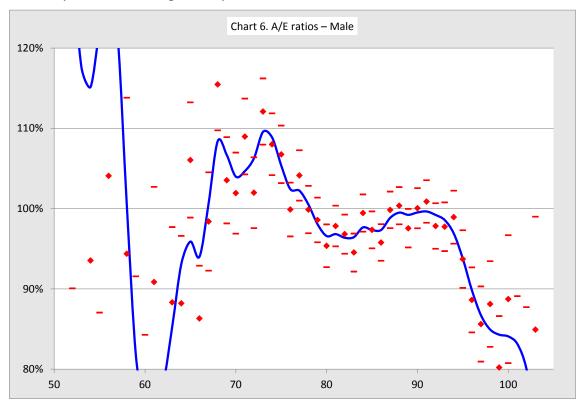
The first check is what ceiling, if any, to put on the amount of income included. Table 11 shows exposure and deaths for various ranges of annualized income. The ranges are shown by the lower limit; the highest range is unlimited at the upper end. The column labeled "Percent" is the percentage of total income in the respective range.

It is possible that very large annuities could cause anomalies because their impact on the total is so large. That does not appear to be the case with the set of data. However, it is still wise to limit the maximum impact of any one annuity. Note that less than 0.25% of income lies in any of the ranges shown over \$120k of income. It was decided, admittedly arbitrarily, to set a ceiling at \$120k. Thus any annuity larger than \$120k of annualized income is included in the construction of the table for only \$120k.

Table 11. Distribution by range of annualized income. (income in thousands)						
Lower		Exposure			Deaths	
Limit	Count	Income	Percent	Count	Income	Percent
0	3,765,165	21,123,134	96.70%	177,631	840,983	96.34%
60,000	4,701	319,423	1.46%	187	12,713	1.46%
80,000	1,567	140,187	0.64%	77	6,878	0.79%
100,000	758	82,860	0.38%	32	3,494	0.40%
120,000	377	49,955	0.23%	25	3,327	0.38%
150,000	264	42,942	0.20%	19	3,017	0.35%
200,000	150	36,565	0.17%	4	997	0.11%
300,000	87	49,700	0.23%	2	1,485	0.17%
All	3,773,070	21,844,766	100.00%	177,977	872,893	100.00%

# 8.1.2 Ages to include

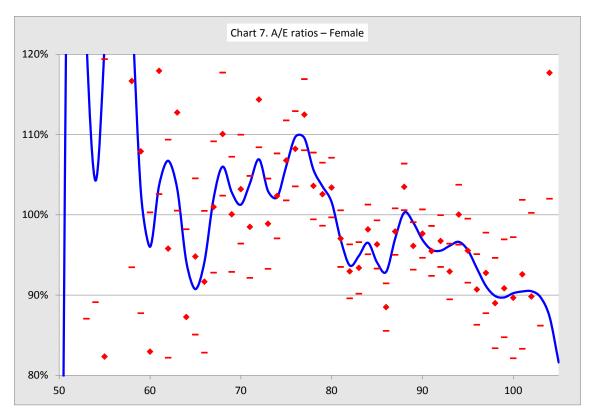
The next issue to consider is what ages to include in the graduation of the data. Charts 6 and 7 show A/E (on CPM2014priv with CPM-B) ratios. The red diamonds indicate the observed ratio. The red ticks above and below indicate the observed ratio plus or minus one standard deviation. The blue lines represent the ratio of graduated actual, with relatively little smoothing<sup>1</sup>, to expected.



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<sup>&</sup>lt;sup>1</sup> Graduated A/E ratios with Whittaker-Henderson, order of difference 4, balancing factor 0.1. Weights are normalized expected by income.



It is evident that the ages in the middle of the range have good support in the data, but there are too little data under age 60 to have much certainty about the estimated ratios from the experience.

Table 12 shows an unexpected pattern. Mortality rates are surprisingly flat below age 62. There is not sufficient information to discern the reason for this pattern. It is possible that retirements at younger ages are disproportionately for those in ill health. It is not clear whether the final table should reflect this flattening of the mortality curve. Because of the uncertainty, graduated rates will be used only down to age 65.

In tables 12 and 13, the columns marked "Smoothed" are derived from the same graduation as used the blue line in charts 6 and 7.

Table 12. Mortality rates at various ages.					
	Raw M	ortality	Smoo	othed	
Age	Male	Female	Male	Female	
55	0.0058	0.0019	0.0058	0.0029	
56	0.0055	0.0044	0.0067	0.0039	
57	0.0090	0.0048	0.0071	0.0044	
58	0.0061	0.0038	0.0065	0.0041	
59	0.0053	0.0039	0.0058	0.0037	
60	0.0054	0.0033	0.0058	0.0038	
61	0.0076	0.0052	0.0064	0.0045	
62	0.0062	0.0046	0.0071	0.0051	
63	0.0086	0.0060	0.0083	0.0055	
64	0.0091	0.0051	0.0096	0.0055	
65	0.0117	0.0061	0.0106	0.0058	

Over age 95 there is a different problem. The higher the age, the lower the mortality rates are relative to any reasonable expectation. This is a typical pattern that is observed when there is incomplete reporting of deaths or deaths are reported very late (Howard, 2014). It is likely that the insurance companies, despite their best efforts, have lost contact with some of the very elderly, some of whom have already died. The result of the lack of reporting is that deaths are understated and exposure is overstated. The problem is large enough to notice only at very high ages.

The pattern can be seen clearly in table 13, especially for males. It is not reasonable for the mortality curve to turn downward over age 100, but that is what we observe.

Table 13. Mortality rates at various ages.					
	Raw M	ortality	Smoothed		
Age	Male	Female	Male Femal		
95	0.234	0.187	0.234	0.187	
96	0.243	0.198	0.246	0.204	
97	0.256	0.226	0.259	0.222	
98	0.284	0.240	0.274	0.242	
99	0.277	0.268	0.291	0.264	
100	0.327	0.285	0.310	0.287	
101	0.309	0.316	0.324	0.308	
102	0.316	0.325	0.330	0.327	
103	0.369	0.277	0.319	0.342	
104	0.249	0.469	0.282	0.348	
105	0.201	0.258	0.210	0.342	

Because of concerns about the accuracy of data at the highest ages, the final table will use graduated rates only up to age 95.

#### 8.1.3 Central Year

It has become traditional to base annuitant mortality tables on the year of publication by employing mortality improvement to that year. Some actuaries who have been involved in constructing recent tables have concluded that this choice is unfortunate. When a new improvement scale is developed, that scale cannot be conveniently applied directly to the published table because the experience underlying the table was from many years earlier. It seems wiser to base the table on the central year of the underlying data. Then the application of a new improvement scale is straightforward.

Table 14 shows the average year of experience for various age groups and for all ages combined. In this case, the average year is close to the same for all age groups. That is not always the case. For all ages, the average year is 2011.70; that is, the average year of experience started approximately on September 14, 2011. It therefore seems reasonable to use 2012 as the base year of the table.

Table 14. Average year of experience				
by age group.				
	Avg. Year of	Exposure		
Ages	Experience	\$billions		
under 50	2011.49	0.07		
50–59	2011.73	0.95		
60–69	2011.79	5.10		
70–79	2011.49	7.89		
80–89	2011.75	6.56		
90–99	2012.46	1.20		
over 99	2012.75	0.02		
All	2011.70	21.77		

#### 8.1.4 Adjusting to 2012

If 2012 is to be the base year, how do we get data to represent 2012? There are many reasonable approaches, but the one best suited to the task is to adjust deaths using an accepted improvement scale, MI-2017 in our case, to 2012. The reason why this method is preferred is that we understand mortality improvement to be a multiplicative process, but averaging data over several years is an additive process. A simple example should clarify the issue.

Suppose there is a population that has had one million lives exposed in each of 2010–2016, that mortality has improved by 5% each year over that time, that the mortality rate for 2013 is 0.040, and that statistical fluctuation does not happen. In those circumstances the average year of experience is 2013. The deaths are 46,654, 44,321, 42,105, 40,000, 38,000, 36,100, and 34,295, for 2010–2016, respectively. We want a

mortality table with a base year of 2013, and we already know that we want the mortality rate to be 0.040. However, if we simply divide deaths by exposure, we get 0.040211, an error of about 0.5%. The error is in line with the standard deviation in the GAMS data, but in an idealized world without statistical fluctuation, we should be able to do better.

We have a problem because the experience of 2010–2012 and 2014–2016 does not represent the experience of 2013, which we seek. If indeed mortality is decreasing exponentially, the method of averaging experience over several years will always overstate the mortality rate at the centre of those years. Averaging is unbiased only if mortality decreases linearly. The bias is negligible if mortality rates are small or the period of years is short, neither of which is going to be true for an annuitant table.

If instead we adjust each year's deaths for improvement in mortality, we get a better result. For example, we ask how many deaths would we have recorded from the 2012 data if they had happened in 2013 instead of 2012. The answer is that the number of deaths would have been 5% less. We multiply the deaths of 2010–2012 by 0.95³, 0.95² and 0.95, respectively, and we divide the deaths of 2014–2016 by 0.95, 0.95², and 0.95³, respectively. Then for each of the seven years we have death counts consistent with 2013, and we can average the deaths to get the mortality rate. We get 0.040000 as expected.

One may object that in real life we are dependent on guessing the improvement rate correctly. True, but as long as we are close, we get a much better estimate than if we ignored mortality improvement (that is, assume the rate of mortality improvement is zero). In the above example, if we adjusted deaths assuming 4% improvement, the mortality rate would be calculated as 0.040009, which is much more accurate than 0.040211.

Our real-life data has an additional problem. The central year of experience is not integral, and it varies by age. Some might prefer to calculate the average mortality rate, and then adjust that rate with MI-2017 for the period of time from the actual average year of experience to the desired base year. However, that averaging still leaves us with the problem of using an additive process when we believe mortality improvement to be multiplicative. The method used avoids even having to calculate a central year, and it avoids the problem with the additive process.

All experience is adjusted on MI-2017 from the actual year of experience to 2012. (For example, experience of 2015 is increased to reflect the expected improvement from 2012–2015, and experience of 2010 is decreased to reflect the expected improvement from 2010–2012. Data for 2012 are not adjusted.) Thus all 10 years of experience are adjusted to be consistent with mortality in 2012.

Incidentally, CPM-2014 and CIP2014 both used a similar adjustment for mortality improvement. In the case of those tables, the adjustment was made to 2014, that being the year of publication. In the current case, adjustment is to the approximate central

year of experience. The latter seems preferred because the adjustments are smaller and the published table is less influenced by a particular improvement scale.

#### 8.1.5 Outliers

It is possible that there are some raw mortality rates that are not representative. For example, if there were a death for a very large amount at an age in which the exposure was light, the raw mortality rate might be unrepresentatively high. Setting a ceiling of \$120,000 of annualized income should help prevent such a problem, but it is still prudent to look.

Of course, simple statistical fluctuation is to be expected and is not considered an outlier. It is handled by the graduation process.

There are two tests employed.

The first is to look at successive A/E ratios. One would not normally see a ratio for one age fall outside of the ratio for the next age plus or minus two standard deviations.

The second test is to compare the mortality rate for all data (with the ceiling at \$120,000) with the mortality rate for data including annuities of \$60,000 or less. One would expect the rate for all data to be a little lower, but if it is more than 4% different, that might be a concern.

Only male age 63 failed both tests. The first test failed in comparing 62 with 63, but not 63 with 64. The mortality rate for male 63 on all data was 5% higher than the rate on data only to \$60,000.

There is good reason to conclude that there are no material outliers in the data which could unreasonably influence the table construction. No adjustments are made for outliers.

# 8.2 Graduation for main ages

As mentioned in the previous section, graduated rates are used without modification for ages 65–95. However, because graduation tends to be less reliable at the ends of the graduation, the graduation is applied to data for ages 61–100, and the results for the extreme ages are not used.

Those not familiar with Whittaker-Henderson graduation (WH) may wish to get an overview of the method from Howard (2007) or a more detailed description from London (1985) which was formerly on the Society of Actuaries (SOA) exam syllabus.

There are three parameters to choose, separately for male and female: the order of difference for calculating smoothness (referred to as "order"), the balancing factor to combine fit and smoothness (referred to as "balance"), and the exponent as specified in Lowrie's variation of WH (referred to as "exponent").

Many will be unfamiliar with exponent in WH. Classical WH has exponent = 0; then WH calculates smoothness as the sum of squared finite differences of the specified order. Perfect smoothness is a polynomial of degree, order -1. For example, if order is 3, then

third differences are used and a parabola is perfectly smooth. However, when exponent is non-zero, WH (using Lowrie's variation) has perfect smoothness if the graduated rates lie on an exponential with base (1 + exponent) plus a polynomial of degree order -2. For example, if order is 3 and exponent is 0.1, then an equation of the form  $a(1.1^x)+bx+c$  is perfectly smooth. Because mortality rates tend to increase exponentially, particularly over age 50, having a non-zero exponent may be an advantage.

From past experience, order 3 or 4 is likely to be most appropriate.

A good value for balance, when there is plenty of data, is typically near 100.

It is reasonable to test exponent = 0 and another value of exponent suggested by the raw data. The value was calculated by comparing the average mortality rate for ages 65–69 to that for ages 85–89. The twentieth root of the ratio of the latter divided by the former is 1.109 for males and 1.119 for females. Exponent was tested with these two values.

Table 15 shows the results of some of the tests for males. The column marked "Fit" shows the sum of squared differences between the raw and graduated rates for ages 65–95, multiplied by normalized weights. The columns marked "nth diff" show the sum of squared differences of the degree indicated for ages 65–95.

Table 15. Testing graduation parameters, for male ages 65-95 only.						
Exponent	Order	Balance	Fit	2nd diff	3rd diff	4th diff
0	3	30	4.97E-05	1.37E-05	4.13E-07	3.65E-08
0	3	100	5.52E-05	1.31E-05	2.34E-07	1.44E-08
0	3	300	6.35E-05	1.31E-05	1.31E-07	7.16E-09
0	4	30	4.57E-05	1.47E-05	6.73E-07	5.64E-08
0	4	100	4.68E-05	1.42E-05	5.08E-07	2.61E-08
0	4	300	4.81E-05	1.39E-05	4.20E-07	1.81E-08
0.109	3	30	5.00E-05	1.37E-05	3.95E-07	3.18E-08
0.109	3	100	5.70E-05	1.30E-05	2.15E-07	1.36E-08
0.109	3	300	7.03E-05	1.30E-05	1.05E-07	7.15E-09
0.109	4	30	4.58E-05	1.46E-05	6.42E-07	5.06E-08
0.109	4	100	4.66E-05	1.42E-05	4.89E-07	2.38E-08
0.109	4	300	4.74E-05	1.40E-05	4.25E-07	1.85E-08

The first thing to note is that there is not a bad choice among the nine sets of parameters shown in the table. Exponent 0.109, order 4, and balance 100 were chosen as giving the best compromise of fit and smoothness. The results for exponent 0, order 4, and balance 100 were almost as good.

For females, the same parameters were chosen except that the exponent is 0.119.

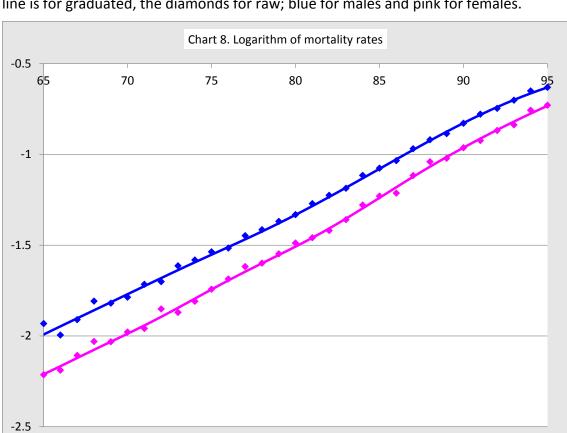


Chart 8 shows the logarithm of graduated and raw mortality rates for ages 65–95. The line is for graduated, the diamonds for raw; blue for males and pink for females.

#### 8.3 Extension to younger ages

Grad M

A table starting at age 65 will not do, but there is not enough exposure for ages under 65 to construct the rates. Clearly, rates under age 65 must be derived from some other source. However, it is not necessary to have the same degree of certainty for rates under age 65 as for older ages. If this table were used by an insurance company, it would be for pricing and valuing business that is almost entirely on retired lives. Therefore, rates under age 55 would be very rarely used.

Grad F

Raw M

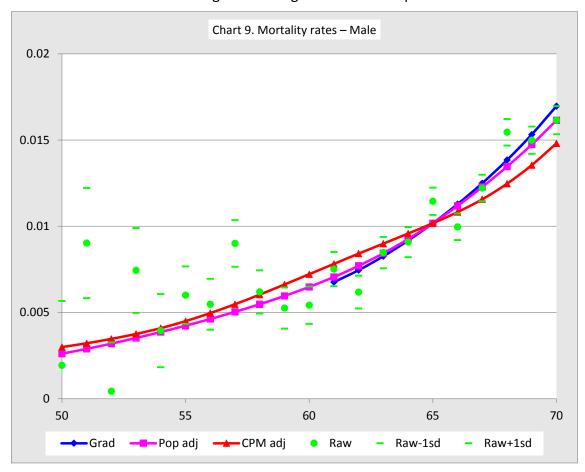
Raw F

One possibility is to fit a curve, like Makeham, to the graduated data. Testing shows that although the results appear good down to age 50, they are unacceptable by age 20. No curve was tried in this research that appeared to be reasonable over ages 20–65.

Another possibility is to use a multiple of CPM2014priv, which in turn uses a multiple of rates from an insurance table. Although an insurance table may be a good choice, it is unproven, and the interplay of select and ultimate mortality and of smoker and non-smoker status makes the appropriateness uncertain.

The method which ultimately was chosen is based on population data. The raw population mortality rates are taken as an average of the last five years available from the <u>Human Mortality Database</u>, years 2007–2011. The raw rates are adjusted to 2012 using MI-2017. The adjusted mortality rates are graduated<sup>2</sup> by WH. Then a multiple of those rates is used for ages 20–65 so that the age 65 rate is the same as produced by the graduation.

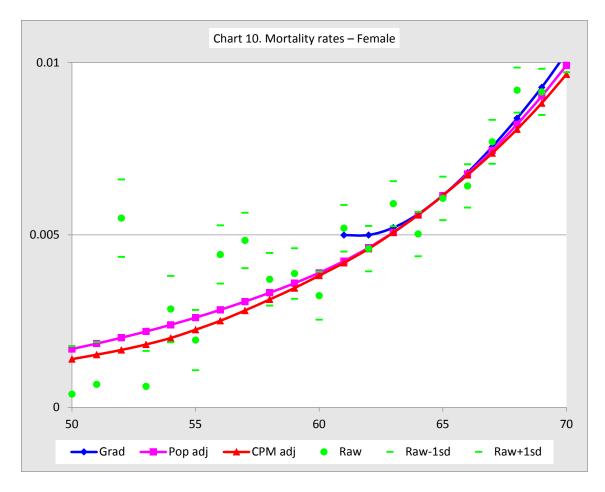
Charts 9 and 10 show the extension of mortality rates to age 50 using the latter two methods. The population curve is closer to parallel to the graduated curve and is therefore preferred. The curves called "CPM" are a multiple of CPM2014priv adjusted to 2012 with MI-2017 so that the graduated age 65 rates are reproduced.



25

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<sup>&</sup>lt;sup>2</sup> The graduation was done over age 15–109 with order 4, balance 20, and exponent 0.1 for males and 0.11 for females. The weights are the average exposures normalized.



One surprising feature in the data is that the slope of mortality rates under age 65, particularly under age 60, is much less steep than is typically observed. The flattening seems to extend to a higher age for females than for males, as can be seen in chart 10. By using a multiple of the population table, the slope over ages 55–65 is steeper than the raw mortality rates. There do not seem to be enough data to justify reflecting what may be an anomaly at ages under 65, but the emerging experience will bear watching.

# 8.4 Extension to older ages

There are enough data to continue for a few more ages at the older end, but as mentioned above, at those ages the data is of questionable accuracy.

Curve fitting holds more promise in this case. Gavrilova (2011) has observed that experience over age 100, when data are very carefully scrubbed, fits closely to Gompertz. Howard (2011) found a similar confirmation of Gompertz. Some actuaries prefer Kannisto; it was used recently by the SOA to finish its table RP-2014. Another possibility is Beard. The general expression for each of these models is given below.

Gompertz 
$$\mu_x = Bc^x$$

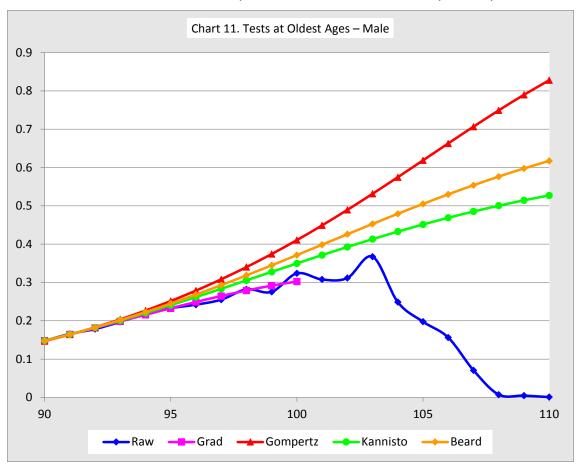
Kannisto 
$$\mu_x = \frac{Bc^x}{1+Bc^x}$$

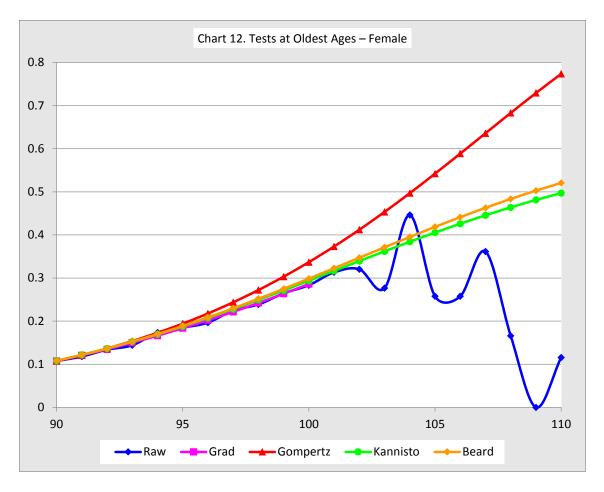
Beard 
$$\mu_{x} = \frac{Bc^{x}}{1+Dc^{x}}$$

Because the data at very high ages are less reliable, fitting to our data must not go to too high an age. Testing showed that the choice of age range was not particularly important for Gompertz and Kannisto. The results are much more sensitive to the age range for Beard. Because of its sensitivity and because for some age ranges the *D* parameter went negative, giving unreasonable mortality rates at very high ages, Beard was rejected.

The models were fitted with data adjusted to 2012, for ages 80–94, minimizing the weighted sum of squared errors between the raw mortality rates and those calculated on each of the models. Charts 11 and 12 show the resulting mortality rates for ages 90–110.

The blue line shows the raw mortality rates from our data set. The pink line shows the graduated mortality rates from our data set, up to age 100. The red, green, and orange lines are for the three models, Gompertz, Kannisto, and Beard, respectively.





The charts are interesting, but how does one decide which model fits best? The weighted sum of squared errors is very similar across all three models. It would be best to have some external data for a reasonableness check.

That check can be made using the data posted by the <u>Gerontology Research Group</u> of supercentenarians. This webpage lists dates of birth and death (unless still alive) for the 1739 individuals then known to have lived at least to age 110 and to have dates of birth verified. This information has been compiled to calculate mortality rates, age last birthday. Records are excluded for those who died prior to 1980. The results are shown in table 16, for ages 110–115.

Table 16. Mortality rates and deaths for global					
supercent	enarians.				
	Mortali	ty Rates	Count o	f Deaths	
Age	Male	Female	Male	Female	
110	0.457	0.456	69	702	
111	0.598	0.434	49	364	
112	0.516	0.469	16	206	
113	0.533	0.477	8	102	
114	0.571	0.618	4	63	
115	0.667	0.563	2	18	

The Kannisto model generates mortality rates much closer to those of table 16 than Gompertz. However, there are some cautions:

- 1. The supercentenarian data is for all countries of the world rather than just Canada.
- 2. There may be effects of heterogeneity. These very long-lived individuals may follow a different mortality curve than the rest of the population.
- 3. The data are age last birthday, but all the rest of the mortality rates in this report are calculated as age nearest birthday on January 1.

The final table uses the fitted Kannisto curve for ages 100–114. The rates for ages 96–99 are on the cubic curve passing through the otherwise obtained rates for ages 94, 95, 100, and 101. Incidentally, the fitted parameters *B* and *c* are 7.5407E-7 and 1.1474 for males and 3.5016E-7 and 1.1522 for females.

#### 8.5 Final table

The final rates for the table, called GAC2012, are available in an Excel workbook.

Table 17 compares mortality rates as of the beginning of 2018 on the new table with rates from CPM2014priv. GAC2012 is higher at age 70, but lower at other ages for males. The tables are fairly close for females except at the highest and lowest ages shown.

Table 17. Mortality rates as of January 1, 2018.						
	GAC2012 c	n MI-2017	CPM2014priv on CPM-B		CPM2014priv on MI-2017	
Age	Male	Female	Male	Female	Male	Female
50	0.00232	0.00157	0.00286	0.00136	0.00278	0.00135
60	0.00577	0.00354	0.00668	0.00364	0.00674	0.00361
70	0.01508	0.00938	0.01344	0.00915	0.01380	0.00918
80	0.04094	0.02837	0.04096	0.02817	0.04158	0.02831
90	0.13852	0.10210	0.14098	0.10592	0.13932	0.10513
100	0.34192	0.28776	0.36683	0.31669	0.36251	0.31349

Table 18 shows that annuity values are higher on GAC2012 for age 80 and over and
fairly close otherwise.

Table 18. Present value of monthly life annuities-due of 1 p.a. at 4% as of January 1, 2018.						
	GAC2012 c	on MI-2017	CPM2014pri	v on CPM-B	CPM2012priv on MI-2017	
Age	Male	Female	Male	Female	Male	Female
50	18.63	19.47	18.48	19.37	18.55	19.48
60	15.70	16.78	15.65	16.66	15.68	16.75
70	12.02	13.19	12.04	13.12	12.07	13.18
80	7.88	8.96	7.71	8.79	7.78	8.82
90	4.14	4.90	3.95	4.67	4.03	4.74
100	2.07	2.39	1.94	2.22	1.95	2.23

Some actuaries may find it instructive or useful to review or use the software that constructs GAC2012. Accordingly, the CIA is making available a <u>zip file</u> that contains the Excel workbook that was used in this project and the associated binary file of summarized annuitant data. The calculations are done in VBA. See the worksheet "Describe" for instructions.

# 8.6 Excluding secondary annuitants

It is common for the experience of pension funds to exclude lives not "in pay". It is less appropriate for insured lives; insurance companies have a stronger incentive than pension funds to seek timely reporting of the deaths of secondary annuitants because they are able to release actuarial liability and free up associated capital.

Some will object to including the experience of secondary annuitants because the IBNR factors are significantly higher than for single life annuitants or joint primary annuitants. That is true, particularly for the most recent year and the next most recent year. The size of the factors need not be a concern if the estimate is reliable. All companies reviewed their IBNR factors this year in anticipation of the study being produced.

It seems better practice to include the experience of secondary lives if the data can be reliably obtained, and GAC2012 was so constructed. However, because some actuaries may prefer to exclude secondary, a variation of the table was constructed using only single life and joint primary data; that is, it includes experience of lives "in pay" only. That table is referred to as GAC2012xs (for eXclude Secondary), and is available on the same worksheet as GAC2012, referred to in section 8.5. The method used is identical to that of GAC2012; the only difference is that the underlying data set is smaller because of the exclusion of secondary annuitants.

The exclusion of secondary annuitants has very little impact on the male table because less than 4% of the exposure is for secondary annuitants, and the A/E ratio (on GAC2012) for secondary annuitants is only slightly over 100%. The exclusion of secondary annuitants is material for the female table because over half of the female experience is secondary, and A/E ratio is 93%, significantly lower than the overall A/E ratio.

Table 19 shows mortality rates and annuity present values at 4%, comparable to the values of tables 17 and 18, but for GAC2012xs rather than for GAC2012. The annuity values for males are virtually the same as for GAC2012, but generally lower for females. They are almost 2% lower at age 70 and 1% lower at age 80.

Table 19. GAC2012xs on MI-2017 as of January 1, 2018.					
	Mortali	ty Rates	Life Annu	iities-due	
Age	Male	Female	Male	Female	
50	0.00233	0.00175	18.62	19.28	
60	0.00582	0.00396	15.69	16.54	
70	0.01525	0.01146	12.02	12.95	
80	0.04085	0.03039	7.88	8.88	
90	0.13846	0.10220	4.14	4.92	
100	0.34192	0.28091	2.07	2.45	

GAC2012xs may not be an appropriate table to use for joint annuities. A separate assumption would be needed for the secondary annuitant, and GAC2012xs would not be appropriate in that case because the experience on which it is based includes no secondary annuitants.

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