Member's Paper

Going Concern Discount Rate for Pension Plans With High Asset Allocation to Bonds

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January 2022

Document rp222002

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Executive summary	Résumé
Due to the increased importance of going concern valuations in the funding of Canadian pension plans, there is an increased emphasis on the assumptions used for these valuations, including the going concern discount rate. This paper addresses considerations when selecting the going concern discount rate for a pension plan with a high allocation to bonds, with a particular emphasis on the component of the discount rate which is based on the long-term expected rate of investment return of the plan's bond portfolio. If the hedge ratio of a pension plan on a going concern basis is important to and is being managed by the plan sponsor,[1] it is likely preferable to assume for the purpose of selecting the going concern discount rate that the long-term expected rate of investment return on the plan's bond portfolio will equal the internal rate of return (IRR) of the portfolio on the valuation date, *without* reflecting the effect of reinvestment and future changes in interest rates. However, it is only appropriate to make	En raison de l'importance accrue des évaluations sur base de continuité dans le provisionnement des régimes de retraite canadiens, l'accent est davantage mis sur les hypothèses utilisées pour ces évaluations, y compris le taux d'actualisation sur base de continuité. Ce document aborde les réflexions lors de la sélection du taux d'actualisation sur base de continuité pour un régime de retraite avec une forte allocation aux obligations, avec un accent particulier sur la composante du taux d'actualisation basée sur le taux de rendement attendu à long terme du portefeuille d'obligation du régime. Si le ratio de couverture d'un régime de retraite sur base de continuité est important pour le promoteur du régime et est géré par ce dernier[2], il est probablement préférable de supposer, aux fins de la sélection du taux d'actualisation sur base de continuité, que le taux de rendement attendu à long terme du portefeuille d'obligations du régime sera égal au taux de rendement interne (TRI) du portefeuille à la date d'évaluation, sans tenir compte de l'effet du réinvestissement et des variations futures des taux d'intérêt. Cependant, il n'est approprié de faire cette hypothèse que sous certaines conditions :

[1] While this paper refers to the plan sponsor as the entity responsible for certain oversight and actions related to pension plans, for some plans it is the plan administrator which is the entity responsible for the oversight and actions.

[2] Bien que ce document désigne le promoteur du régime comme l'entité responsable de certaines surveillances et actions liées aux régimes de retraite, pour certains régimes, c'est l'administrateur du régime qui est l'entité responsable de la surveillance et des actions.

this assumption under certain conditions. • One such condition is that a "buy-and-hold" strategy is being employed with the plan's bond portfolio. • In the case of a bond portfolio that is periodically rebalanced (i.e., the plan sponsor is not employing a buy-and-hold strategy), it is generally not appropriate to assume that the long-term rate of investment return on the portfolio will be equal to the IRR of the portfolio. However, it may be appropriate to make this assumption when certain conditions are assumed to hold. Two key conditions are the assumption that the future interest rate environment will remain unchanged, and that the makeup of the bond portfolio (i.e., the pattern of future cash flows) will remain approximately the same over time. • A third potential situation is when the investment strategy of a pension plan sponsor is to maintain a specific hedge ratio over time (e.g., 100% or 80%). The author recommends further research into the expected long-term rates of investment returns on bond portfolios for this type of strategy. When an actuary is using the IRR of a bond portfolio to select the going concern discount rate, without reflecting the effect of reinvestment and future changes in interest rates, there are a number of important	• L'une de ces conditions est qu'une stratégie d'achat à long terme soit utilisée avec le portefeuille d'obligations du régime. • Dans le cas d'un portefeuille d'obligations qui est périodiquement rééquilibré (c'est-à-dire que le promoteur du régime n'emploie pas de stratégie d'achat à long terme), il n'est généralement pas approprié de supposer que le taux de rendement à long terme du portefeuille sera égal au TRI du portefeuille. Cependant, il peut être approprié de faire cette hypothèse lorsque certaines conditions sont censées être vérifiées. Deux conditions clés sont l'hypothèse que l'environnement futur des taux d'intérêt restera inchangé et que la composition du portefeuille d'obligations (c'est-à-dire la configuration des flux de trésorerie futurs) reste approximativement la même au fil du temps. • Une troisième situation potentielle est lorsque la stratégie de placement d'un promoteur de régime de retraite est de maintenir un ratio de couverture spécifique au fil du temps (par exemple, 100 % ou 80 %). L'auteur recommande des recherches plus poussées sur les taux de rendement à long terme attendus des portefeuilles

considerations that should be taken into account which are discussed in this paper. While the focus of this paper is on Canadian going concern funding valuations, the concepts discussed should be relevant to any situation in which an actuary is estimating the long-term rate of investment return on a bond portfolio. This includes the assumptions used to calculate the funding requirements of certain pension plans in countries other than Canada, and the assumptions used to calculate the pension accounting cost in accordance with certain accounting standards. Also, the concepts in this paper have been addressed from the perspective of a pension actuary selecting the assumptions for the actuarial valuation of a pension plan. However, a person who is responsible for establishing the investment strategy for a pension plan should consider these concepts from the perspective of how the strategy for the plan's bond mandate (e.g., buy and hold versus periodic rebalancing) will affect the financing strategy for the plan. For example, the strategy for the plan's bond mandate will likely affect the expected rate of investment return of the bond portfolio, the hedge ratio of the pension plan on a going concern basis, the expected evolution of the plan's allocation to bonds over time, and the trade-off between investing in bonds and other risk management solutions, such as the purchase of a group annuity from an insurance company. It also is helpful for both the plan sponsor and the actuary to be forward-looking and consider how,	d'obligations pour ce type de stratégie. Lorsqu'un actuaire utilise le TRI d'un portefeuille d'obligations pour sélectionner le taux d'actualisation sur base de continuité, sans tenir compte de l'effet du réinvestissement et des variations futures des taux d'intérêt, il y a un certain nombre de considérations importantes qui doivent être prises en compte et qui sont discutées dans ce document. Bien que le présent document se concentre sur les évaluations de provisionnement sur base de continuité au Canada, les concepts abordés devraient être pertinents pour toute situation dans laquelle un actuaire estime le taux de rendement à long terme d'un portefeuille d'obligations. Cela comprend les hypothèses utilisées pour calculer les besoins de provisionnement de certains régimes de retraite dans des pays autres que le Canada et les hypothèses utilisées pour calculer le coût comptable des régimes de retraite conformément à certaines normes comptables. De plus, les concepts de ce document ont été abordés du point de vue d'un actuaire du domaine des régimes de retraite qui choisit les hypothèses pour l'évaluation actuarielle d'un régime de retraite. Cependant, une personne chargée d'établir la stratégie de placement d'un régime de retraite devrait considérer ces concepts du point de vue de la façon dont la stratégie du mandat des obligations du régime (p. ex., achat à long terme par rapport au rééquilibrage périodique) affectera probablement la stratégie financière du régime. Par exemple, la stratégie du mandat des obligations du régime aura sûrement des répercussions sur le taux de rendement prévu du portefeuille d'obligations, le ratio de couverture du régime de retraite sur base de continuité, l'évolution prévue de l'allocation

depending on the sponsor’s pension risk management strategy, the investment strategy and asset mix allocation of a pension plan are expected to evolve over time.

The author would like to thank all the reviewers of this paper, including Félix Jean, Josephine Marks, Tommy Ouellet, and Gus Van Helden, whose feedback led to significant improvements to its content. However, the conclusions drawn in the paper are those of the author, and any errors it may contain are the responsibility of the author alone.

The opinions expressed in this paper are those of the author alone and are not intended to reflect or represent guidance from the CIA, any other actuarial organization, or the author’s employer.

du régime aux obligations au fil du temps, et le compromis entre investir dans des obligations et d’autres solutions de gestion des risques telles que l’achat d’une rente collective auprès d’une société d’assurance. Il est également utile que le promoteur du régime et l’actuaire soient tournés vers l’avenir et examinent comment, selon la stratégie de gestion du risque de retraite du promoteur, la stratégie de placement et la répartition de l’actif d’un régime de retraite devraient évoluer au fil du temps.

L’auteur tient à remercier tous les relecteurs de ce document, dont Félix Jean, Josephine Marks, Tommy Ouellet et Gus Van Helden, dont les commentaires ont permis d’améliorer considérablement son contenu. Cependant, les conclusions tirées sont celles de l’auteur et toute erreur pouvant être contenue dans le présent document est de la responsabilité de l’auteur.

Les opinions exprimées dans ce document sont celles de l’auteur et ne visent pas à refléter ou à représenter l’orientation de l’Institut canadien des actuaires, de toute autre organisation actuarielle ou de l’employeur de l’auteur.

Background

Since the eighties, pension legislation in most Canadian jurisdictions has required that the minimum employer contributions to a private sector registered defined benefit (DB) pension plan be based on the financial position of the plan using two types of actuarial valuations:

- Going concern valuation: The going concern valuation is premised on the pension plan continuing to exist indefinitely. For the purpose of calculating the plan liabilities, the actuary projects the expected benefit stream payable from the pension plan using long-term assumptions, such as member withdrawal rates, retirement rates, and mortality rates. If pension benefits are based on the future earnings of plan members, the actuary also makes an assumption about future increases to member earnings.

  To calculate the plan liabilities, the actuary must select the discount rate used to discount the expected benefit payment stream back to the valuation date. In accordance with the Canadian actuarial *Standards of Practice*, the actuary typically selects the discount rate based on the long-term expected rate of investment return on the pension plan assets.[3] Since the future benefit stream payable from a pension plan can continue for many decades following the valuation date, the discount rate assumption is usually the assumption that has the most material effect on the going concern liabilities of the plan.

- Solvency valuation: The solvency valuation is premised on the pension plan winding up on the valuation date, and the settlement of the liabilities of the plan on that date. For the purpose of calculating a plan's solvency liabilities, the actuary typically assumes that, upon plan windup, a portion of the plan's liabilities is settled by paying lump sum commuted values to plan members, while the remainder of the liabilities is settled through the purchase of a group annuity from an insurance company. Since both commuted values and group annuity premiums are sensitive to prevailing bond yields, the discount rates used to measure solvency liabilities are based on bond yields on the date of the actuarial valuation, with little discretion available to the actuary when selecting these assumptions.

Although the funding rules differ in each Canadian jurisdiction, if the going concern actuarial valuation revealed that plan liabilities exceeded plan assets, the deficiency usually had to be funded over a maximum period of 15 years. If the solvency actuarial valuation revealed a deficiency, the deficiency usually had to be funded over a maximum of five years.

[3] Paragraph 3230.02 of the Canadian actuarial *Standards of Practice* also permits an actuary who is selecting the going concern discount rate to "reflect the yields on fixed income investments" regardless of the actual investment strategy for the pension plan assets. While this approach may be appropriate for a pension plan with assets entirely invested in bonds, the approach is likely not ideal for a plan with a portion of assets allocated to return-seeking assets (such as equities). This is because this approach does not appear to accommodate the reflection of the risk premium expected to be earned by any return-seeking assets when selecting the discount rate.

During the eighties and nineties, most private sector pension plans had a surplus on a solvency basis, and therefore the going concern valuation was the driver of the minimum employer contribution requirements. However, the decades-long trend of decreasing bond yields resulted in an increase in solvency liabilities over time, which led to the solvency valuation becoming the driver of minimum contributions for most pension plans.

The combination of decreasing bond yields and the "marking to market" of solvency liabilities' discount rates based on current bond yields resulted in large and volatile solvency funding requirements for many private sector employers who sponsor DB pension plans. This led to pension plan sponsors undertaking various actions to reduce the financial risk in their pension plans. One such de-risking action was to reduce the allocation of pension plan assets to return-seeking assets, such as equities, and increase the allocation to liability-hedging assets, such as bonds. In many cases, the strategy of increasing the asset allocation to bonds was done with a view to hedging changes in the plan's solvency liabilities. Since solvency liabilities' discount rates are marked to market based on current bond yields, the change in the value of solvency liabilities due to changes in bond yields is very similar to the change in the value of an appropriately constructed bond portfolio. By increasing the allocation to bonds, pension plan sponsors increased the hedge ratio[4] of their pension plan on a solvency basis. This reduced the volatility of the plan's solvency funded position as bond yields changed, with a resulting decrease in the volatility of the plan sponsor's solvency funding requirements.

The large and volatile pension solvency funding requirements over the past couple of decades became a significant financial challenge for many private sector employers who sponsor DB pension plans. Over time, many pension policymakers concluded that the prevailing funding regimes were not sustainable and that funding reform was needed. This led to private sector pension funding reform in several Canadian jurisdictions that increases the focus on going concern funding, and decreases the focus on solvency funding. For example, sponsors of pension plans registered in British Columbia, New Brunswick, Nova Scotia, and Ontario now only need to fund up to 85% of a plan's solvency liabilities. For plans registered in Québec, no solvency funding is required. For the Canadian jurisdictions that have implemented funding reform, the reduction in solvency funding requirements has been at least partially offset by more stringent funding requirements on a going concern basis.

An important consequence of the pension funding reform described above is that the going concern valuation has replaced the solvency valuation as the key driver of the minimum funding requirements for many Canadian DB pension plans.[5] As previously mentioned, the discount rate is the most material assumption when calculating the going concern liabilities of a pension plan. The importance of the going concern discount rate has led plan sponsors with a

[4] The hedge ratio measures the sensitivity of a pension plan's asset value to a change in bond yields as a proportion of the sensitivity of the plan's liabilities to the same change in bond yields.

[5] Although the going concern valuation may now be the key driver of minimum funding requirements for many pension plans, this does not mean that the solvency valuation has become irrelevant. For example, a plan sponsor that has a short-term objective of winding up its pension plan may focus on the plan's solvency liabilities as the key funding target.

pension plan that has a high allocation of investments in bonds, and its actuaries, to consider such questions as:

- In most cases, the high allocation to bonds was originally based on a strategy of increasing the pension plan's hedge ratio on a solvency basis. How well hedged is the plan on a going concern basis? This may be important since the going concern valuation will likely be the driver of the funding requirement for the plan going forward.
- What is an appropriate approach for selecting the discount rate for the plan, especially the component of the discount rate based on the long-term expected rate of investment return of the plan's bond portfolio?
- Is there an investment strategy and/or approach for selecting the going concern discount rate that improves the plan's going concern hedge ratio, and therefore reduces the volatility of the sponsor's minimum contribution requirements?

The remainder of this paper seeks to address the above questions, with an emphasis on the premises underlying an approach for selecting the discount rate that improves the hedge ratio of a pension plan on a going concern basis.

Approaches for selecting the discount rate

Two approaches for selecting the going concern discount rate based on the long-term expected rate of return on plan assets are described in the revised educational note *Determination of Best Estimate Discount Rates for Going Concern Funding Valuations*, published in 2015 by the CIA Committee on Pension Plan Financial Reporting:

i. Building block approach

ii. Stochastic methodology

Building block approach

The building block approach for selecting the going concern discount rate can be summarized as follows:

- Step 1: Estimate the long-term expected investment return for each asset class in which the pension plan's assets are invested.

- Step 2: Calculate the preliminary expected investment return on total plan assets as the weighted average of the expected return for each asset class estimated in Step 1. The weight assigned to each asset class is set equal to the asset class' target allocation percentage as established in the investment policy for the plan.

- Step 3: If appropriate, the expected return on plan assets calculated in Step 2 may be increased for the effect of diversification and rebalancing. This reflects the fact that the long-term expected geometric return on a diversified investment portfolio that is regularly rebalanced will exceed the weighted average of the expected returns for each of the portfolio's asset classes.

- Step 4: Reduce the expected return on plan assets calculated in Step 3 to make provision for future investment and administrative expenses expected to be paid from plan assets to the extent that a provision for these expenses has not been reflected elsewhere in the actuarial basis.

- Step 5: Reduce the expected return on plan assets calculated in Step 4 by a margin for adverse deviations (i.e., a margin for conservatism), if appropriate.

The following example illustrates the application of the building block approach:

Example 1: Building block approach

A pension plan has the following policy with respect to the allocation of its assets:

Table I

Asset Class	Target Allocation
• Canadian equities	10%
• Global equities	10%
• Long bonds	80%
• Total	100%

By applying the building block approach, the actuary selects the going concern discount rate assumption as follows:

- Step 1: The actuary estimates the long-term expected rate of investment return on the plan's Canadian equities, global equities, and long bonds portfolios as 7.0%, 7.5%, and 3.5% per year, respectively.
- Step 2: Calculate the preliminary expected investment return on plan assets as:

  (7.0% x 10%) + (7.5% x 10%) + (3.5% x 80%) = **4.25%** per year
- Step 3: The investment policy of the plan requires the periodic rebalancing of the assets to the target allocation, and the actuary estimates that the diversification and rebalancing effect will add 0.25% per year to the long-term expected investment return on plan assets. Therefore, the expected return is adjusted as follows:

  4.25% + 0.25% = **4.50%** per year
- Step 4: The actuary estimates that future expenses related to the investment of the plan assets and administration of the plan will amount to approximately 0.40% of assets per year over the long term. The expected investment return on plan assets is further adjusted as follows:

  4.50% - 0.40% = **4.10%** per year
- Step 5: Based on the funding rules in the jurisdiction in which the pension plan is registered, and direction from the plan sponsor, the actuary concludes that there is no need to reduce the discount rate to reflect a margin for adverse deviations.

Therefore, in this example, the actuary selects a discount rate of **4.10%** per year.

Stochastic methodology

The educational note describes the stochastic methodology for selecting the going concern discount rate as follows:

> a logically constructed stochastic asset model that calculates a probability distribution of long-term investment returns by asset class. The asset model requires inputs of the assumed investment policy and assumptions about investment returns and standard deviations on each of the asset classes in that policy (and correlations between the investment returns on different asset classes). Such a model directly incorporates the effects of diversification and rebalancing. The best estimate asset return assumption to be used would normally be based on a percentile at or near the median of the distribution of long-term investment returns of the portfolio (page 11).

For the purpose of establishing the discount rate, the actuary would adjust the estimated asset returns produced by the model to reflect investment and administrative expenses and a margin for adverse deviations, as appropriate (i.e., similar to steps 4 and 5 of the building block approach).

In Canada, many actuarial firms have stochastic asset models that can be used to select the discount rate assumption. Within an actuarial firm, one would expect the building block approach and a stochastic asset model to produce consistent results.

Observations regarding approaches for selecting the discount rate

The following are a few observations regarding the common approaches described above for selecting the going concern discount rate assumption:

- The educational note indicates that, in order to estimate the expected rate of investment return on plan assets, a time horizon of 20 to 30 years is typically used. However, the time horizon for the future benefit stream payable from a pension plan is often much longer, and the size of the expected annual benefit stream changes over time.
- Unless the pension plan sponsor has formally adopted an investment glide path,[6] the actuary typically assumes that the asset allocation for the plan does not change in the future. This is the case even though the demographic makeup of most DB plans is expected to change over time (particularly for plans that are closed to new entrants), which could be expected to lead to a change in asset allocation over time. It will be suggested later in this paper that, in certain circumstances, when selecting the discount rate it may be appropriate for the actuary to reflect expected future changes in the asset allocation.

[6] A glide path is an investment strategy that provides for specified changes to the plan's asset allocation in the future. Depending on the nature of the glide path strategy adopted by the sponsor of the pension plan, the changes in asset allocation may occur at specified future dates or could occur when certain "trigger" events occur (e.g., the funded ratio of the plan increases to certain levels).

- Paragraph 1620.29 of the Canadian actuarial *Standards of Practice* requires that the material assumptions selected by an actuary be "independently reasonable and appropriate in the aggregate." Therefore, the assumed underlying economic metrics, such as future price inflation, bond yields, and gross domestic product (GDP) growth used to determine the expected investment return on various asset classes (e.g., bonds and equities) and to select other going concern valuation assumptions (e.g., price inflation and wage increases) should be consistent.

Estimating expected rate of investment return on bonds

Whether the building block approach or a stochastic methodology is used to select the going concern discount rate, the estimated long-term expected rate of investment return on a plan's bond portfolio embedded in the approach becomes more important as the allocation to bonds increases.

With respect to estimating the long-term expected investment return on a plan's bond portfolio, the educational note says: "For a plan where assets are invested in part in treasury bills or bonds and are expected to be invested that way indefinitely, the best estimate of the long-term investment return on that class of assets may be reasonably viewed as the market yield on the particular investments or the yield on a market index representative of such investments at the calculation date, adjusted to reflect an allowance for reinvestment and the effect of possible changes in interest rates on future investments, if appropriate" (page 6).

The following are a few comments with respect to estimating the long-term expected rate of investment return on a plan's bond portfolio:

- Unless the bond portfolio is intended to replicate a market index, it is generally preferable to base the expected rate of investment return on the actual internal rate of return (IRR)[7] of the plan's bond portfolio on the valuation date, instead of the average yield to maturity on a market index. One of the reasons for this is that the yield of a particular bond portfolio can differ materially from the yield on a market index if the characteristics of the bonds making up the portfolio are significantly different from the characteristics and mix of the bonds that are reflected in the index.
- If there is material credit risk in the portfolio, the expected investment return on the bond portfolio should be adjusted (i.e., decreased) to reflect the risk of the future default and downgrades of some of the bonds in the portfolio.
- If the expected return is based on the actual IRR of the plan's bond portfolio on the valuation date, a key consideration is whether to "reflect an allowance for reinvestment and the effect of possible changes in interest rates on future investments." With respect to the effect of interest rate changes on the investment return of a bond portfolio, in an

[7] The IRR of a bond portfolio is the single discount rate for which the market value of the bond portfolio is equal to the discounted value of the cash flows paid by the portfolio. It should be noted that the IRR differs from the average yield to maturity of a market index, which is typically determined as the average of the market-value-weighted yields to maturity of each bond included in the index. When selecting the going concern discount rate, an actuary who is using the average yield to maturity of a bond portfolio in lieu of the IRR may wish to consider whether using the average yield to maturity has a material effect on the discount rate assumption chosen.

environment of low bond yields, such as the environment prevailing at the time this paper was written, it is common to assume that bond yields will increase in the future. When the yield increases are assumed to occur, the market value of a bond portfolio will decrease (i.e., the portfolio will incur capital losses). However, after the increases in bond yields, the portfolio will earn a higher yield (i.e., investment return). These offsetting effects could result in either an increase or a decrease in the long-term expected rate of investment return of a bond portfolio due to increasing bond yields. In the case of decreases in bond yields, there will be similar but opposite offsetting effects on the investment rate of return (i.e., immediate capital gains offset by lower yields on the portfolio after the yield decreases).

For the purpose of selecting the going concern discount rate, using the IRR of a bond portfolio without reflecting "an allowance for reinvestment and the effect of possible changes in interest rates on future investments" will likely result in a better hedge of a plan's funded position on a going concern basis. The following examples illustrate why this is the case:

Example 2: Internal rate of return without adjustment approach

On the date of an actuarial valuation, a pension plan has assets of $100 million invested entirely in a bond portfolio. The bond portfolio has an IRR of 3.5% per year and a duration[8] of 15 years. The actuary for the pension plan assumes that the long-term expected rate of investment return on the bond portfolio will equal the IRR of the bond portfolio and therefore selects a going concern discount rate of 3.5% per year.[9] In this example, based on a discount rate of 3.5% per year, the going concern liabilities of the plan are equal to $100 million and the liabilities have a duration of 15 years.

Assume that there is a shock in the financial markets on the valuation date that causes a 1% (i.e., 100 basis points, or bps) decrease in the IRR of the plan's bond portfolio (i.e., from 3.5% to 2.5% per year). Since the actuary is assuming that the long-term investment return on the bond portfolio will be equal to the IRR, the actuary lowers the going concern discount rate from 3.5% to 2.5% per year. The following table summarizes the going concern financial position of the pension plan before and after the financial market shock:

[8] For the purposes of this paper, "duration" refers to the modified duration of a bond portfolio or pension liabilities. The modified duration measures the sensitivity of the price of a bond portfolio or pension liabilities when there is a change in the IRR of the bond portfolio or liability discount rate. For example, if a bond portfolio has a modified duration of 15 years, it has been assumed that a 1% decrease (increase) in the bond portfolio's IRR will result in a 15% increase (decrease) in the price or value of the bond portfolio.

[9] In examples 2 and 3, for the purpose of simplicity no adjustments have been made to the going concern discount rate for factors such as expenses expected to be paid from the pension plan assets and the risk of future defaults, although such adjustments would typically be made.

Table II

(000s)	Before Shock (discount rate of 3.5%)	After Shock (discount rate of 2.5%)[10]
• Assets	$ 100,000	$ 115,000
• Going concern liabilities	100,000	115,000
• **Reported surplus/ (deficit)**	**$ 0**	**$ 0**

In this example, both the IRR of the plan assets and the going concern discount rate decrease by 100 bps due to the decrease in bond yields, and both assets and liabilities have the same duration (i.e., 15 years). Therefore, both assets and liabilities increase by 15% and there is no change to the reported going concern financial position of the pension plan due to the decrease in bond yields (i.e., in this example, the plan assets provide a perfect hedge to the going concern liabilities in the case of a change in bond yields).

Example 3: Internal rate of return with adjustment approach

As in Example 2, on the date of an actuarial valuation, a pension plan has assets of $100 million invested entirely in a bond portfolio. The bond portfolio has an IRR of 3.50% per year and a duration of 15 years. However, the actuary for this pension plan assumes that the long-term expected rate of investment return on the bond portfolio will equal the IRR of the bond portfolio, adjusted for the effect of reinvestment and future changes in interest rates on future investments. Based on the actuary's assumption regarding future changes in interest rates, the actuary calculates that the effect of reinvestment and future changes in interest rates will add 25 bps per year to the long-term expected rate of return on plan assets, and therefore selects a going concern discount rate of 3.75% per year. For this plan, based on a discount rate of 3.75% per year, the going concern liabilities of the plan are equal to $100 million and the liabilities have a duration of 15 years.

If the same shock in the financial markets as in Example 2 occurs, the actuary calculates that, after reflecting the shock, the effect of reinvestment and future changes in interest rates will add 60 bps per year to the long-term expected rate of return on plan assets.[11] Therefore, the actuary lowers the going concern discount rate from 3.75% to 3.10% per year. (The discount rate of 3.10% is determined as the IRR of the bond portfolio after the

[10] For the purposes of examples 2 and 3, the effect of convexity on the changes in plan assets and liabilities due to changes in bond yields and the going concern discount rate has been ignored.

[11] In this example, the effect of reinvestment and future changes in interest rates is assumed to add 60 bps per year to the long-term expected rate of return on plan assets after the financial shock compared to 25 bps per year prior to the financial shock, because even though the actuary has not changed the assumption regarding the level of interest rates in the long-term, the actuary calculates that as bond yields rise in the future to transition to these long-term rates, the effect on the bond portfolio's rate of investment return of the portfolio earning a higher yield in the future due to the future increase in bond yields relative to the effect of the capital losses incurred on the portfolio will be greater after the financial shock than before.

shock of 2.50%, plus the adjustment of 0.60%.) The following table summarizes the going concern financial position of the pension plan before and after the financial market shock:

Table III

(000s)	Before Shock (discount rate of 3.75%)	After Shock (discount rate of 3.10%)
• Assets	$ 100,000	$ 115,000
• Going concern liabilities	100,000	109,500
• **Reported surplus/ (deficit)**	**$ 0**	**$ 5,500**

One can see from Table III that the change to the going concern discount rate does not equal the change in the IRR of the bond portfolio, due to the effect of future reinvestment and changes in interest rates. This creates volatility in the going concern funded position (i.e., an improvement in the reported funded position of the plan of $5.5 million) when the IRR of the bond portfolio decreases by 100 bps. While in this scenario the volatility improves the funded position of the pension plan, in other scenarios the volatility can cause a deterioration in the funded position and can lead to volatility in the funding requirement for the plan. It should be noted that in this example volatility in the going concern funded position can also be caused by changes in the actuary's outlook with respect to future interest rate levels, as changes in the actuary's outlook may result in a change to the going concern discount rate assumption.

As discussed above, for pension plan sponsors who have a high allocation to bonds in their plan assets and whose primary objective is to reduce the volatility of their required pension contributions, it is likely preferable to select the going concern discount rate based on the assumption that the long-term expected rate of investment return on their plan's bond portfolio will equal the IRR of the portfolio, *without* reflecting the effect of reinvestment and future changes in interest rates (which can be referred to as a "mark-to-market" approach). This is because using a mark-to-market approach to estimate the expected return on a plan's bond portfolio results in a better hedge ratio of the plan on a going concern basis. Given the merits of using the mark-to-market approach for some pension plans, much of the remainder of this paper focuses on the conditions under which use of this approach is appropriate.

Calculating the rate of investment return on a bond portfolio

In order to assess whether it is appropriate to assume that the long-term expected rate of investment return on a plan's bond portfolio will be equal to the IRR of the portfolio on the valuation date, without reflecting the effect of reinvestment and future changes in interest rates, it is useful to examine some of the factors affecting the annual rate of return of an illustrative bond portfolio.

For this purpose, assume that the assets of a pension plan include an allocation to a portfolio of Government of Canada (GoC)[12] bonds with the following characteristics:[13]

> $\boldsymbol{MV_t}$ is the market value of the bond portfolio at time $\boldsymbol{t}$.
>
> $\boldsymbol{CF_s}$ is the sum of the cash flows (coupons and redemption values) paid by the portfolio at time $\boldsymbol{s}$.
>
> $\boldsymbol{N}$ is the time at which the final cash flows from the portfolio are paid.

Also, define the one-year forward rate at time $\boldsymbol{t}$ for GoC bonds for the maturity period $\boldsymbol{u}$ as $\boldsymbol{{}_tf_{u-1}}$. The relationship between the market value[14] of the bond portfolio at time $\boldsymbol{t}$ and the forward rates at time $\boldsymbol{t}$ can be expressed as follows:

$$\boldsymbol{MV_t = \sum_{s=t+1}^{N} CF_s \prod_{u=t+1}^{s} (1 + {}_tf_{u-1})^{-1}} \qquad \textbf{\textit{(A)}}$$

$$\boldsymbol{= \sum_{s=t+1}^{N} {}_tPV_s} \qquad \textbf{\textit{(B)}}$$

where $\boldsymbol{{}_tPV_s = CF_S \prod_{u=t+1}^{s} (1 + {}_tf_{u-1})^{-1}}$ for $\boldsymbol{s = t + 1}$ to $\boldsymbol{N}$.

In formula ***(B)***, $\boldsymbol{{}_tPV_s}$ represents the value at time $\boldsymbol{t}$ of the sum of the cash flows that will be paid by the portfolio at time $\boldsymbol{s}$.

Based on formula ***(A)***, the value of the portfolio at time $\boldsymbol{t+1}$ can be expressed as follows:

$$\boldsymbol{MV_{t+1} = \sum_{s=t+2}^{N} CF_s \prod_{u=t+2}^{s} (1 + {}_{t+1}f_{u-1})^{-1}} \qquad \textbf{\textit{(C)}}$$

If $\boldsymbol{R_{t+1}}$ is defined as the rate of investment return of the portfolio between times $\boldsymbol{t}$ and $\boldsymbol{t+1}$, then $\boldsymbol{R_{t+1}}$ can be calculated as follows:

$$\boldsymbol{R_{t+1} = (CF_{t+1} + MV_{t+1})/MV_t - 1} \qquad \textbf{\textit{(D)}}$$

Using formulas ***(A), (B), (C),*** and ***(D)***, it can be shown that:

---

[12] In this paper, a portfolio of GoC bonds is used as an example of a portfolio of homogeneous bonds for which the pricing of all the bonds in the portfolio is reflective of the same underlying yield curve. However, the concepts in this paper also apply to other types of bond portfolios.

[13] Although bond coupons are usually paid semi-annually and yields are quoted on a semi-annual basis, a simplifying assumption has been made in this paper that all bond cash flows are paid annually, yields are quoted on an annual basis, and any sale or purchase of bonds in a portfolio only occurs at the end of a year.

[14] $\boldsymbol{MV_t}$ is determined after the cash flows payable at time $\boldsymbol{t}$ have been paid.

$$\boldsymbol{R_{t+1}} = \sum_{s=t+1}^{N} \frac{{}_t\boldsymbol{PV_s}}{\boldsymbol{MV_t}} [(\mathbf{1} + {}_t\boldsymbol{f}_t)\,{}_t\boldsymbol{GL_s} - \mathbf{1}] \qquad \textbf{\textit{(E)}}$$

$$\text{where } {}_t\boldsymbol{GL_{t+1}} = \mathbf{1}, \text{and } {}_t\boldsymbol{GL_s} = \frac{\prod_{u=t+2}^{s}(1+{}_tf_{u-1})}{\prod_{u=t+2}^{s}(1+{}_{t+1}f_{u-1})} \text{ for } \boldsymbol{s = t+2} \text{ to } \boldsymbol{N}.$$

Formula ***(E)*** can be interpreted as follows:

- The rate of investment return on the bond portfolio between times ***t*** and ***t+1*** is equal to the sum of the weighted average of the rates of investment return on each set of portfolio cash flows $\boldsymbol{CF_s}$.
- The weight assigned to the rate of investment return of the set of cash flows payable at time ***s*** ($\boldsymbol{CF_s}$) is ${}_t\boldsymbol{PV_s}/\boldsymbol{MV_t}$. This is the ratio of the value at time ***t*** of the cash flows payable by the portfolio at time ***s*** over the value of the entire bond portfolio at time ***t***.
- The rate of investment return on the set of cash flows payable at time ***s*** is equal to the forward rate for the one-year maturity period ***t*** (i.e., ${}_t\boldsymbol{f}_t$), adjusted by ${}_t\boldsymbol{GL_s}$.
- ${}_t\boldsymbol{GL_s}$ is the capital gain or loss adjustment factor in respect of the cash flows payable at time ***s*** due to the change in the forward curve between times ***t*** and ***t+1*** (the capital gain or loss adjustment factor is the adjustment due to discounting the cash flows payable at time ***s*** using the forward curve at time ***t+1*** instead of the forward curve at time ***t***).

The following simplified numerical examples illustrate the application of formula ***(E)***:

Assume the following with respect to a bond portfolio's future cash flows and the forward rates (i.e., forward curve) at time ***0***:

Table IV

s	CF_s	${}_0f_{s-1}$	${}_0PV_s$	${}_0PV_s$ / MV_0
1	1,000.00	0.5%	995.02	11.50%
2	1,500.00	1.0%	1,477.76	17.08%
3	2,000.00	1.5%	1,941.23	22.44%
4	2,500.00	2.0%	2,378.96	27.50%
5	2,000.00	2.5%	1,856.75	21.47%
		MV_0 =	**$ 8,649.72**	
		IRR_0 =	**1.20%**	

In the above table, formula ***(B)*** can be used to derive the values in the ${}_0\boldsymbol{PV_s}$ column. For example:

${}_0\boldsymbol{PV_3}$ = \$2,000 x 1.005^{-1} x 1.01^{-1} x 1.015^{-1} = \$1,941.23

In this example, the IRR at time ***0*** is 1.20%. There is no closed-form solution for calculating the IRR, but it can be derived by either trial and error or using an IRR function available in certain software programs.

Example 4: Shifting forward curve

In order to assess how the evolution of the forward curve over time affects the rate of return on the above portfolio, assume that the forward curve "shifts" over time in such a way as to preserve the original shape of the remainder of the time t forward curve. In other words, ${}_{t+1}f_{u-1} = {}_{t}f_{u-1}$, in which case there are no capital gains and losses due to changes in the forward curve, and the capital gain or loss adjustment factor (${}_{t}GL_{s}$) is equal to 1 at all times s, for $s = t+1$ to N.

In this example, formula ***(E)*** can be simplified as follows to calculate the investment return over the time period ***0*** to ***1***:

$$R_{t+1} = {}_{t}f_{t} = {}_{0}f_{0} = 0.5\%$$

This can be verified by determining the value of the bond portfolio at time 1.

Table V

s	CF_s	${}_1f_{s\text{-}1}$	${}_1PV_s$
2	1,500.00	1.0%	$ 1,485.15
3	2,000.00	1.5%	1,950.93
4	2,500.00	2.0%	2,390.85
5	2,000.00	2.5%	1,866.03
		MV_1 =	**$ 7,692.96**

Using formula ***(D)***, the investment return of the portfolio over the time period ***0*** to ***1*** can be calculated as:

$(CF_1 + MV_1) / MV_0 - 1$ = ($1,000 + $7,692.96) / $8,649.72 – 1 = 0.5%.

In this example, where the forward curve is assumed to shift each year, the investment return over the time period ***1*** to ***2*** will be the forward rate for maturity period s = 2 (i.e., 1.0% per year), the investment return for the time period ***2*** to ***3*** will be the forward rate for maturity period s = 3 (i.e., 1.5% per year), etc.

Example 5: Forward curve that resets

Another assumption regarding the change in the forward curve over time that is worth considering is that the forward curve "resets" to its original shape at the end of each year. In other words, ${}_{t+1}f_{u} = {}_{t}f_{u-1}$, in which case the capital gain or loss adjustment factor (${}_{t}GL_{s}$) is equal to $(1 + {}_{t}f_{s-1})/(1 + {}_{t}f_{t})$ for $s = t+2$ to N, which leads to:

$$R_{t+1} = \sum_{s=t+1}^{N} \frac{{}_{t}PV_{s}}{MV_{t}} ({}_{t}f_{s-1}) \qquad \textbf{\textit{(F)}}$$

Applying formula ***(F)*** to Example 5 results in the following:

R_1 = (11.50% x 0.5%) + (17.08% x 1.0%) + (22.44% x 1.5%) + (27.50% x 2.0%) + (21.47% x 2.5%)

= 1.65%

This can be verified by determining the value of the bond portfolio at time ***1***.

Table VI

s	CF_s	${}_1f_{s\text{-}1}$	${}_1PV_s$
2	1,500.00	0.5%	1,492.54
3	2,000.00	1.0%	1,970.35
4	2,500.00	1.5%	2,426.53
5	2,000.00	2.0%	1,903.16
		MV_1 =	**$ 7,792.58**

Using formula ***(D)***, the investment return of the portfolio over the time period ***0*** to ***1*** can be calculated as:

$(CF_1 + MV_1) / MV_0$ – 1 = ($1,000 + $7,792.58) / $8,649.72 – 1 = 1.65%.

Buy-and-hold investment strategy

In order to assess whether it is appropriate to assume that the long-term expected rate of investment return on a plan's bond portfolio will be equal to the IRR of the portfolio on the valuation date, consider a portfolio of bonds that is held to maturity without reinvesting the cash flows from the bonds. In other words, the cash flows from the portfolio are not reinvested but instead are used to pay the pensions of all or a portion of the pension plan's retirees as they come due, thereby "immunizing" a portion of the pension plan's obligations. This type of investment strategy will be referred to as a "buy-and-hold" strategy.

The IRR of a bond portfolio y_t at time t is defined as the discount rate for which the market value of the bond portfolio at time t is equal to the discounted value of the cash flows paid by the portfolio. In the case of the portfolio of GoC bonds discussed earlier:

$$MV_t = \sum_{s=t+1}^{N} CF_s\,(1 + y_t)^{-(s-t)} \qquad \textit{(G)}$$

It can also be noted from formulas ***(A)*** and ***(G)*** that:

$$\sum_{s=t+1}^{N} CF_s\,(1 + y_t)^{-(s-t)} = \sum_{s=t+1}^{N} CF_s \prod_{u=t+1}^{s} (1 + {}_tf_{u-1})^{-1} \qquad \textit{(H)}$$

If both sides of formula ***(H)*** are multiplied by **-1**, and the sum of all the cash flows payable by the bond portfolio after time ***t*** is added to both sides of the formula, the following formula results:

$$\sum_{s=t+1}^{N} CF_s - \sum_{s=t+1}^{N} CF_s\,(1+y_t)^{-(s-t)} = \sum_{s=t+1}^{N} CF_s - \sum_{s=t+1}^{N} CF_s \prod_{u=t+1}^{s} (1+{}_tf_{u-1})^{-1} \qquad \textit{(I)}$$

Both sides of formula ***(I)*** represent the total amount of investment return (in dollars) that will be earned on the bond portfolio over the period starting at time ***t*** and ending at the time the final cash flows from the bond portfolio are paid.

It can be shown that formula ***(I)*** is equivalent to the following formula:

$$\sum_{s=t}^{N-1} {}_yMV_s \times y_t = \sum_{s=t}^{N-1} MV_s \times R_{s+1} \qquad \textit{(J)}$$

$$\text{where } {}_yMV_s = \sum_{u=s+1}^{N} CF_u\,(1+y_t)^{-(u-s)}$$ for ***s*** = ***t*** to ***N-1***

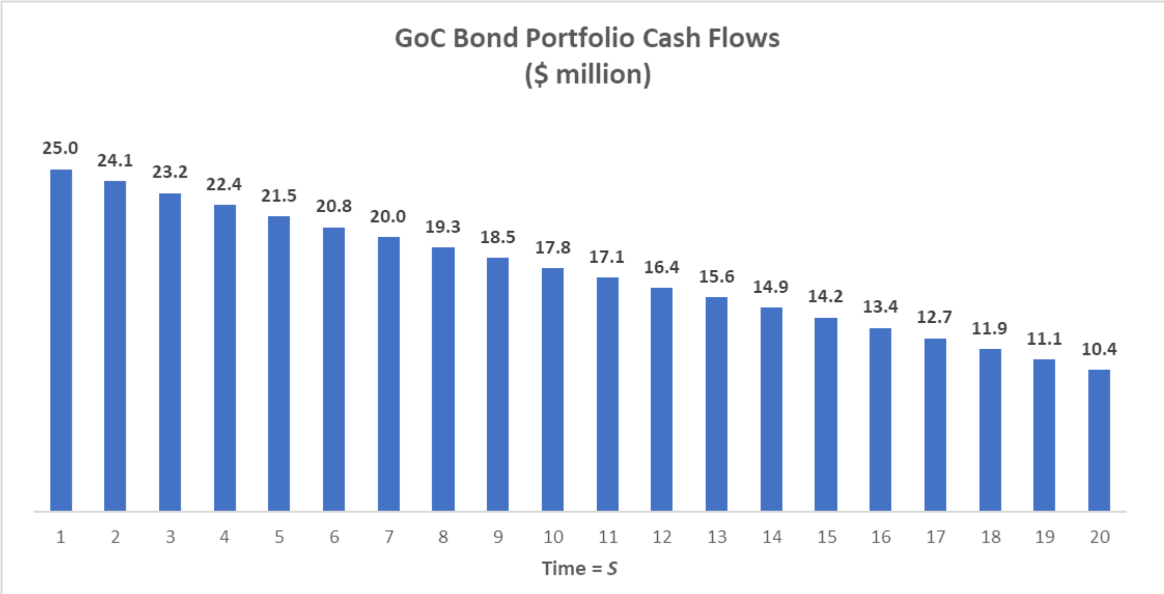
The left-hand side of formula ***(J)*** represents the sum of the annual investment returns earned on the bond portfolio over the period ***t*** to ***N*** determined by valuing the portfolio using the IRR of the portfolio at time ***t***. The right-hand side of the formula also represents the sum of the annual investment returns earned on the portfolio over the same period, but using the market value of the portfolio determined using the prevailing forward curve to calculate each annual return.

Formula (***J***) implies that if a buy-and-hold strategy is employed with respect to a bond portfolio in a pension plan, it is appropriate to assume at time ***t*** that the long-term expected rate of investment return on the portfolio will be equal to the IRR of the portfolio at time ***t***, without reflecting the effect of reinvestment and future changes in interest rates. This is the case regardless of changes in forward rates between times ***t*** and ***N*** and regardless of whether any of the bonds in the portfolio are downgraded (as long as none of the bonds default).

The application of formula ***(J)*** can be illustrated through a number of examples:

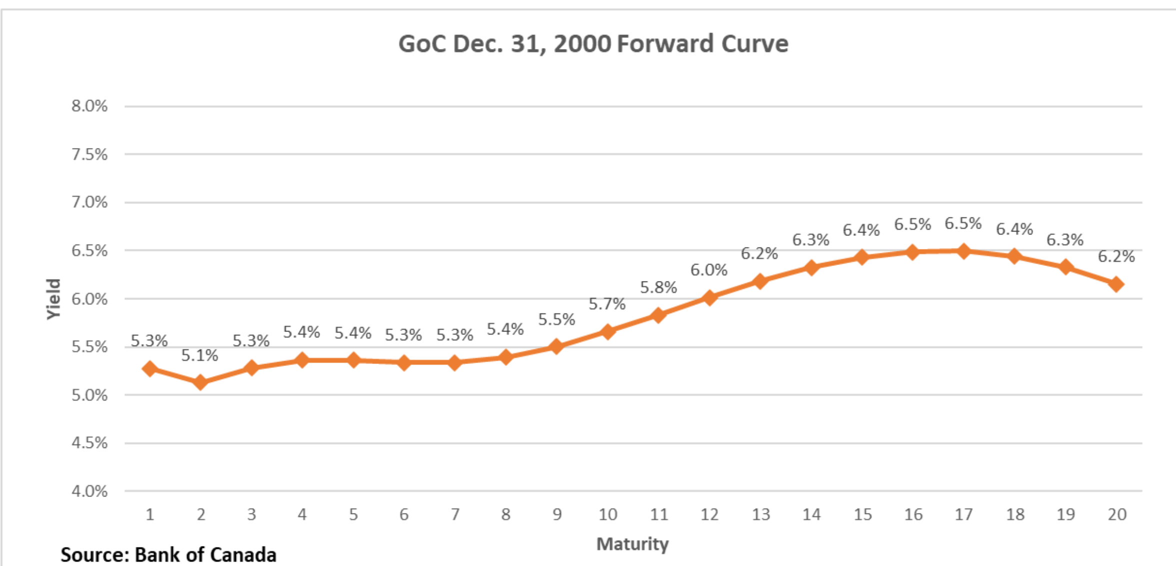
> On December 31, 2000 (which is assumed to be equal to time ***t*** = ***0*** for the examples that follow), the sponsor of a pension plan uses $225.5 million of plan assets to purchase a portfolio of GoC bonds paying the following annual cash flows for the next 20 years:

Chart 1



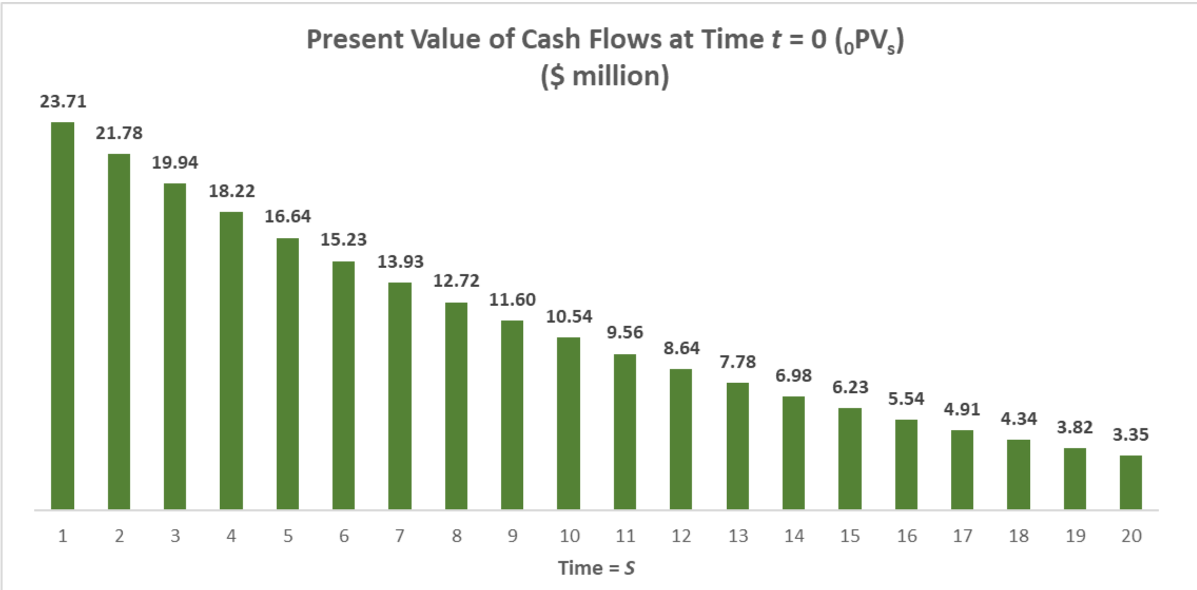
The daily history of GoC yield curves for zero-coupon bonds can be found on the Bank of Canada website at www.bankofcanada.ca/rates/interest-rates/bond-yield-curves/. By converting the GoC zero-coupon yields in the December 31, 2000, GoC yield curve to forward rates, the one-year forward rates as of December 31, 2000, for the following 20 annual maturity periods, are as follows:

Chart 2



Using formula ***(B)*** and the above forward curve $_{0}PV_{s}$, the present value at time ***0*** of the cash flows that will be paid by the portfolio at time ***s*** can be calculated for ***s*** = ***1*** to ***20***. The following chart summarizes these present values:

Chart 3



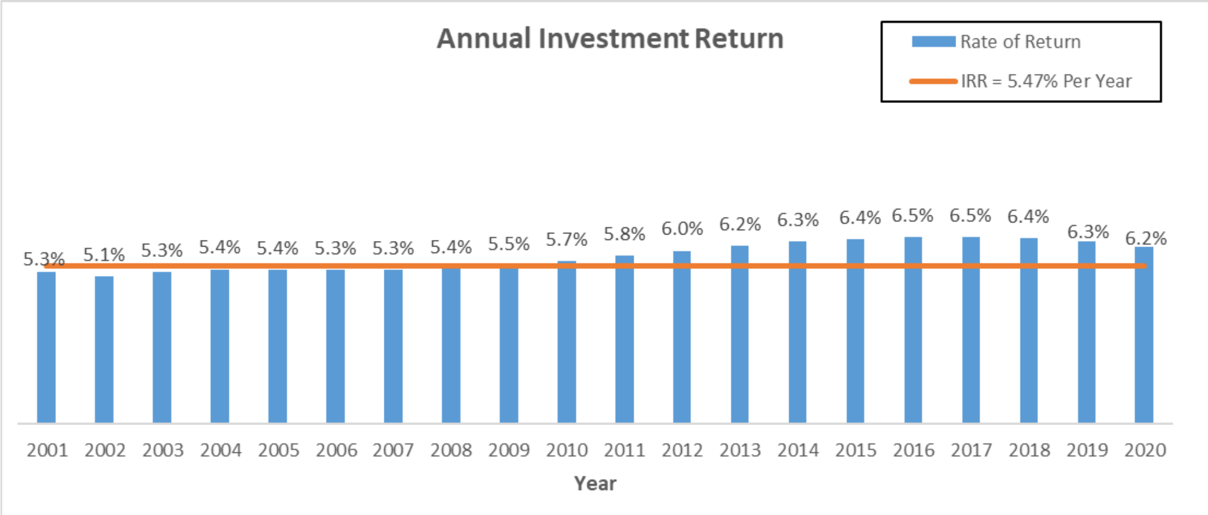
Also, using formula ***(B)*** the market value of the portfolio MV_0 can be calculated as **$225.5** million.

Using formula ***(G)***, the IRR of the portfolio y_0 (as of December 31, 2000) is calculated as **5.47%** per year.

Example 6: Buy and hold – shifting curve

If the plan sponsor is employing a buy-and-hold strategy with respect to the GoC bond portfolio and, similar to Example 4, the forward curve shifts each year in such a way as to preserve the original shape of the remainder of the curve for the 20-year period following December 31, 2000, the chart below summarizes the rate of investment return earned on the portfolio for each year in the 20-year period following December 31, 2000, until the final cash flows are paid from the portfolio. The annual rates of investment return are compared to the December 31, 2000, IRR of the portfolio of **5.47%** per year:

Chart 4



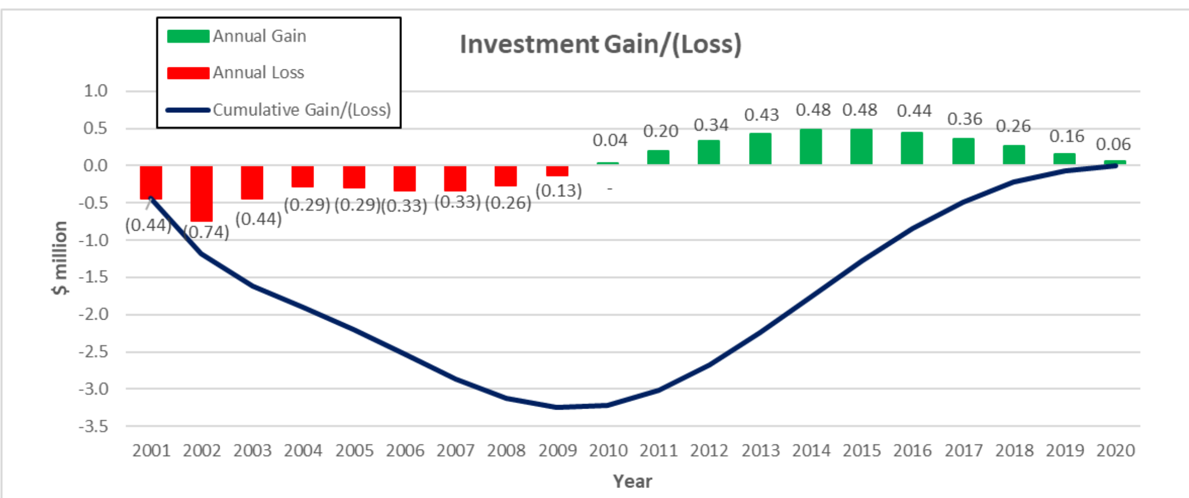
As discussed in Example 4, in the case of the shifting forward curve the annual rate of investment return for a given year is equal to the forward rate, as of December 31, 2000, in respect of that year. Therefore, the returns shown in Chart 4 match the forward rates summarized in Chart 2.

Using formula ***(J)***, the investment gain or loss (relative to the annual expected investment return based on the IRR y_t of the portfolio as of December 31, 2000, of **5.47%** per year) for the one-year period following time ***s*** can be calculated as follows:

$$Gain_s = MV_s \times R_{s+1} - {}_yMV_s \times y_t \qquad \textit{(K)}$$

The following chart summarizes the annual and cumulative investment gain/(loss) for the bond portfolio for the 20-year period following December 31, 2000:

Chart 5



It should be noted that, as anticipated by formula ***(J)***, the cumulative gain/(loss) is equal to 0 at the end of the 20-year period. Therefore, the average rate of investment return over the 20-year period is the IRR of the portfolio as of December 31, 2000, of **5.47%** per year.

Example 7: Buy and hold – curve resets

Consistent with Example 6, assume that the plan sponsor is employing a buy-and-hold strategy with respect to the GoC bond portfolio. However, similar to Example 5, the forward curve resets each year to its original shape for the 20-year period following December 31, 2000, which is analogous to assuming that there is no change to the current or forecast interest rate environment.

The following charts summarize the rates of investment return and investment gains/(losses) for the portfolio for the 20-year period following December 31, 2000. Again, it should be noted that the cumulative gain/(loss) is equal to ***0*** at the end of the 20-year period:

Chart 6

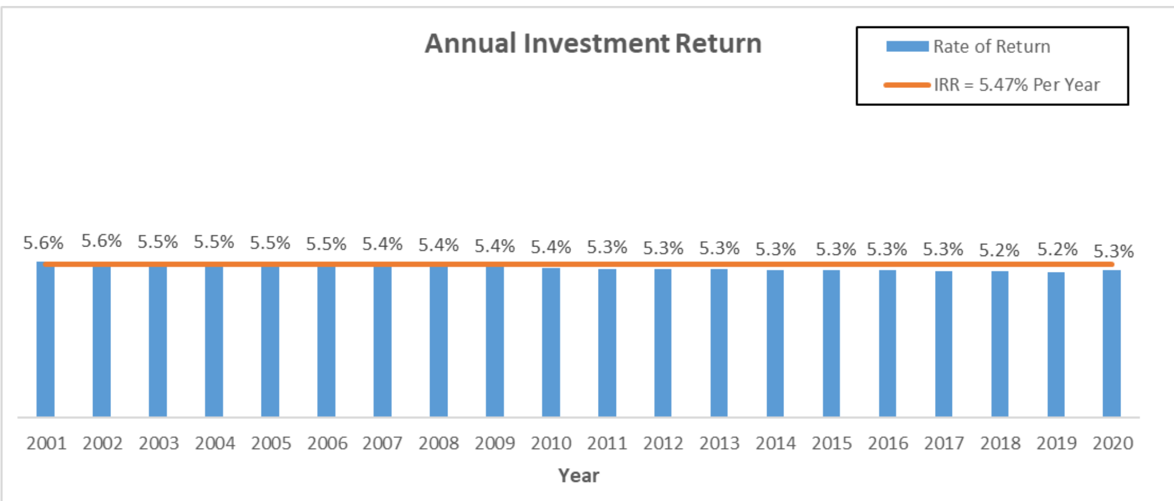
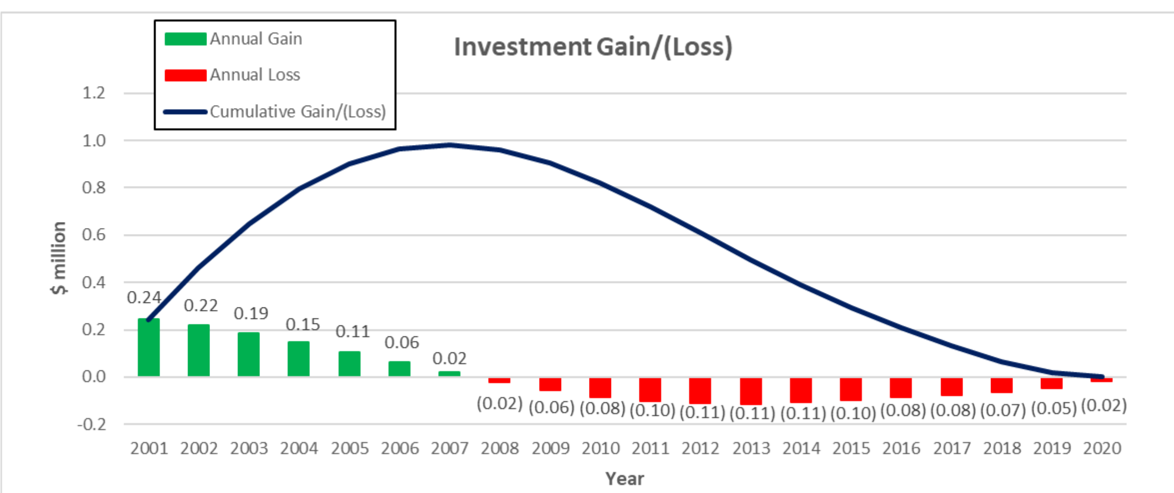


Chart 7



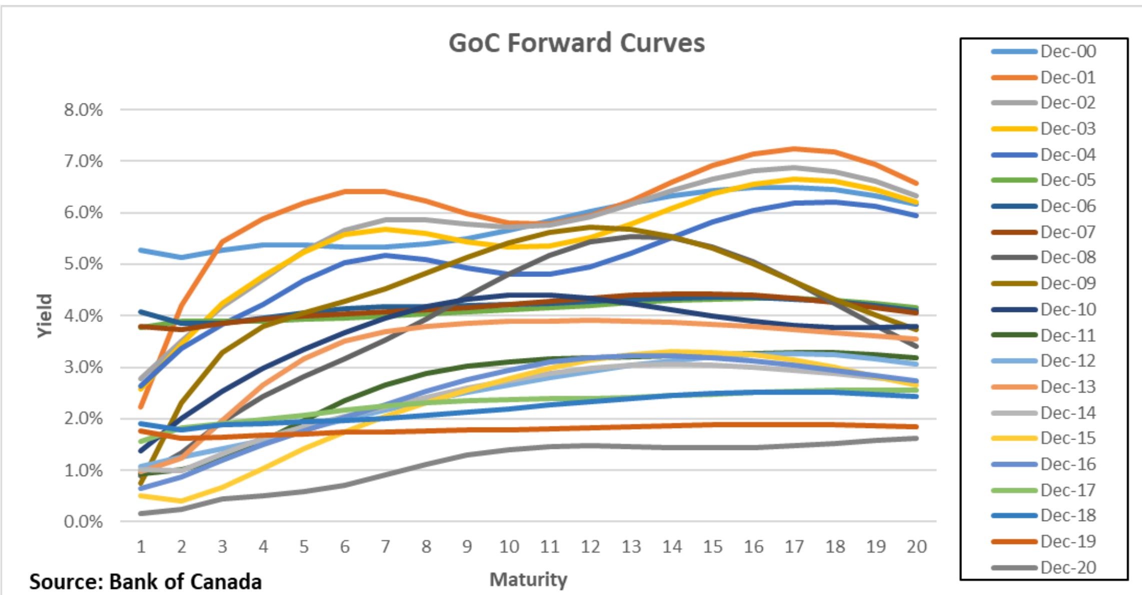
Example 8: Buy and hold – historical experience

While the examination of the pattern of investment returns of a bond portfolio in the case of shifting or resetting forward curves provides useful information, actual forward curves do not behave in such a smooth and consistent manner over time.

In this example, the same investor as in examples 6 and 7 employs a buy-and-hold strategy with the bond portfolio. However, the annual investment returns on the portfolio reflect the actual historical changes to the GoC forward curve over the 20-year period commencing December 31, 2000. (The forward curves at each year-end are based on the historical zero-coupon yield curves posted on the Bank of Canada website.)

The following chart shows the historical forward curves used in this example:

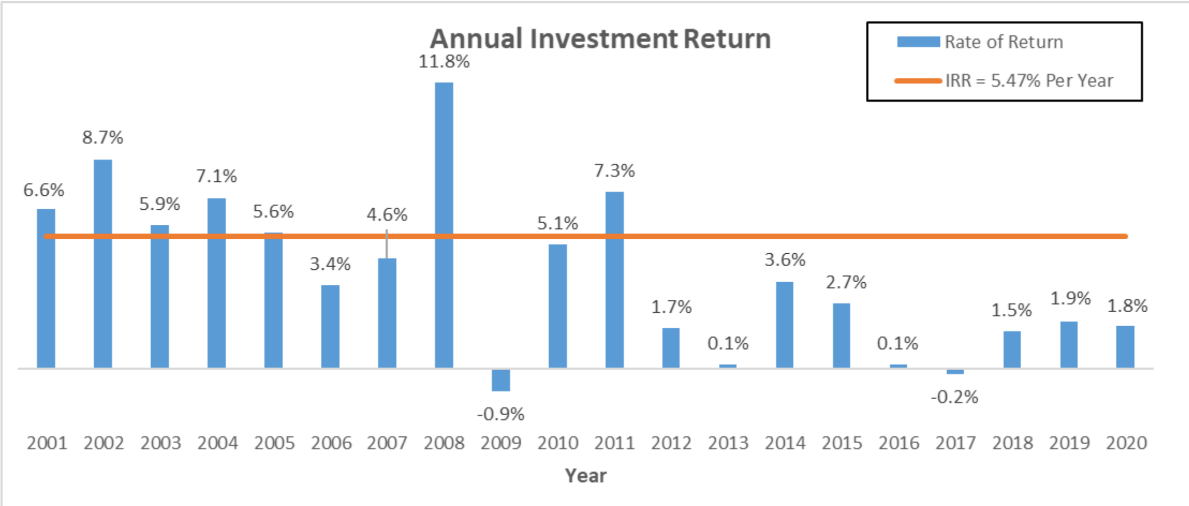
Chart 8



From Chart 8, it is evident that during the 20-year period commencing on December 31, 2000, GoC bond yields decreased significantly.

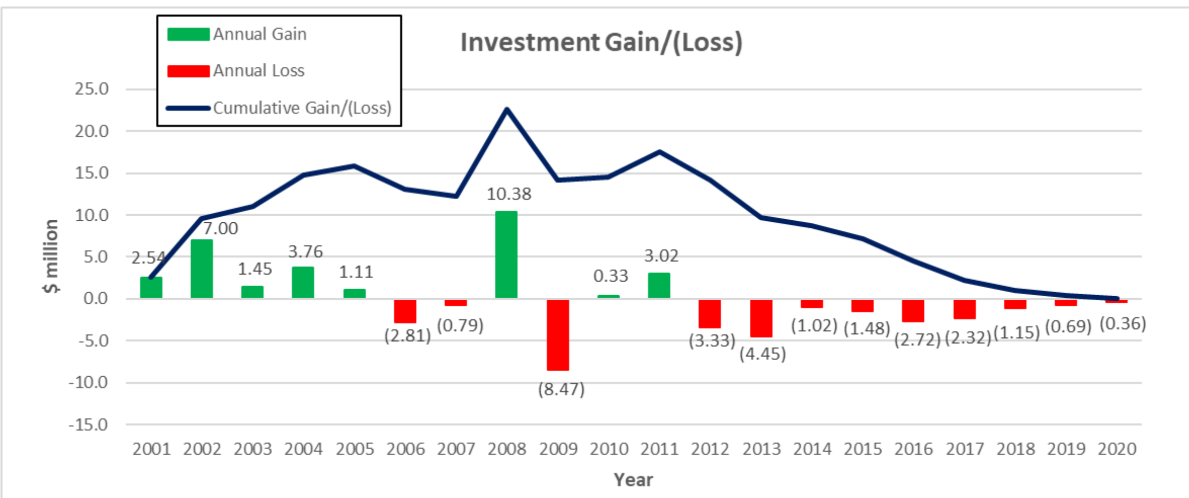
The following chart summarizes the annual rates of investment return of the portfolio, which are generally positive, as expected in a period of generally declining bond yields, but quite volatile from year to year:

Chart 9



The following chart summarizes the investment gains/(losses) for the portfolio:

Chart 10



Consistent with examples 6 and 7, the cumulative gain/(loss) is equal to 0 at the end of the 20-year period and, therefore, the average rate of investment return over the 20-year period is the IRR of the portfolio as of December 31, 2000, of **5.47%** per year. This is the case despite the fact that the 20-year period following December 31, 2000, was one in which bond yields decreased significantly and, in some periods, the forward curve exhibited considerable volatility.

As anticipated by formula ***(J)***, in the case of a buy-and-hold strategy, regardless of the movement of the forward curve during the 20-year period, capital gains/(losses) will be offset by subsequent lower/(higher) yields on the portfolio such that the cumulative gain or loss relative to the IRR of **5.47%** per year as of December 31, 2000, will amount to zero at the point when the last cash flows from the portfolio are paid.

Therefore, if a buy-and-hold strategy is employed with a portfolio of bonds in a pension plan, it is appropriate to assume at time ***t*** that the long-term expected rate of investment return on the portfolio will be equal to the IRR of the portfolio at time ***t***, without reflecting the effect of reinvestment and future changes in interest rates.

However, the following are considerations regarding the selection of the going concern discount rate in the case of a buy-and-hold strategy for a pension plan's bond portfolio:

a) If there is material credit risk in the portfolio, the expected rate of investment return of the bond portfolio should be adjusted (i.e., decreased) to reflect the risk of the future default of some of the bonds in the portfolio and the risk of downgrades if it is expected that future downgrades would trigger rebalancing by the plan sponsor in order to maintain the credit quality of the portfolio.

b) Ideally, there would be no need to reinvest any cash flows paid by the bond portfolio because all cash flows are immediately used to pay pension benefits to plan members. However, for a number of reasons, it is not feasible to create a perfect match between cash flows payable from a bond portfolio and pension benefit payments:

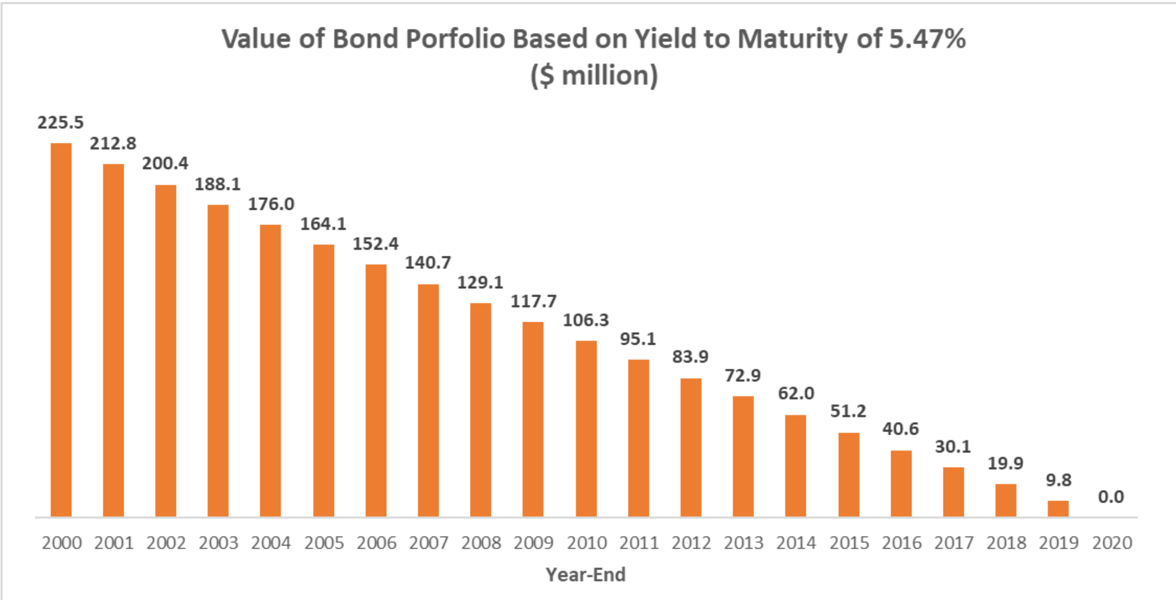
    - The future benefit payment stream from a pension plan is usually not known with certainty as it can depend on a number of contingencies, including the mortality experience of plan members, future price inflation in the case of indexed pensions, and the proportion of members who may elect to receive their pension entitlement in the form of a lump sum payment.
    - The bond portfolio may include certain features, such as call provisions, which make the cash flows from the bond portfolio uncertain.
    - There may not be bonds with sufficiently long maturities to cover some of the long-term benefit payments from the pension plan that the bond portfolio is intended to match.

    If future reinvestment of a portion of the cash flows from the bond portfolio is expected to be material, when selecting the discount rate the actuary will need to make an assumption about the manner in which these assets will be reinvested and the expected rate of investment return on the reinvested assets. Insight into approaches for doing this may be found in other actuarial practice areas, such as insurance.

c) If the bond portfolio represents only a portion of the total assets of a pension plan, it would not be appropriate to increase the expected rate of investment return on the pension plan assets to reflect the effect of diversification and rebalancing between the bond portfolio and the other plan assets. This is because there will not be regular rebalancing of the asset allocation between the buy-and-hold bond portfolio and the other assets of the plan.

d) If the bond portfolio represents only a portion of the total assets of a pension plan, the proportional asset allocation between the bond portfolio and the other plan assets will likely change over time. In order to illustrate why this is the case, below is the value of the GoC bond portfolio used in examples 6 through 8 valued at each year-end over the period December 31, 2000, through December 31, 2020, using the portfolio's IRR as of December 31, 2000, of **5.47%** per year:

Chart 11



As time passes and cash flows are paid from the bond portfolio, the value of the portfolio will tend to decrease because the number of future cash flows payable by the portfolio decreases. Unless the other plan assets are expected to exhibit a similar pattern, the bond portfolio will likely become a smaller and smaller proportion of plan assets over time.

Unless the pension plan sponsor has formally adopted an investment glide path, for the purpose of selecting the going concern discount rate Canadian actuaries usually assume that the asset allocation of the plan will not change in the future. However, given the nature of a buy-and-hold bond strategy, the author suggests that the actuary consider reflecting the expected future change in the asset allocation when establishing the going concern discount rate.

In this case, it is important for the actuary and pension plan sponsor to assess how the asset allocation is likely to evolve over time depending on the expected evolution of the plan liabilities over time and the sponsor's risk management and investment strategies. For example, the expected future change in the asset allocation would reflect the decrease over time of the proportion of plan assets attributable to the bond portfolio that is subject to the buy-and-hold strategy. However, if it is expected that the plan sponsor will allocate additional assets to bonds in the future, the actuary would consider reflecting these additional expected allocations, which will require an assumption about the expected rates of return on these future investments in bonds.

Rebalancing effect

Many pension plan sponsors in Canada do not explicitly employ a buy-and-hold strategy for the allocation to bonds in their pension plan assets. For example, an investment strategy may allocate a target percentage (e.g., 70%) of plan assets to a pooled fund comprised of long bonds with the objective of improving the plan's hedge ratio. In the case of this type of strategy, the investment manager of the pooled fund may rebalance the fund periodically to maintain a consistent mix of bonds and portfolio duration over time. Also, if the mandate of the bond portfolio is to replicate a market index, the investment manager will rebalance the portfolio over time in an attempt to replicate changes in the makeup of the market index. In addition, to maintain the allocation to bonds in line with the target percentage over time, the pension plan sponsor will periodically sell or buy units of the pooled fund as part of the rebalancing process.

Because of the rebalancing performed by both the investment manager and pension plan sponsor, the long-term investment return on the bond portfolio will be affected by the investment returns on future cash flows from the bond portfolio that are reinvested in the same portfolio and units of the pooled fund that are purchased. These effects can be referred to as the "reinvestment effect." The long-term investment return will also be affected by bond yields prevailing at the time that units of the pooled fund are sold as part of the periodic rebalancing process.

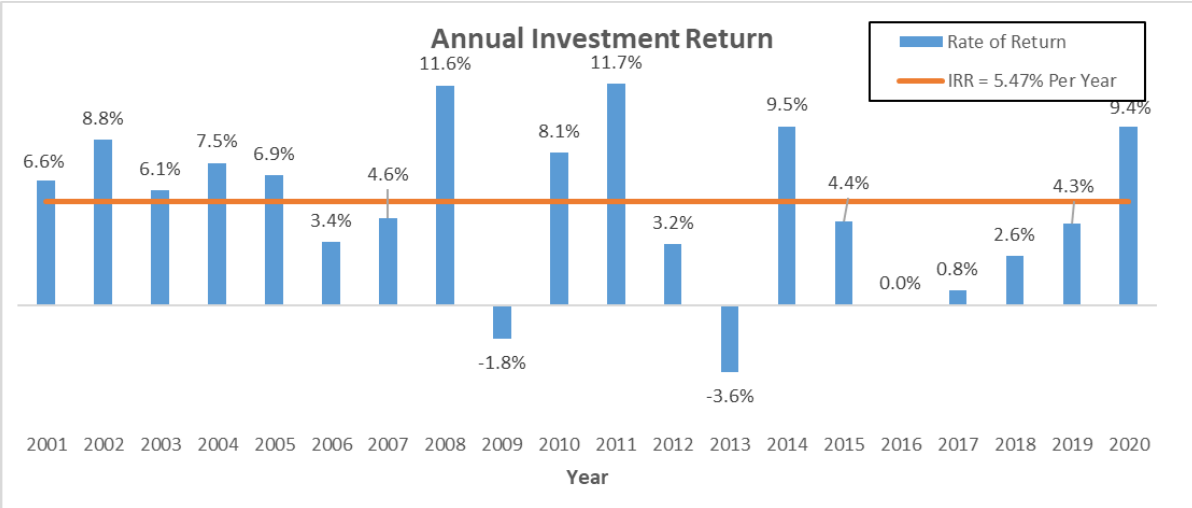
For such an investment strategy, is it appropriate for an actuary to assume that the long-term rate of investment return on the bond portfolio will be equal to the IRR of the portfolio on the valuation date? Because the investment return will be affected by the return on reinvested cash flows and by the effects of selling and buying units in the portfolio, the long-term return will be affected by the future level and shape of the applicable forward curves. When these effects are considered, it becomes apparent that the long-term rate of investment return on the portfolio will differ from the IRR on the valuation date under most future bond yield outcomes.

Example 9: Rebalancing – historical experience

In order to demonstrate how a simple rebalancing strategy may work, we return to the **$225.5** million GoC bond portfolio with the cash flows summarized in Chart 1 and an IRR of **5.47%** per year purchased by a plan sponsor on December 31, 2000. However, instead of employing a buy-and-hold strategy with the bond portfolio, at the end of each year the plan sponsor rebalances the asset allocation of the pension plan and the bond portfolio such that the future cash flows payable from the bond portfolio are the same as they were at the previous year-end. In other words, at any time t after the annual rebalancing of the portfolio, CF_{t+1} = **$25.0** million, CF_{t+2} = **$24.1** million . . ., and CF_{t+20} = **$10.4** million. It should be noted that this example reflects only one of many ways in which a bond portfolio within a pension plan may be rebalanced over time.

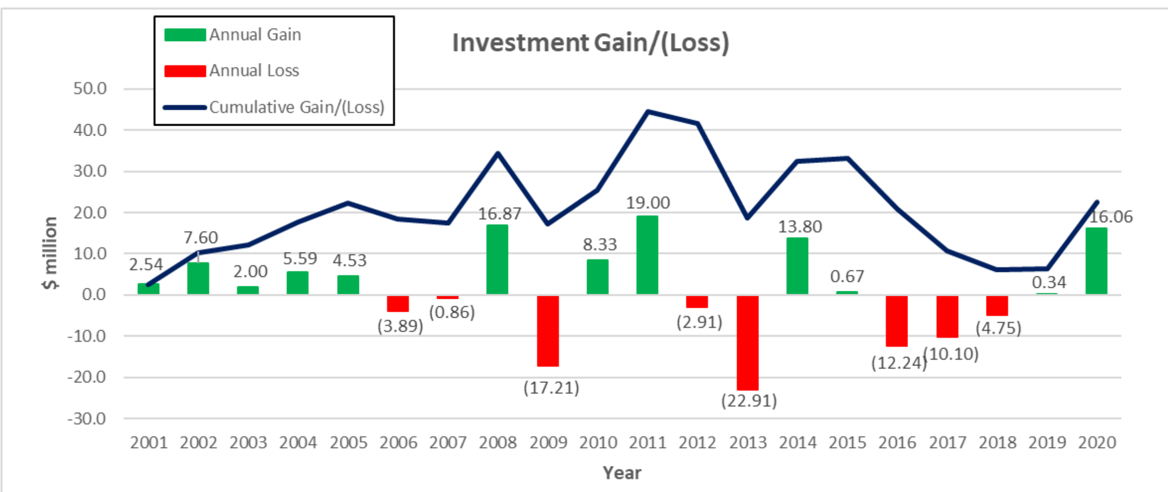
The following chart summarizes the annual rates of investment return of the portfolio based on the actual historical changes to the GoC forward curve over the 20-year period commencing December 31, 2000:

Chart 12



The following chart summarizes the investment gains/(losses) for the portfolio:

Chart 13



It should be noted that, as expected, the cumulative gain/(loss) relative to an expected rate of investment return of **5.47%** per year does not trend toward zero and is **$22.5** million as of December 31, 2020.

Example 10: Rebalancing – analysis of cumulative investment gain/(loss)

In order to explore the cumulative gains/(losses) for bond portfolios that are rebalanced in more detail, the analysis in Example 9 was repeated nine times using a bond portfolio with the same cash flows. However, the portfolio was assumed to be purchased by the pension plan sponsor on different dates – at each year-end from 1991 through 1999. The market value of the bond portfolio and the initial IRR on the purchase date were based on the GoC forward curve on that date (using historical zero-coupon yield curves posted on the Bank of Canada website). The following table summarizes the cumulative investment gain/(loss) on each of the bond portfolios 10 years and 20 years after the purchase date based on actual historical changes to the GoC forward curve:

Table VII (millions)

Date of Purchase	Market Value at Purchase	Initial IRR	Cumulative Gain/(Loss) After 10 Years	Cumulative Gain/(Loss) After 20 Years
Dec-91	$ 186.0	8.35%	$ 24.8	$ 34.7
Dec-92	189.1	8.09%	28.4	30.2
Dec-93	206.9	6.71%	24.4	15.4
Dec-94	179.3	8.94%	32.3	10.0
Dec-95	201.2	7.13%	29.5	20.2
Dec-96	210.4	6.47%	22.1	11.9
Dec-97	221.9	5.70%	18.8	9.1
Dec-98	234.3	4.93%	34.6	14.0
Dec-99	213.5	6.25%	19.3	(1.7)
Dec-00	225.5	5.47%	25.5	22.5

From Table VII it is apparent that in the case of a bond portfolio that is periodically rebalanced, future cumulative investment gains/(losses) relative to the portfolio's IRR at any point in time will not necessarily trend toward zero. For this type of investment strategy, it is not necessarily appropriate to assume that the long-term rate of investment return on the bond portfolio will be equal to the IRR of the portfolio. When selecting the going concern discount rate, the actuary should consider the expected effect of reinvestment and future changes in interest rates after the valuation date.

Assuming rate of investment return equal to internal rate of return: Curve reset

As mentioned in the previous section, in the case of a bond portfolio that is periodically rebalanced it is generally not appropriate to assume that the long-term future rate of investment return on the portfolio will be equal to the IRR of the portfolio. However, a situation for which this assumption may (under certain conditions) be appropriate is when it is assumed, as in Example 5, that each year in the future the forward curve resets to its original shape (which is analogous to assuming that there is no change to the current or forecast interest rate environment). In other words, ${}_{t+s+1}f_u = {}_{t+s}f_{u-1}$.

In order to demonstrate why this is the case, assume that y_t is the IRR of a GoC bond portfolio at time t, and y'_{t+1} is the IRR of the same portfolio at time $t+1$ before the bond portfolio is rebalanced.

Using formulas ***(D)*** and ***(G)***, it can be shown that R_{t+1}, the rate of investment return of the bond portfolio over the period t to $t+1$, can be calculated as:

$$R_{t+1} = (1 + y_t)\frac{CF_{t+1} + \sum_{s=t+2}^{N} \frac{CF_s}{(1 + y'_{t+1})^{s-t-1}}}{CF_{t+1} + \sum_{s=t+2}^{N} \frac{CF_s}{(1 + y_t)^{s-t-1}}} - 1 \qquad \textit{(L)}$$

The following are a few observations regarding formula ***(L)***:

- If y'_{t+1} = y_t, the rate of investment return over the period ***t*** to ***t+1*** is equal to y_t, the IRR of the portfolio at time ***t***.
- Even when y'_{t+1} is not equal to y_t, if they have similar values then the numerator and denominator in formula ***(L)*** will have similar values and the rate of investment return over the period ***t*** to ***t+1*** will be approximately equal to y_t. The value of y'_{t+1} will be similar in value to y_t under the following conditions:
  - The forward curve is reset at time ***t+1*** to its original shape one year earlier.
  - The size and period of cash flows payable from the bond portfolio are large enough such that removal of CF_{t+1} does not have a material effect on the pattern of cashflows of the portfolio. In other words, the fact that the calculation of y_t reflects CF_{t+1}, while the calculation of y'_{t+1} does not reflect these cash flows payable at time ***t+1***, does not result in a material difference between the values of y_t and y'_{t+1}.
- y'_{t+1} is based on one year less of cash flows compared to the determination of y_t. Therefore, if the forward curve is reset at time ***t+1*** and is generally upward-sloping, then $y'_{t+1} < y_t$. Under these conditions, the numerator in formula ***(L)*** will be larger than the denominator, in which case $R_{t+1} > y_t$. This gain in the rate of investment return relative to y_t can be referred to as the "curve reset gain."

Based on the above observations, in the case of a bond portfolio that is periodically rebalanced, it may be appropriate to assume that the long-term future rate of investment return on the portfolio will be equal to the IRR of the portfolio on the valuation date when the following conditions hold:

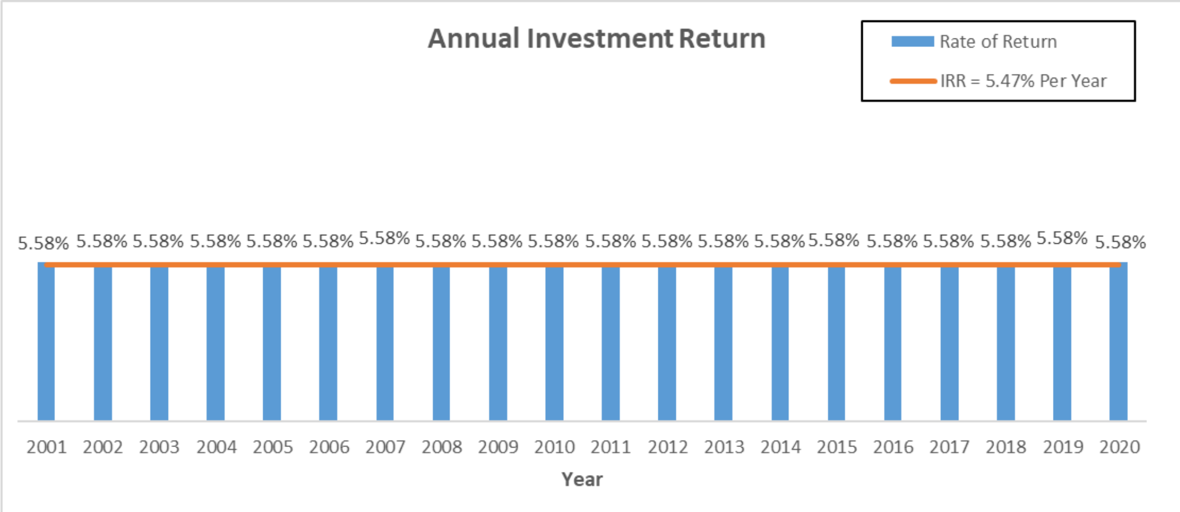
i. The forward curve is expected to reset at the end of each year to its original shape (which is analogous to assuming that there is no change to the current or forecast interest rate environment).
ii. The bond portfolio is expected to be rebalanced periodically such that its makeup (i.e., the pattern of future cash flows) remains approximately the same over time.
iii. The size and period of cash flows payable from the bond portfolio are significant enough such that removal of the cash flows payable in one year's time does not have a material effect on the pattern of cash flows of the bond portfolio.

Example 11: Assuming rate of investment return equal to IRR – curve reset

For the purposes of this example, on December 31, 2000, a pension plan sponsor invests in the same bond portfolio and rebalances it in the same manner as described in Example 9. However, at the end of each year following December 31, 2000, the forward curve is assumed to reset to its original shape.

The following chart summarizes the annual rates of investment return of the portfolio during the 20-year period following December 31, 2000:

Chart 14



The rate of investment return on the bond portfolio is **5.58%** per year during each year of the 20-year period following December 31, 2000. The rate of return is the same each year because at the beginning of each year the future cash flows payable from the bond portfolio and the forward curve are the same as they were as of December 31, 2000.

The annual rate of return of **5.58%** per year is close to the IRR of the portfolio of **5.47%** per year as of December 31, 2000 (i.e., the curve reset gain is only **0.11%** per year). Therefore, under this scenario, it would likely have been appropriate for an actuary conducting a December 31, 2000, actuarial valuation of the pension plan to assume that the long-term rate of investment return on the bond portfolio would be equal to the portfolio's IRR of **5.47%** per year.

Example 12: Curve reset additional historical examples

In order to provide additional examples of scenarios in which the forward curve resets annually, the analysis in Example 11 has been repeated nine times, but assuming that the bond portfolio was purchased by the pension plan sponsor on different dates – at each year-end from 1991 through 1999. The market value of the bond portfolio and the initial IRR on the purchase date were based on the GoC forward curve on the purchase date.

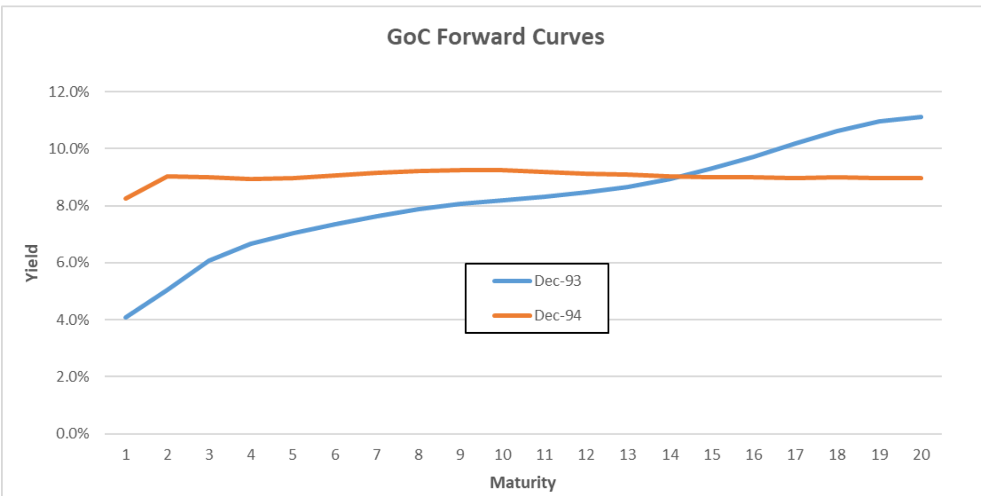
The following is a summary, for each purchase date, of the initial IRR of the bond portfolio, the curve reset gain, and the annual rate of investment return on the portfolio:

Table VIII

Date of Purchase	Initial IRR	Curve Reset Gain	Annual Portfolio Return
Dec-91	8.35%	0.19%	8.54%
Dec-92	8.09%	0.18%	8.27%
Dec-93	6.71%	0.36%	7.07%
Dec-94	8.94%	0.02%	8.96%
Dec-95	7.13%	0.16%	7.29%
Dec-96	6.47%	0.24%	6.71%
Dec-97	5.70%	0.12%	5.82%
Dec-98	4.93%	0.09%	5.02%
Dec-99	6.25%	0.05%	6.30%
Dec-00	5.47%	0.11%	5.58%

It should be noted that a steeper upward-sloping forward curve will tend to be associated with a larger curve reset gain. For example, Chart 15 shows the forward curves as of December 31, 1993 (which produces a relatively large curve reset gain), and as of December 31, 1994 (which produces a small curve reset gain):

Chart 15



The following are some considerations in the case that an actuary is using the IRR of a bond portfolio (without reflecting the effect of reinvestment and future changes in interest rates) to select the going concern discount rate based on the assumption that conditions i., ii., and iii. above hold:

a) If there is material credit risk in the portfolio, the expected investment return on the bond portfolio should be adjusted (i.e., decreased) to reflect the risks of the future default and downgrade of some of the bonds in the portfolio.

b) The actuary should consider whether it is reasonable to assume that there will be no change to the current or forecast interest rate environment for at least 10 to 20 years following the valuation date. For example:

- Since the natural shape of a yield curve is upward-sloping, would the assumption of no changes in the interest rate environment over the long term be appropriate if the yield curve is downward-sloping on the valuation date?
- At the end of 2020, interest rate levels were likely significantly affected by the actions being taken at the time by central banks, such as quantitative easing, to address the economic challenges caused by the COVID-19 pandemic. Would it be appropriate to assume, for the purpose of a December 31, 2020, actuarial valuation, that no future changes to the interest rate environment will occur after the end of 2020?

c) The actuary should consider whether the curve reset gain is material enough to reflect in the going concern discount rate. As mentioned, a steeper upward-sloping forward curve will tend to be associated with a larger curve reset gain. The actuary will need sufficient detail regarding the cash flows payable from the bond portfolio and the value of these cash flows in order to calculate the curve reset gain.

d) The material assumptions selected by an actuary for a valuation should be "independently reasonable and appropriate in the aggregate." If the actuary is assuming that the interest rate environment will remain unchanged after the valuation date, then the other valuation assumptions, such as price inflation, wage increases, and the expected rate of investment return on other asset classes (e.g., equities) should be consistent with an economic environment in which no future changes in interest rates occur. Demographic assumptions, such as member withdrawal and retirement rates, may also be affected by an economic environment in which no future changes in interest rates occur. In addition, this type of economic scenario may affect the expected correlation between the investment returns of different asset classes, which can impact the diversification and rebalancing component of the going concern discount rate assumption.

   It should be noted that capital market models commonly used by actuaries often assume that over time key economic measures such as GDP, inflation, and interest rates will, on average, revert to means which are different from current conditions. Therefore, it may not be appropriate for an actuary to assume that the future interest rate environment will remain unchanged for the purpose of estimating the expected rate of investment return of a pension plan's bond portfolio while at the same time basing other valuation assumptions, including the expected rate of investment return of other asset classes, on a capital market model that assumes future mean reversion.

Targeting a specific hedge ratio

In some cases, the investment strategy of a pension plan sponsor may be to maintain a specific hedge ratio over time (e.g., a hedge ratio of 100% or 80% on a solvency valuation basis). If an investment strategy of using a bond portfolio to maintain a specific hedge ratio is being implemented in the following manner:

A. the targeted hedge ratio is expected to remain the same over time;

B. the bond portfolio is adjusted frequently in order to maintain the targeted hedge ratio, recognizing that the duration of the plan liabilities will evolve over time;

C. as the bond portfolio is adjusted over time, the overall characteristics of the portfolio, in terms of factors such as sector mix and credit quality, are expected to remain consistent over time; and

D. the strategy is implemented in such a way that, ignoring (i) the effect on the duration of the plan liabilities of any future service accruals of active plan members and (ii) sales from the bond portfolio to source the portfolio's share of pension benefits payable from the plan, it is unlikely that a significant portion of the bond portfolio will need to be sold or a significant amount of new bonds will need to be purchased in the future in order to maintain the targeted hedge ratio,

then it may be appropriate to assume that the long-term future rate of investment return on the portfolio will be equal to the IRR of the portfolio on the valuation date (without reflecting the effect of reinvestment and future changes in interest rates). The rationale for this assertion is that the market value of a bond portfolio over time that is established and maintained consistent with the characteristics described in A. to D. above may reasonably track the value of a bond portfolio with a similar market value and IRR that is held to maturity.

The author recommends further research into the expected long-term future rates of investment return of a bond portfolio in the case of a strategy of maintaining a targeted hedge ratio in order to better understand when it may be appropriate to assume that the rate of return will be equal to the IRR. The research should include assessing the practicality of establishing an investment strategy under which the conditions described in A. to D. above will hold and scenarios in which leverage is being used to achieve the desired hedge ratio.

An actuary who concludes that, for the purpose of selecting the going concern discount rate, it is appropriate to assume that the rate of return on the bond portfolio will be equal to the IRR of the bond portfolio on the valuation date should also consider factors such as those described in a) to d) of the "Buy-and-hold investment strategy" section of this paper.

Conclusion

Due to the increased importance of going concern valuations in the funding of Canadian pension plans, there is an increased emphasis on the assumptions used for these valuations, including the going concern discount rate.

This paper addressed considerations when selecting the going concern discount rate for a pension plan with a high allocation to bonds, with a particular emphasis on the component of the discount rate which is based on the long-term expected rate of investment return of the plan's bond portfolio.

If the hedge ratio of a pension plan on a going concern basis is important to and is being managed by the plan sponsor, it is likely preferable to assume for the purpose of selecting the going concern discount rate that the long-term expected rate of investment return on the plan's bond portfolio will equal the IRR of the portfolio on the valuation date, *without* reflecting the effect of reinvestment and future changes in interest rates. However, it is only appropriate to make this assumption under certain conditions.

One such condition is that a buy-and-hold strategy is being employed with the bond portfolio. However, there are a number of considerations when selecting the going concern discount rate and a buy-and-hold strategy is being employed:

a) If there is material credit risk in the portfolio, the expected rate of investment return on the bond portfolio should be adjusted (i.e., decreased) to reflect the risk of the future default and downgrades of some of the bonds in the portfolio.

b) If future reinvestment of a portion of the cash flows from the bond portfolio is expected to be material, the actuary will need to make an assumption about the manner in which the assets will be reinvested, and the expected rate of investment return on the reinvested assets.

c) If the bond portfolio represents only a portion of the total assets of the pension plan, it would not be appropriate to increase the expected return on the pension plan assets to reflect the effect of diversification and rebalancing between the bond portfolio subject to the buy-and-hold strategy and the other plan assets.

d) The author suggests that the actuary consider reflecting the expected future change in the asset allocation when establishing the going concern discount rate.

In the case of a bond portfolio that is periodically rebalanced (i.e., the plan sponsor is not employing a buy-and-hold strategy), it is generally not appropriate to assume that the long-term rate of investment return on the portfolio will be equal to the IRR of the portfolio. However, it may be appropriate to assume that the rate of investment return on the portfolio will be equal to the IRR of the portfolio on the valuation date when the following conditions hold:

i. The interest rate environment will remain unchanged following the valuation date.

ii. The bond portfolio will be rebalanced periodically such that the makeup of the bond portfolio remains approximately the same over time.

iii. The size and period of cash flows payable from the bond portfolio are significant enough such that removal of the cash flows payable in one year's time does not have a material effect on the pattern of cashflows of the bond portfolio.

However, the following are some considerations when an actuary is using the IRR of a bond portfolio that is being periodically rebalanced to select the going concern discount rate based on the assumption that conditions i. to iii. above will hold:

a) If there is material credit risk in the portfolio, the expected rate of investment return on the bond portfolio should be adjusted (i.e., decreased) to reflect the risks of the future default and downgrades of some of the bonds in the portfolio.

b) The actuary should consider whether it is reasonable to assume that the interest rate environment will remain unchanged for at least 10 to 20 years after the valuation date.

c) The actuary should consider whether the curve reset gain is material enough to reflect in the going concern discount rate.

d) The actuary should ensure that all material valuation assumptions are consistent with an economic environment in which no future changes in the interest rate environment occur.

A third potential situation where it may be appropriate to assume that the long-term rate of investment return on a bond portfolio will be equal to the IRR of the portfolio is when the investment strategy of a pension plan sponsor is to maintain a specific hedge ratio over time (e.g., 100% or 80%). The author recommends further research into the expected long-term rates of investment returns on bond portfolios for this type of strategy.

The examples in this paper focus on portfolios of GoC bonds. In reality, a bond portfolio often includes bonds from different sectors (e.g., government and corporate) and bonds that differ with respect to their credit rating. For a bond portfolio with a heterogeneous makeup, each category of bonds in the portfolio implicitly has a forward curve associated with it. Since the concepts outlined in this paper apply to each category of bonds in the portfolio, the concepts also apply to the portfolio overall.

While the focus of this paper is on Canadian going concern funding valuations, the concepts discussed should be relevant to any situation in which an actuary is estimating the long-term rate of investment return on a bond portfolio. This includes the assumptions used to calculate the funding requirements of certain pension plans in countries other than Canada, and the assumptions used to calculate the pension accounting cost in accordance with certain accounting standards.

Also, the concepts in this paper have been addressed from the perspective of a pension actuary selecting the assumptions for the actuarial valuation of a pension plan. However, a person who is responsible for establishing the investment strategy for a pension plan should consider these concepts from the perspective of how the strategy for the plan's bond mandate (e.g., buy and hold versus periodic rebalancing) will affect the financing strategy for the plan. For example, the strategy for the plan's bond mandate will likely affect the expected rate of investment return of the bond portfolio, the hedge ratio of the pension plan on a going concern basis, the expected evolution of the plan's allocation to bonds over time, and the trade-off between investing in bonds and other risk management solutions such as the purchase of a group annuity from an insurance company. It also is helpful for both the plan sponsor and the actuary to be forward-looking and consider how, depending on the sponsor's pension risk management strategy, the investment strategy and asset mix allocation of a pension plan are expected to evolve over time.

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