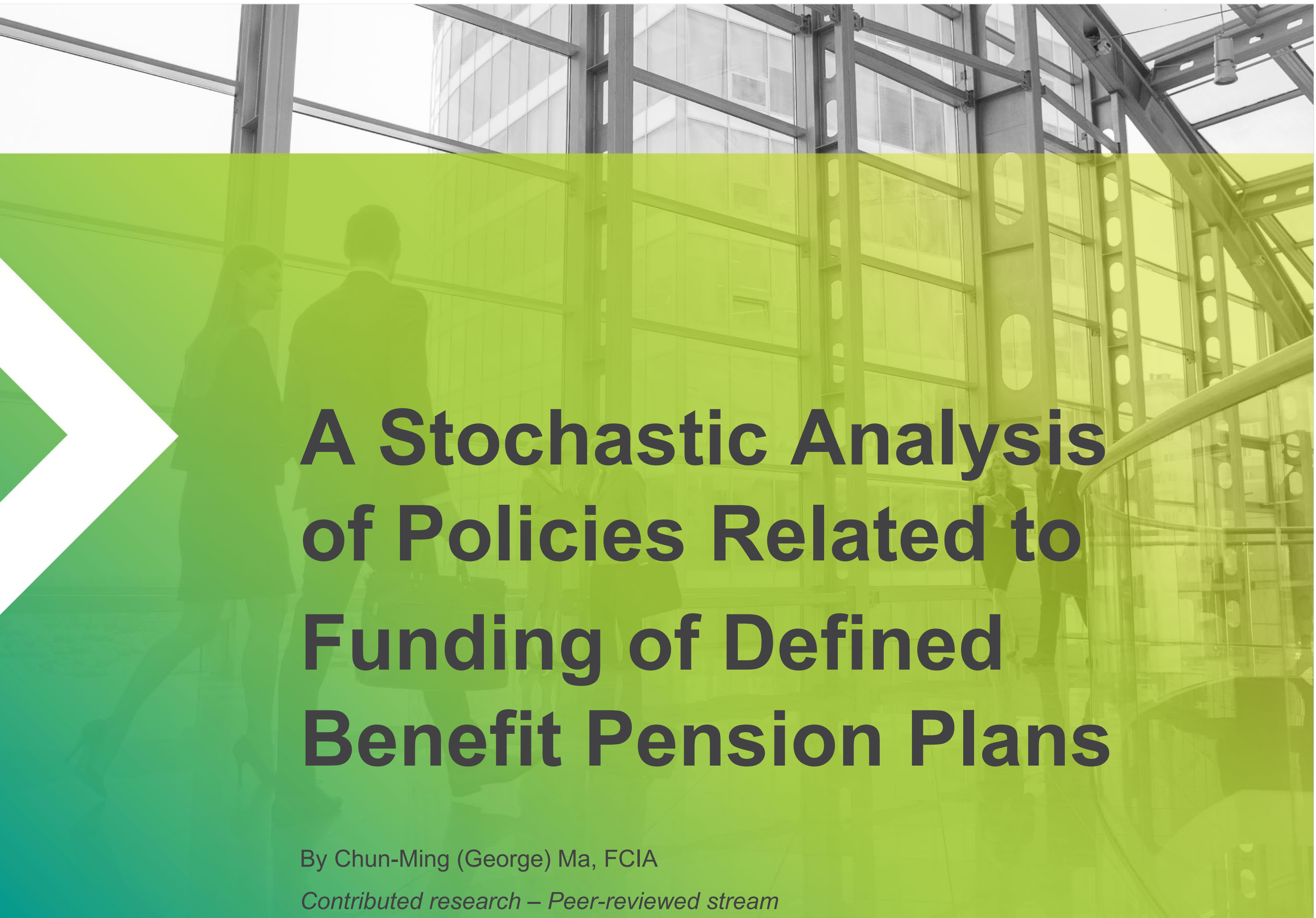
Canadian Institute of Actuaries
Institut canadien des actuaires



A Stochastic Analysis of Policies Related to Funding of Defined Benefit Pension Plans

By Chun-Ming (George) Ma, FCIA

Contributed research – Peer-reviewed stream

Document 223051

Study in brief

Many Canadian jurisdictions have reformed their pension legislation that governs the funding of defined benefit (DB) pension plans, moving to a new funding regime known as "going concern plus." One of the key objectives of the funding reform is to reduce the level and volatility of required contributions to DB plans, mainly through the elimination or easing of the solvency funding requirements. At the same time, going concern funding requirements are strengthened. Plans are required to keep reserves to buffer against adverse deviations in experience versus assumptions made in the actuarial valuation. This regulatory funding regime is new, and untested.

This research uses a stochastic modelling technique to analyze the impact of funding-related policies on the risks and costs of a DB plan under the new funding regime. Going concern funded ratio (ratio of assets to liabilities) is employed as the key metric for evaluating risk. The policy decisions under investigation are amortization period, target asset mix, limits on use of surplus, and provision for adverse deviations (PfAD).

Key findings of our exploration are as follows:

- A long amortization period would help to stabilize the annual contribution requirements of a plan, but could expose the plan to a significant downside risk, thereby requiring additional contributions from the employer in the future.
- Adoption of an investment policy with a high allocation of the pension plan fund to return-seeking assets could potentially lower the plan's costs, but would entail a greater volatility in the funding level of the plan and a greater risk of underfunding.
- Unrestricted use of surplus, as a reduction in required contributions or a refund to the employer, could increase a plan's exposure to the risk of underfunding over time.
- Including a PfAD in the funding requirements could help improve the funding level of a plan and reduce its risk of being underfunded in the long term.

We have devised a "funding reserve map" to facilitate the search for a risk-based PfAD to meet the long-term funding goal of a plan. The PfAD reflects the investment risk being taken by a plan. Plans with a higher proportion of their pension fund invested in equities would require a higher level of PfAD than plans with a more conservative investment strategy, all else being equal.

Another lesson captured from our modelling is that the required PfAD to fully fund a plan over the long term should reflect the current funding level of the plan – the lower the current funded ratio, the higher the PfAD required.

Once policy-makers have established an investment policy and a deficit amortization rule for their pension plan, they could control the risk of the plan's future funding level, either upside or downside, by setting appropriate values for the following two parameters: (1) the level of PfAD to be included in the plan's funding target and (2) the limit on the amount of surplus that is available for use as a contribution holiday or for other purposes.

For sponsors and/or trustees trying to deliver benefits promised under a DB plan while managing the volatility of funding requirements, the policy insights provided in this paper may help them put plans on a sustainable funding path.

Acknowledgments

This work was carried out with the assistance of Ms. Wendy Pui Yi Lai, while she was an actuarial science student at the University of Hong Kong (HKU) and supported by a research grant provided through Professor Kam C. Yuen of the Department of Actuarial Science and Statistics at HKU.

The author would like to thank Robert L. Brown and Doug Chandler for their peer review and comments on the paper. The author retains responsibility for any errors and the views expressed.

Table of contents

1 Introduction

Since 2016, many Canadian pension jurisdictions have moved gradually to a new funding regime for defined benefit (DB) pension plans, known as "going concern plus." This new model puts less emphasis on solvency funding while strengthening the requirements for going concern funding. For example, in Ontario, British Columbia, New Brunswick, and Nova Scotia, DB plan sponsors are now only required to fund up to 85% of a plan's solvency liabilities. And in Québec, no solvency funding is required.

Solvency funding prescribed under the old funding regime essentially evaluates whether a plan had sufficient assets to support the benefits promised under the plan at current market interest rates if the plan were to wind up. Going concern funding, on the other hand, puts focus on rational and orderly accumulation of assets to support the payments of promised benefits over the long term.

The new DB funding regimes being implemented in various jurisdictions have several common features:

- A funding reserve, also known as a "provision for adverse deviations" or "PfAD," must now be included in the going concern liabilities and/or normal cost for determining a plan's funding requirements.
- The amortization period for funding going concern deficits is shortened from 15 years to 10 years. All outstanding unfunded liabilities at a valuation date are allowed to be consolidated into a single amount and re-amortized over a period of not more than 10 years from the valuation date (i.e., a "fresh-start" basis).
- The employer of a pension plan is permitted to take a contribution holiday or a refund of surplus, if the plan permits, as long as the actuary determines that the plan will remain fully funded. A contribution holiday or surplus refund may only be taken in respect of surplus exceeding a certain threshold, which is equal to 5% of liabilities (going concern or solvency) in some jurisdictions.
- Several jurisdictions (for example, federal, Alberta, British Columbia, Ontario, Québec, and Nova Scotia) have amended their legislation to permit employers to use an irrevocable letter of credit or solvency reserve account, instead of funding solvency deficiencies on a permanent basis.

As of January 1, 2021, over 4.4 million Canadians were covered by a DB plan. Membership in a DB plan accounted for two-thirds of the total membership in a registered pension plan. The public sector dominated the DB membership at 72.5%.[1] The number of open DB plans in the private sector is expected to continue its downward trend as more employers in that sector either wind up or freeze their DB plans, or merge them into a consolidation vehicle in the public sector (such as the "jointly sponsored pension plan" in Ontario). Public sector pension plans in Canada are for the most part exempt from the volatile solvency funding requirement.

Given the pension landscape and the foregoing development of the regulatory funding frameworks, the going concern valuation is likely to be the key funding driver going forward for many Canadian DB plans.

Ma (2021) presented a mathematical model for assessing the impacts of policies under the new funding regime on a DB plan over a 20-year period from 2000 to 2019, inclusive. The economic environment of that period was characterized by a trend of declining long-term bond yields and volatile equity returns. This paper extends the application of that model to a stochastic economic environment. We focus our attention on those DB plans which are not subject to the funding of solvency deficiencies, and whose employers have the financial ability to support the plan now and in the future.[2] Buyout with an insurer is not the endgame of their funding and investment strategy.

We use the stochastic model described in Appendix A to assess how different funding-related policies would impact the risks and costs of a DB plan under the new funding regime. The measures under investigation are amortization policy, investment policy, surplus policy, and funding reserve policy. Our focus is on investment risk while the demographic risk is examined in the context of different membership profiles (stationary vs. non-stationary).

Unless otherwise specified, the term "cost" used throughout this paper is the contribution required on a going concern funding basis to make provision for the plan's liabilities. It is not the cost in the accounting or economic sense.

The remainder of this paper is organized as follows. In Section 2, we demonstrate the uncertain nature of pension funding dynamics through a stochastic simulation on a plan with stationary membership. In Section 3, we explore the long-term funding implications arising from different policy measures applied to a DB plan, and capture lessons learned. Section 4 uses a "funding reserve map" to establish a risk-based PfAD for a plan that aims to meet a stated

[1] Source: Statistics Canada. https://www150.statcan.gc.ca/n1/daily-quotidien/220718/dq220718a-eng.htm

[2] Most pension plans in the public sector fall into this category. Generally speaking, the government as a sponsor of public sector pension plans has the power to tax and the ability to survive the ups and downs of the business cycle. Thus, public sector employers tend to be less concerned with insolvency risk than their private sector counterparts and so can follow a long-term approach in funding their pension plans.

long-term funding goal. Section 5 examines the long-term funding risk pertaining to a plan with either declining or growing membership, in contrast to a plan with stationary membership. In Section 6, we contend that the PfAD level for a plan with a pre-set investment policy should not be held constant, but should vary according to the evolving funding position of the plan. Section 7 discusses the implications of funding-related policies under the new funding regime and proposes a risk management approach to developing a long-term funding strategy for DB plans. Section 8 concludes.

2 Baseline pension plan

For the purposes of our analysis, we consider a baseline DB plan with the following characteristics:

- Its membership is stationary.
- Fifty per cent of the plan liabilities are related to pensions in pay.[3]
- It adopts a "Balanced" investment strategy, where 50% of the plan assets are invested in bonds, with a hedge ratio[4] of 1.0, and 50% in equities.
- It does not include a PfAD for determining the funding requirement.
- It does not require a surplus threshold[5] before amortization of surplus begins.
- Its starting funded ratio is 1.0.

Similar to the amortization approach being implemented in Canada, we assume that any emerging funding deficit would be amortized over 10 years, on a fresh-start basis at each valuation date. For simplicity, we use a 0% interest rate to calculate the amortization amount. This means that if a valuation reveals a funding deficit, an amount equal to 10% of that deficit is paid into the pension fund each year before the next valuation date.[6] Symmetrically, 10% of any funding excess, over and above any applicable surplus threshold, is applied as a reduction in contributions or for other purposes (e.g., refund to the employer).

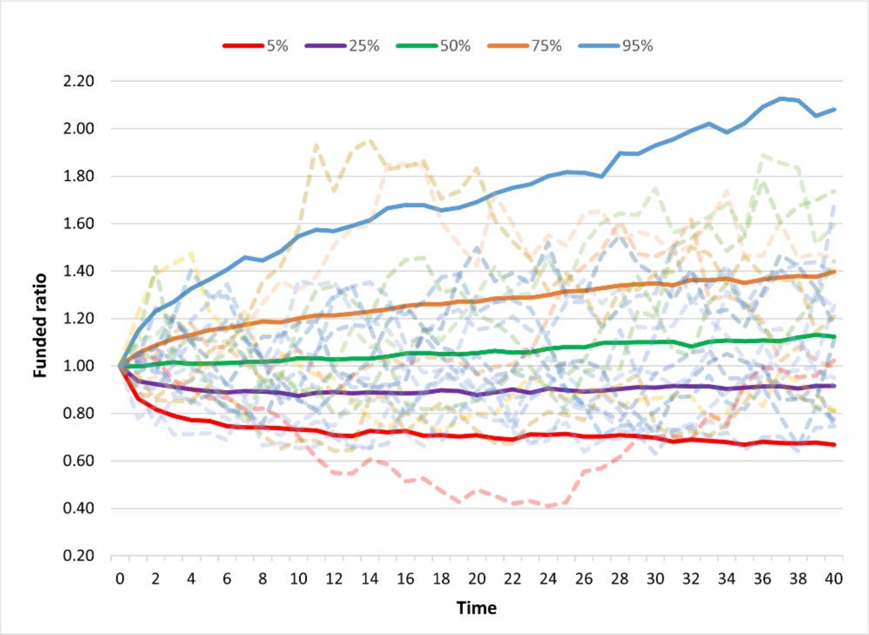
We apply a stochastic model to project the assets, liabilities, and funded ratio of the baseline pension plan, in order to capture the variability of outcomes with probabilities. The methodology and assumptions used in stochastic simulations are set out in Appendix A.

The going concern funded ratio (ratio of assets to liabilities[7]) is used as the key metric for evaluating the underlying risk of a funded pension plan. A funded ratio below 1.0 indicates that the plan is underfunded, requiring an elimination of deficits via additional contributions, lower benefits, or both. A funded ratio above 1.0, on the other hand, indicates that the plan has assets in excess of its funding target (i.e., surplus). In a DB plan that promises a fixed benefit, variability in its funded ratio would lead to unstable and unpredictable contributions, potentially with adverse financial consequences to the plan sponsor. On the other hand, the extent to which the funded ratio falls below 1.0 is a significant factor for assessing the risk of costs being transferred from current to future generations of stakeholders (which may include shareholders, taxpayers, or employees).

Figure 1 shows the 5th, 25th, 50th, 75th, and 95th percentiles for the funded ratio distribution of the baseline pension plan at each time period and, in addition, 20 individual paths generated from stochastic simulations. The graph illustrates the uncertain nature of pension funding dynamics. It shows an increasing range of possible levels of the funded ratio over time. The distribution of the funded ratio would become more dispersed and more skewed to the right as we go further in the future.[8]

[3] See Appendix A for a description of the liability model.
[4] Hedge ratio is defined as the ratio of the duration of the bond portfolio of the pension plan fund to the duration of the plan liabilities. It measures the proportion of plan liabilities that is protected from parallel shifts in bond yield curve. Neither yield curve convexity nor default risk are considered in our analysis.
[5] "Surplus threshold" refers to the amount of funding excess (i.e., surplus) that must be kept in the plan as a reserve, before any use of surplus is permitted. Under Canadian pension legislation, this threshold is typically set at 5% of plan liabilities (on either a going concern or a solvency basis, or both).
[6] Our modelling assumes that payment is made at the middle of each year.
[7] Assets are measured at market value. Liabilities are calculated using actuarial assumptions consistent with the plan's investment allocation strategy.
[8] This funded ratio distribution was generated based on our bond yield model, which incorporates a long-run mean of 4.93% and an initial bond yield of 2%. Had the initial bond yield been set equal to 4.93%, the distributions of funded ratios in future time periods would be more symmetrical and less right-skewed.

Figure 1: Evolution of distribution of funded ratios



All the figures and estimates of funding levels presented throughout this paper are based on the author's calculations.

3 Impacts analysis

In each of the next four subsections, we will examine how a policy change applied to the baseline pension plan would impact the long-term funding level of the plan. Four funding-related policies are considered (Table 1). We will vary one policy at a time and keep the other policies constant.

Table 1: Pension plan policy decisions

Policy decision	Amortization period	Target asset mix	Surplus threshold	Provision for adverse deviations (PfAD)
1. Amortization policy	1, 5, 7, 10, and 15 years	"Balanced," as described in Section 2 above	Nil	Nil
2. Investment policy	10 years	Different percentage allocations to equity investments	Nil	Nil
3. Surplus policy	10 years	"Balanced"	Nil, 5% and 10% of liabilities (going concern basis)	Nil
4. Funding reserve policy	10 years	"Balanced"	Nil	0% to 25% of liabilities, in 5% intervals

To measure the long-term effect of the policy decisions under consideration, we have chosen a projection period of 20 years for the purposes of our analysis.

We introduce three measures of long-term funding risk based on the projected funded ratio at the end of the 20-year projection period. The first measure is the range within which 90% of the potential funded ratios fall. The wider the range, the more uncertain the future funding level. Risk tolerance for plan funding is defined in terms of the levels of overfunding and underfunding the stakeholders are willing to bear. It is a range of funded ratios with a lower bound below 1.0 and an upper bound above 1.0; we call it a "risk tolerance band" throughout this paper. The probability that the funded ratio will fall below the lower bound or exceed the upper bound is taken as a proxy measure of the risk of underfunding or overfunding (beyond the stakeholders' risk tolerance) being transferred from current to future stakeholders. To support intergenerational fairness, these probability measures should be limited to a level acceptable to plan stakeholders. More importantly, the level of underfunding risk should be kept low to ensure adequate provision for the plan's liabilities.

Results on the first measure of long-term funding risk are presented graphically in the subsections that follow, and those on the last two measures are presented in Appendix B.

3.1 Amortization policy

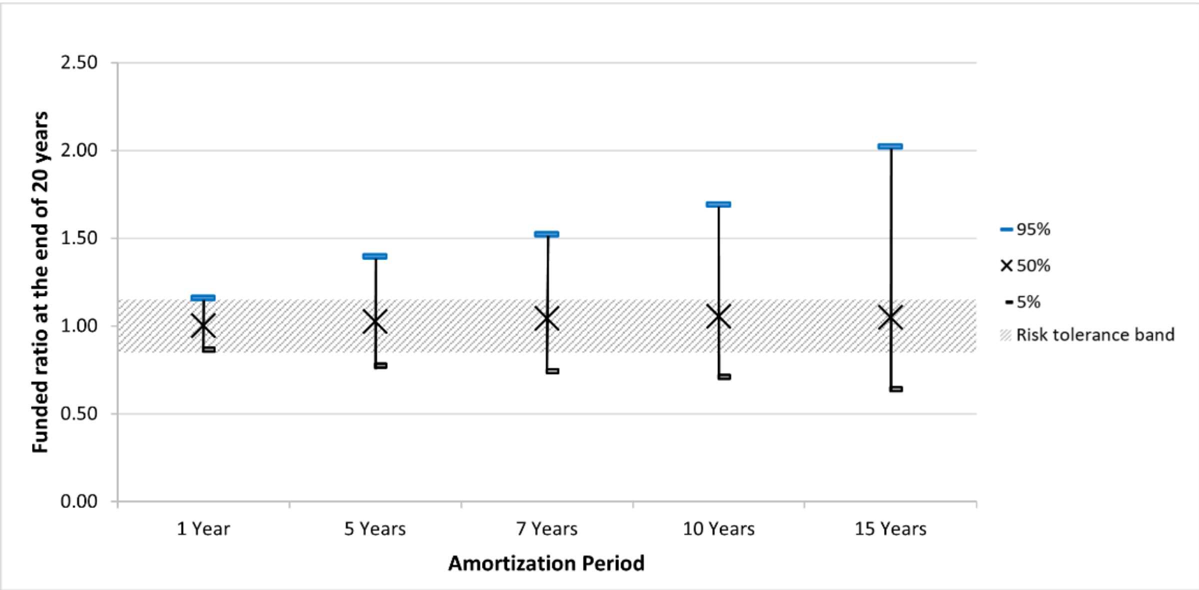
For traditional employer-sponsored DB plans, most pension regulations in Canada now require that any funding deficit of a DB plan be amortized over 10 years, on a fresh-start basis at each valuation date. Amortization of experience gains and losses over a long period would help to stabilize the employer's annual contributions, but would result in more uncertainty in the future funding level of the plan as demonstrated below. The benefits accrued by plan members could potentially become less secure, with funding shortfalls needing to be paid by future contributors. On the other hand, if any funding deficit is made up within a short period of time, the plan's funding level would be maintained but the level of employer contributions would be higher and more volatile.

To examine the effects of amortization on the future funding level of a plan, we apply five amortization rules to the baseline pension plan, with amortization periods of 1, 5, 7, 10, and 15 years. Both deficits and surpluses are subject to the same amortization rule. In our modelling, amortized surplus is used to reduce contributions or as a refund to the employer (treated as a negative contribution).

We use the stochastic model described in Appendix A to project the future funded ratios of the plan under each amortization rule. We then compare the distributions of funded ratios at the end of 20 years for the five rules (Figure 2). The level of the funded ratio is shown for a confidence interval of +/−5%, which means that there is a 90% probability that the level of the funded ratio will be within the bar shown.

Under the one-year amortization rule (leftmost bar, Figure 2), 90% of possible funded ratios fall within a range of 0.86 to 1.16, whereas under the 15-year amortization rule (rightmost bar, Figure 2), the range is from 0.64 to 2.02. The graph shows that the longer the amortization period, the more dispersed (i.e., more uncertain) is the distribution of the plan's funded ratio.

Figure 2: Distribution of funded ratio by amortization policy[9]



As an illustration, we set a risk tolerance band to be a range of funded ratios between 0.85 and 1.15[10] and evaluate the risk of overfunding or underfunding associated with different amortization rules. We calculate the probability that the projected funded ratio at time 20 will be less than 0.85, and the probability that the projected funded ratio will be more than 1.15. These probabilities measure the likelihood of the plan being significantly underfunded or overfunded. Our modelling results show that a longer amortization period would entail a higher risk of both overfunding and underfunding (Table B1, Appendix B).

3.2 Investment policy

Investment risk has important implications for the plan's costs and risks. This subsection aims to assess the impact that different investment policies have on the long-term funding level of a plan.

Setting investment strategy is a trade-off between seeking upside potential and managing downside risk. We consider five investment strategies with varying allocations of the pension fund to long bonds and common equities (Table 2).

[9] Note that the distribution of the funded ratio under the 10-year amortization rule in Figure 2 matches that for time 20 shown in Figure 1.

[10] This interval of funded ratios is close to the 90% range under the one-year amortization rule. Note that it is not necessary for the lower bound and the upper bound to be equidistant from 1.0. For instance, the upper bound could be further away from 1.0 than the lower bound if the plan sponsor is willing to accept more overfunding than underfunding.

Table 2: Investment strategies under consideration

Investment strategy	Allocation to equities (%)	Allocation to bonds (%)
All bonds	0	100
Conservative	25	75
Balanced	50	50
Risky	75	25
All equities	100	0

Where a plan adopts an “All bonds” investment strategy, the discount rate used in the valuation of the plan is equal to the yield on long bonds at the valuation date (see Part 2 of Appendix A). For other investment strategies, the discount rate is equal to the prevailing long bond yield plus an equity risk premium and a diversification return. At the start of the projection period, the yield on long bonds is assumed to be 2%. Table 3 sets out the initial discount rates for various investment strategies.

Table 3: Initial discount rate by investment strategy

Investment strategy	(1) Starting long bond yield (%)	(1) Equity risk premium (%)	(2) Diversification return (%)	Initial discount rate (%) (1)+(2)+(3)
All bonds	2.00	0.00	0.00	2.00
Conservative	2.00	1.25	0.25	3.50
Balanced	2.00	2.50	0.50	5.00
Risky	2.00	3.75	0.25	6.00
All equities	2.00	5.00	0.00	7.00

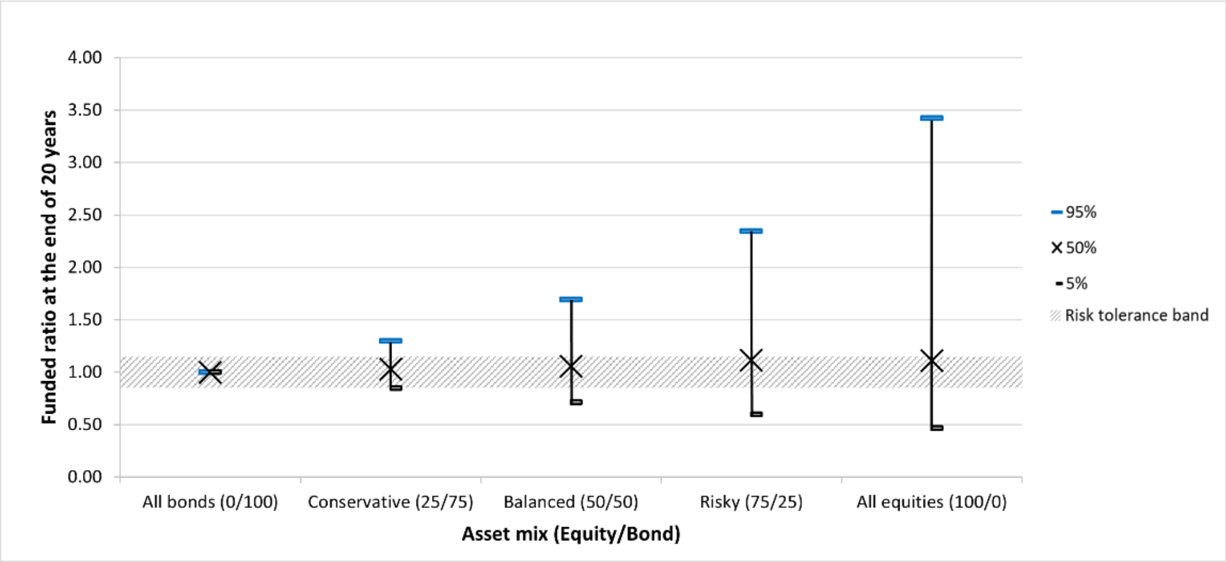
The discount rates depend on the yield on long bonds, which varies from year to year throughout the stochastic simulations. As can be seen, the riskier the investment strategy (i.e., higher allocation to equities) adopted by a plan, the higher the discount rate used in a valuation. A higher discount rate will result in lower amounts of liabilities and normal cost to be funded.

In projecting the future funding level of a plan that adopts different investment strategies, we first consider the case where the duration of the bond portfolio is equal to the duration of the plan liabilities, i.e., the hedge ratio is equal to 1.0. The “All bonds” strategy is the lowest-risk approach, whereby investments are fully matched with the cost of providing the pension benefits.[11] The plan with a “Balanced” investment strategy is the baseline pension plan described in Section 2.

As can be observed from the distributions of funded ratios at the end of the projection period for the five asset mixes (Figure 3), investment risk manifests itself in the dispersion of the projected funded ratio. The riskier the investment strategy, the greater the variability in the projected funded ratio. At one extreme, the plan that invests its assets entirely in a bond portfolio (and is fully hedged) will always be exactly funded. At the other extreme, the plan with 100% allocation to equities is expected to have funded ratios ranging from 0.46 to 3.42. The plans in between are exposed to different levels of equity market risk. They exhibit variability in the funded ratio that falls between the two extreme cases.

[11] The risk associated with bond yield convexity or default is not considered in our analysis.

Figure 3: Distribution of funded ratio by investment policy



Riskier investment strategies exhibit a significantly higher downside and upside risk, as indicated by the level of probability that the funded ratio falls below 0.85 or exceeds 1.15 (Table B2, Appendix B).

Impact of hedge ratio

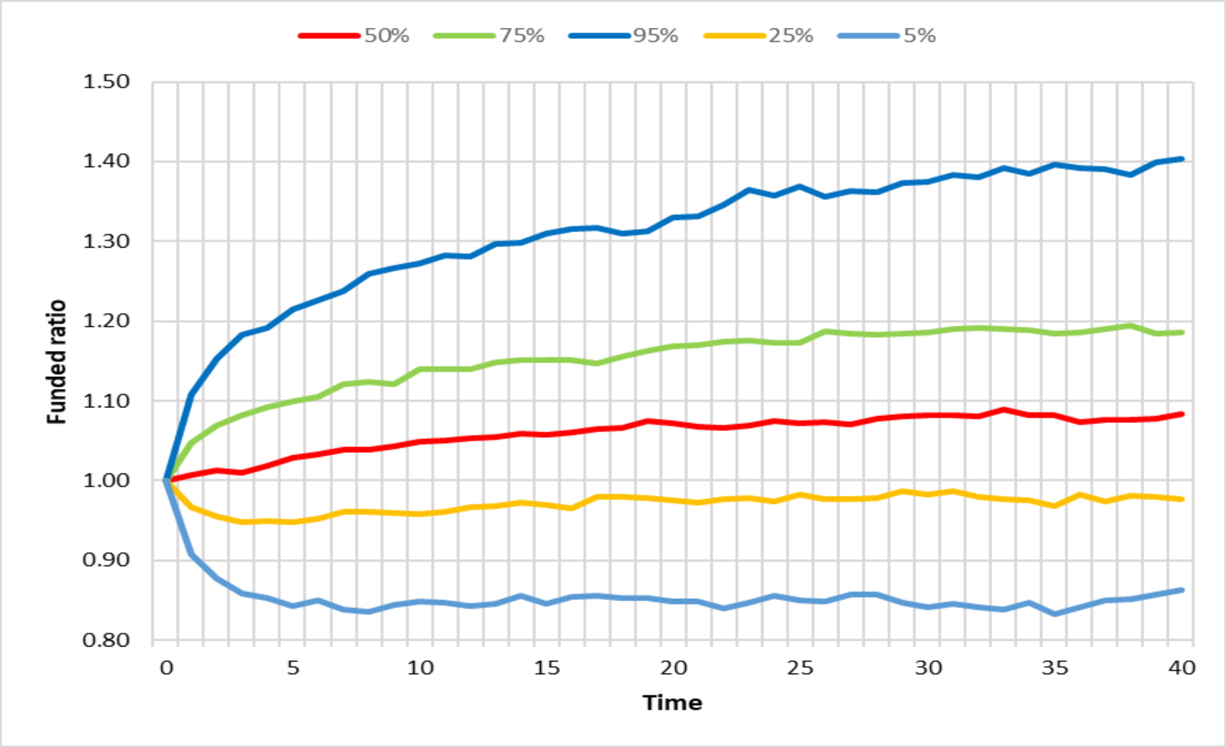
A plan could eliminate its funding risk from investments by investing the plan assets in a bond portfolio with a hedge ratio of 1.0. Any change in the plan liabilities arising from a change in the long-term bond yield would be compensated by the same change in the value of plan assets. As a result, the plan would remain fully funded at all times.

For a plan with assets invested entirely in a bond portfolio, but with a hedge ratio of 0.5,[12] the range within which 90% of possible funded ratios at the end of 20 years fall is between 0.85 and 1.33 (Figure 4). Incidentally, the funded ratios for the "Conservative" investment strategy with a hedge ratio of 1.0 fall within a similar range, from 0.85 to 1.30 (Figure 3). This suggests that the risk of the funded ratio associated with a 50%-hedged "All bonds" investment strategy is comparable to that associated with a fully hedged "Conservative" investment strategy. However, the latter strategy would be accompanied with a higher initial discount rate and, since the projections assume a starting funded ratio of 1.0, a lower initial amount of assets.

[12] For the baseline pension plan, 50% of liabilities are related to pensions in pay. The plan liabilities have a duration at 5.25% of around 13 years, which is close to that of the Canada Long Bond Index (10+ years to maturity). A bond portfolio with a 50% hedge ratio would be more like the Canadian Bond Universe.

Figure 4: Evolution of distribution of funded ratio

("All bonds" strategy with a hedge ratio of 0.5)



In summary, if a plan adopts a riskier investment strategy, it could use a higher discount rate to determine its funding requirements. This would result in lower amounts of normal cost and liabilities to be funded, but would entail a greater variability in the funded ratio (and thus, more unstable and unpredictable funding requirements). On the other hand, a plan could reduce the variability in its funded ratio by investing its pension fund in assets (e.g., bonds) that track the movements of the plan liabilities. However, the lower expected rate of return on such assets would increase the plan's current costs.

3.3 Surplus policy

Where a plan is determined to have a surplus based on the results of a valuation, most pension standards legislation in Canada now permits the employer to use the surplus to reduce contributions or to take a refund of surplus, subject to the withholding of a surplus threshold (typically 5% of liabilities, going concern or solvency) and/or consent requirements.

In our previous analysis of the funding risk associated with different investment strategies, we set the surplus threshold to be nil. Similar to the funding of deficits, any surplus is amortized over 10 years on a fresh-start and straight-line basis. Thus, only 1/10th of the surplus determined in a valuation is applied as a contribution holiday or surplus refund for a year. In this analysis, we incorporate different levels of surplus threshold, defined as a percentage of plan liabilities (going concern basis), in determining the amount of spendable surplus (referred to as "available actuarial surplus" or "AAS"[13]), and simulate the distributions of the funded ratios for the baseline pension plan (which adopts a "Balanced" investment strategy). We look at the case where only 1/10th of the AAS is spent each year as well as the cases where the AAS is fully spent or not spent at all.

We observe the following from the distributions of funded ratios shown in Figure 5:

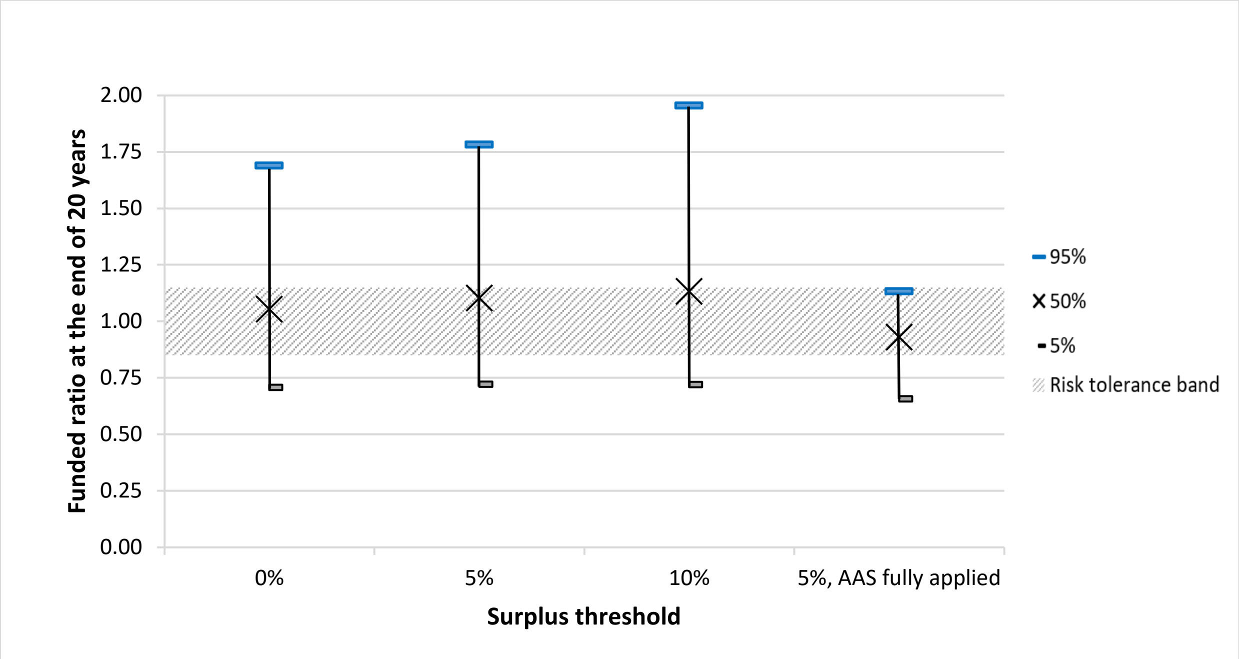
- If only 1/10th of the AAS is spent in a year, inclusion of a surplus threshold up to 10% has only a modest impact on the future funding level of the plan (first three bars on the left). A higher surplus threshold would lead to a somewhat wider distribution of the funded ratio, with volatility mostly on the upside.

[13] The mathematical formula for AAS used in stochastic simulations is given in Part 2 of Appendix A.

- If the entire AAS is spent, the distribution of the funded ratio would be compressed and shift downward (rightmost bar).

If none of the AAS is spent, the distribution of the funded ratio (not shown in Figure 5) would expand on the upside with a 95th percentile funded ratio of 3.5 after 20 years. It would continue to expand on the upside beyond the projection period.

Figure 5: Distribution of funded ratio by surplus threshold



Full consumption of the AAS could lead to a significantly higher risk of underfunding, even with the holdback of a 5% surplus threshold. Compared to the case where only 1/10th of the AAS is applied, the probability of the funded ratio falling below 0.85 is almost doubled (see Columns B and D of Table B3, Appendix B). On the other hand, the plan could be exposed to significant overfunding risk if none of the AAS is amortized (Column E of Table B3, Appendix B). The funded ratio could reach extremely high levels in later years, with a 95th percentile funded ratio of 3.5 after 20 years. Funded ratios significantly higher than 1.0 represent a runaway surplus situation, where the amount of surplus exceeds the amount required to fulfil the plan's obligations for current and future members' future service.

Subsection 3.5 provides a further discussion on the use of surplus in the context of Canadian regulatory funding frameworks.

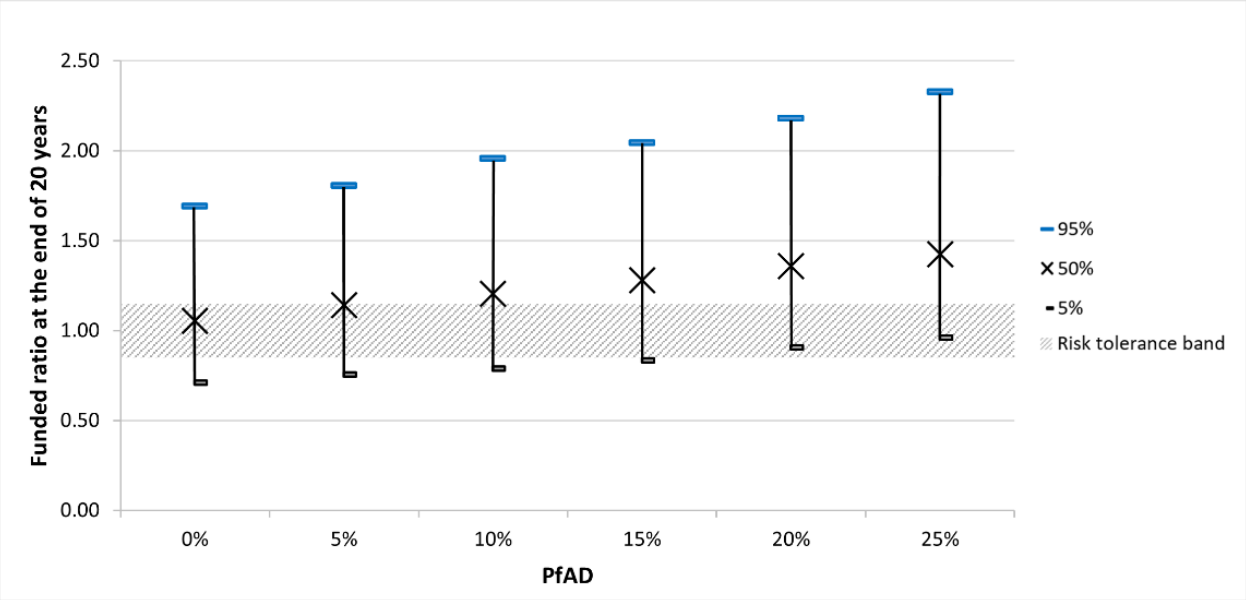
3.4 Funding reserve policy

Many Canadian jurisdictions have mandated the inclusion of a buffer in the funding of a DB plan. In Québec, the buffer is called a "stabilization provision." It is determined as a percentage of the plan's going concern liabilities that varies according to the proportion of plan assets allocated to variable yield investments (e.g., equities) as well as the ratio of the duration of the bond portfolio of the pension fund to the duration of plan liabilities (Québec, 2015). In Ontario, the buffer is called a "provision for adverse deviations" or "PfAD." It is calculated based on the proportion of plan assets allocated to non-fixed income investments, as well as whether the plan is open or closed (Ontario, 2022). The PfAD rule in British Columbia is fundamentally different from those of Québec and Ontario. It has a 5% minimum and is generally calculated as the monthly long-term Government of Canada bond yield as of the valuation date multiplied by five (BCFSA, 2020). The PfAD provisions of other jurisdictions, e.g., Nova Scotia, New Brunswick and Manitoba, are similar in structure to that of Ontario.

This subsection explores the impact of a PfAD on the long-term funding level of a plan. We consider the baseline pension plan with a "Balanced" investment strategy, and project the distributions of its funded ratios to the end of a 20-year period, based on five levels of PfADs: 5%, 10%, 15%, 20%, and 25% of liabilities.

The distributions of funded ratios at the end of 20 years clearly demonstrate that the improvement in the plan's funding level resulted from the inclusion of a PfAD in the funding requirements (Figure 6). However, a PfAD would magnify the volatility in the funded ratio of the plan, though the volatility is mostly on the upside. The risk of the plan being underfunded is significantly reduced by the inclusion of a PfAD while the risk of overfunding is significantly increased, as indicated by the probabilities in the last two rows of Table B4, Appendix B. It is worthy of note that, even with a modest 5% PfAD, there could still be a significant risk of overfunding – there is about a 50% chance that the funded ratio would exceed 1.15 after 20 years.

Figure 6: Distribution of funded ratio by PfAD



PfAD to meet a long-term funding goal

With a PfAD of 10%, the probability that the baseline pension plan will be fully funded at time 20 is equal to 76%.[14] If the funding goal of a plan is to be fully funded at the end of 20 years with 75% probability (or any other level of probability), we are faced with an inverse problem: What level of PfAD will the plan require to achieve that goal? Put another way, what is the level of PfAD that is mostly likely to have produced a distribution of the funded ratio at time 20 such that the 25th percentile funded ratio is equal to 1.0?

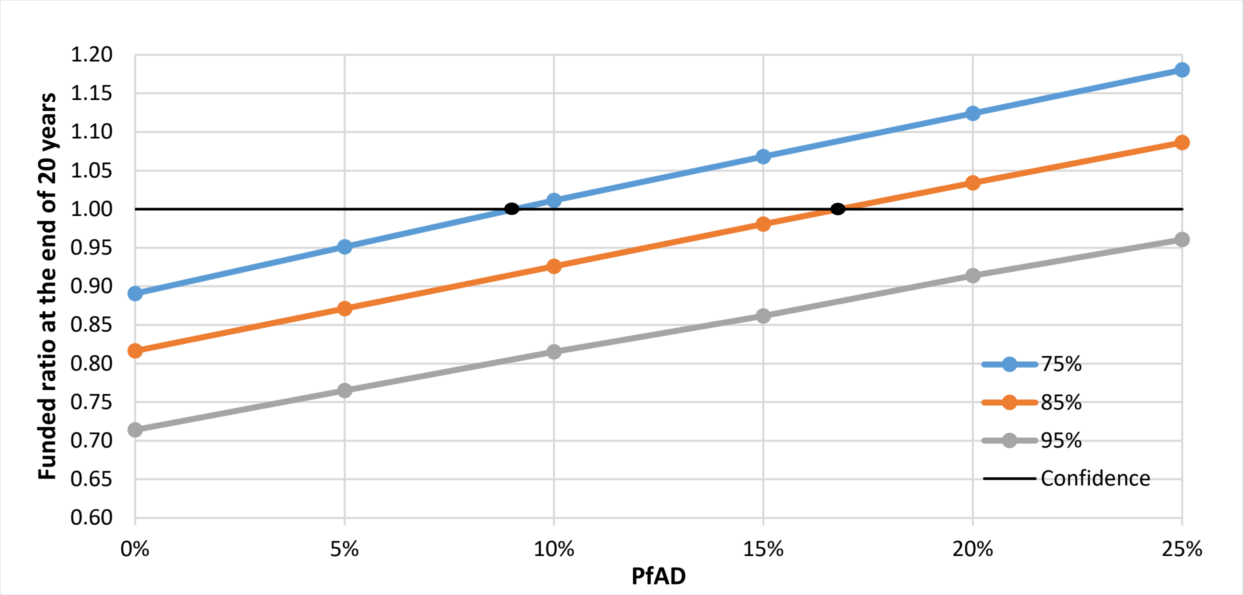
Based on the above simulation results, we plot the funded ratios at time 20, for different confidence levels (75%, 85%, and 95%),[15] against the six PfAD levels (0%, 5%, …, 25%) as illustrated in the following “funding reserve map” (Figure 7). The lines on the map are nearly linear and upward trending. We draw a horizontal line at the funded ratio of 1.0 across the map, and determine a PfAD from the intercept on the line corresponding to the desired confidence level. By linear interpolation, the PfADs corresponding to the three confidence levels are found to be 9.1%, 16.8%, and 29.2%, respectively.[16] The higher the desired level of certainty, the higher the PfAD required.

[14] Source: Author’s calculation.

[15] A confidence level of 75% means that the probability of the plan being fully funded at time 20 is 75%. The confidence levels of 85% and 95% are similarly defined.

[16] We have tested the robustness of this method by running a simulation based on a PfAD of 9.1%. The 25th percentile of the distribution of the funded ratio at time 20 is found to be close to 1.0.

Figure 7: Funding reserve map for baseline plan with "Balanced" investment strategy



For the plan with a 5% surplus threshold, the PfADs corresponding to the three confidence levels (75%, 85%, and 95%) are found to be 7.7%, 15.9%, and 26.6%, respectively. They are somewhat smaller than the PfADs pertaining to the plan with no surplus threshold.

3.5 Use of surplus assets under Canadian regulatory funding regimes

Most Canadian jurisdictions require a plan to maintain a minimum funding threshold based on solvency liabilities before any contribution holiday or refund of surplus is permitted. For example, Ontario permits a plan to take a contribution holiday if (1) the plan is fully funded on a going concern basis (which includes any prescribed PfAD) and (2) after deducting the amount of surplus used to lower the contribution requirements from the assets, the plan's solvency ratio is at least 1.05 (Ontario, 2022).

Under Ontario regulations, the PfAD required for an open plan with a "Balanced" investment strategy (50/50 asset mix) is 7%. We apply this PfAD to the baseline pension plan and examine how different limits on the use of surplus (referred to as "spendable surplus limits") would impact the future funding level of the plan:

1. Only 10% of the plan's available actuarial surplus[17] could be used for a year; no solvency-based limit.
2. Only 20% of the plan's available actuarial surplus could be used for a year; no solvency-based limit.
3. Only 30% of the plan's available actuarial surplus could be used for a year; no solvency-based limit.
4. Any amount of the plan's available actuarial surplus could be used as long as the plan maintains a solvency ratio of at least 1.05.

In all cases, it is assumed that any emerging deficit would be amortized over 10 years, on a fresh-start basis at each valuation date (i.e., at a rate of 10% of the outstanding deficit).

Table 4 summarizes the key statistics on the distributions of funded ratios at time 20 under these spendable surplus limits.

[17] Here, available actuarial surplus is determined without the holdback of a surplus threshold.

Table 4: Statistics on distribution of funded ratio by spendable surplus limit

Funded ratio (α_{20})	**Limit on use of surplus**			
	(1)	**(2)**	**(3)**	**(4)**
Median	1.15	1.08	1.03	1.24
Mean	1.21	1.10	1.04	1.23
Standard deviation	0.34	0.22	0.19	0.28
Pr($\alpha_{20} < 0.85$)	0.11	0.13	0.17	0.10
Pr($\alpha_{20} > 1.15$)	0.51	0.39	0.27	0.59

Notes:

Pr(α_{20} < 0.85) in the above table stands for the probability that the funded ratio at time 20 is less than 0.85; Pr(α_{20} > 1.15) stands for the probability that the funded ratio at time 20 is more than 1.15.

The solvency-based limit (4) would result in the lowest underfunding risk and the highest overfunding risk. This limit on the use of surplus appears to be overly restrictive as it could prevent a plan from taking a contribution holiday even if there is a considerable amount of surplus on the funding basis.[18] The 20% limit (2) is a more balanced measure for controlling the risk of the funded ratio. Compared to the solvency-based limit, the risk of overfunding would be significantly lower while the risk of underfunding would only be modestly higher.

In the view of this author, the solvency-based limit is not an appropriate measure for controlling the funding risk associated with a plan that is not expected to cease operation in the foreseeable future. It is inconsistent with the long-term nature of the plan and its funding objectives. Risk management for such a plan should focus on adequate provision for the plan's liabilities, with consideration to the need for fair distribution of costs, rather than on protecting the solvency position of the plan.

[18] Solvency liabilities are calculated based on long-bond yields at the date of valuation, whereas the liabilities for funding purposes are calculated based on the expected long-term investment return assumption. For a flat-dollar DB plan with significant exposure to equity investments, the value of solvency liabilities would typically be much higher than the value of going concern liabilities.

4. Developing a risk-based PfAD

In Subsection 3.4, we devised a method to establish a PfAD for the baseline pension plan that aims to meet a specific funding target over a time horizon of 20 years. We now employ the method to develop a risk-based PfAD that reflects a plan's exposure to equity market risk. We consider the investment strategies described in Subsection 3.2, except for the "All bonds" strategy. The required PfAD would increase with the investment risk being taken by a plan. A higher confidence level to achieve full funding, which means more protection of members' benefits, would also require a higher level of PfAD (Table 5).

Table 5: Risk-based PfADs

Confidence level	Investment strategy			
	Conservative	Balanced	Risky	All equities
75%	4.8%	9.1%	13.4%	18.3%
85%	8.7%	16.8%	24.5%	30.3%
95%	15.4%	29.2%	42.4%	51.5%

While the PfADs we have established reflect the equity market risk only, we can incorporate different hedge ratios in the modelling to develop a two-dimensional grid based on the level of equity allocation and the portion of interest rate risk being hedged, similar to that adopted in Québec.

5. Plans with non-stationary membership

Ontario regulations (Ontario, 2022) require a plan that is closed to new members to fund for a higher level of PfAD than an open plan, presumably on the grounds that a closed plan would pose a greater funding risk. The foregoing analyses have been carried out for a plan with a steady stream of new members that keeps the membership profile steady. In this section, we explore the implication of a funding reserve policy on a plan with growing or declining membership.[19]

For the plan with declining membership,[20] its liabilities would decrease by approximately 30% after 20 years and 50% after 40 years. The proportion of its liabilities related to pensioners would increase from 0.5 to about 0.7 after 20 years and to 1.0 after 40 years. For the plan with growing membership,[21] its liabilities would increase by approximately 40% after 20 years. The proportion of liabilities related to pensioners would increase from 0.35 to 0.5 after 20 years.

The funding-related policies that apply to each plan are as follows:

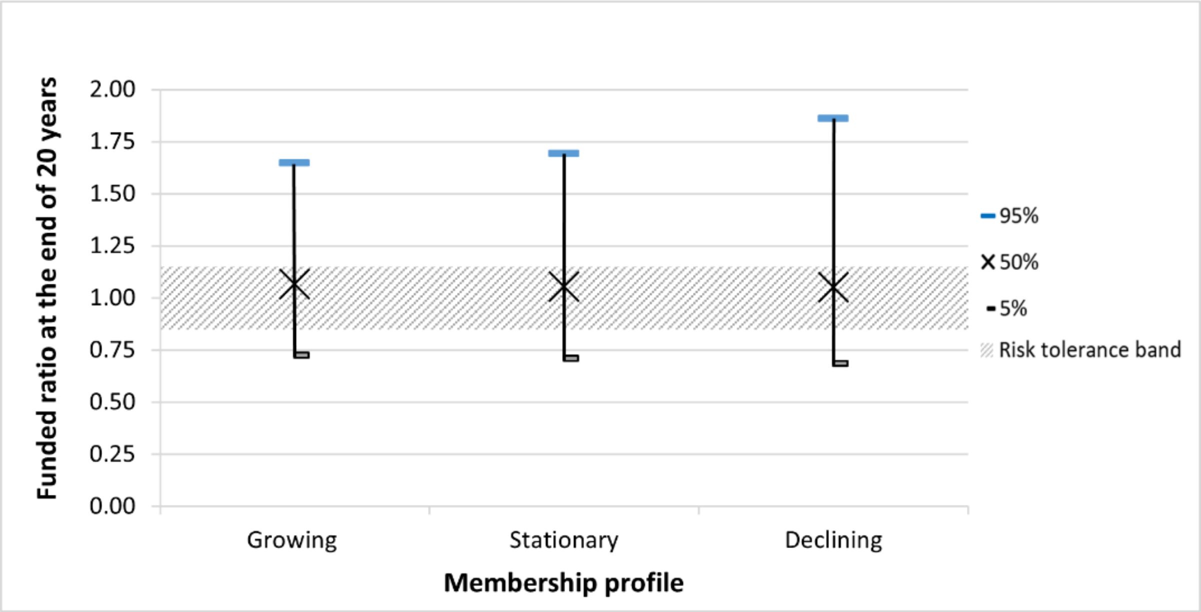
- Investment policy is based on a 50/50 asset mix and a hedge ratio of 1.0 (i.e., “Balanced” investment strategy).
- Funding deficits and excesses are both amortized over 10 years, on a fresh-start and straight-line basis.
- No surplus threshold is included.

We run simulations on the two plans over a 20-year period and compare the projection results with those pertaining to the plan with stationary membership provided in Section 3. We first consider the case where no PfAD is included.

The plan with declining membership exhibits a somewhat higher funding risk, in terms of the volatility in the funded ratio and the probability of underfunding (Figure 8 and Table B5, Appendix B). On the other hand, the plan with growing membership exhibits a lower funding risk. However, the differences in the funding risk metrics stemming from the different membership profiles are relatively small.[22] Of note, the probability of underfunding for the plan with declining membership is only marginally higher than that for the plan with stationary membership (Table B5, Appendix B).

[19] An example of a plan with declining membership is a plan that is closed to new members. The plan’s membership is expected to be declining over time while the proportion of retired members and beneficiaries in the plan will be increasing.
[20] The plan with declining membership is defined by the following liability growth parameters: $p(0) = 0.5;\ \emptyset = -1; \delta_t = 0.0175$ for all t (see Equation (5) in Appendix A).
[21] The plan with growing membership is defined by the following liability growth parameters: $p(0) = 0.35;\ \emptyset = +1; \delta_t = 0.0175$ for all t (see Equation (5) in Appendix A).
[22] The funding risk metrics are not particularly sensitive to the rates of membership growth/decline. We have run simulations on a growing plan and a declining plan with a growth parameter δ_t equal to 2.25%, and found the probabilities of underfunding related to the two plans to be 0.19 and 0.23. The corresponding results for the plans with a 1.75% growth rate are 0.18 and 0.22 (Table B5, Appendix B).

Figure 8: Distribution of funded ratio by membership profile



The required PfAD to achieve full funding at time 20, with 75% probability, for the plan with stationary membership was found to be 9.1% previously (Subsection 3.4). It seems logical to expect that the plan with declining membership (highest-risk plan) will require the highest PfAD and the plan with growing membership (lowest-risk plan) will require the lowest PfAD. Surprisingly, our simulations give the opposite results. The PfADs for the plans with declining and growing membership are found to be 8.6% and 10.1%, respectively. This is counterintuitive.[23]

The required PfADs for the three plans do not differ much from each other. Specifically, the difference in PfADs between the declining plan and the stationary plan (or between the growing plan and the stationary plan) is equal to or less than 1%. This suggests that a plan's membership profile (stationary vs. non-stationary) may not be a significant factor for incorporation in the determination of a PfAD.

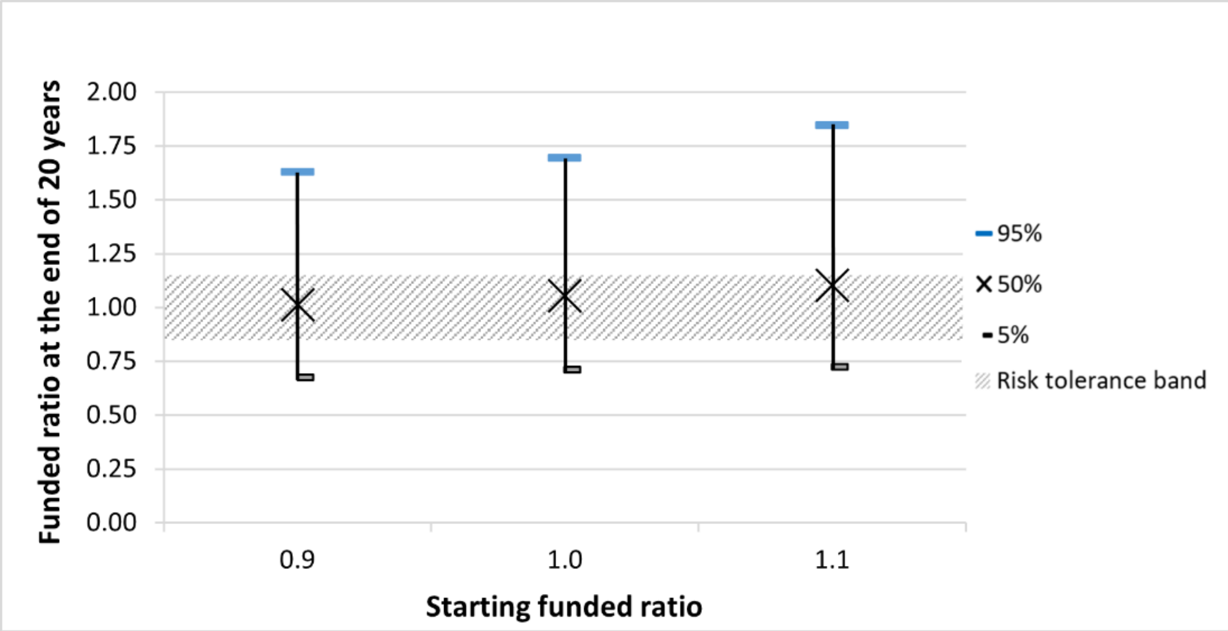
[23] A possible explanation of this phenomenon goes as follows. The PfAD to be funded at a valuation date is defined as a percentage of the liabilities of the plan determined at that date. Special payments made toward the PfAD will help improve the future funding level of the plan. In the case of a declining plan, each dollar of PfAD payment made in the early years of plan operation would improve the funding level of the plan in later years, to a greater extent than a plan with stationary membership. This is because the payments made under the declining plan would amount to a higher percentage of the declining liabilities as time passes, thereby driving up the funded ratio of the plan. Thus, a declining plan would not require a PfAD as high as that for a stationary plan in order to meet its funding goal, all else being equal. A similar reasoning can be applied to explain why a higher PfAD is required for a growing plan than for a stationary plan.

6. Impact of initial funding position

The distributions of funded ratios set out in the foregoing sections are related to a plan with a starting funded ratio of 1.0. In this section, we project the distribution of the funded ratio at time 20, for a plan which starts off with a different funded ratio.

We consider the baseline pension plan with three starting funded ratios: 0.9, 1.0 and 1.1. As can be observed from Figure 9 and Table B6, Appendix B, the *median* funded ratio at time 20 for the plan with a starting funded ratio of 0.9 is the lowest, and that for the plan with a starting funded ratio of 1.1 is the highest. In terms of underfunding risk, the plan with a lower starting funded ratio exhibits a higher level of risk whereas the plan with a higher starting funded ratio exhibits a lower level of risk.

Figure 9: Distribution of funded ratio by starting funded ratio



For the plans with a starting funded ratio of 0.9 and 1.1, we use the funding reserve map method to estimate the required PfADs to achieve full funding at the end of a 20-year period, with 75% probability. They are found to be 11.4% and 6.7%, respectively. The PfAD for the plan with a starting funded ratio of 1.0, estimated to be 9.1% in Subsection 3.4, lies in between these two values.

The current funding level of a plan has an important implication for the design of a PfAD. If the funding goal of a plan is to achieve a certain target funded ratio (e.g., 1.0) over a fixed time horizon from the valuation date (e.g., 20 years), with a given level of confidence (say, 75%), the level of a PfAD will need to change with the current funded ratio of the plan. The lower the current funded ratio, the higher the PfAD required, and vice versa. This means that the PfAD adopted for a plan should not be a static amount. Rather, it should be adjusted upward or downward to reflect any material change in the funding level of the plan (say, +/−10% or more).

7. Policy implications

The “going concern plus” funding regime aims to evaluate whether a DB plan has accumulated assets, together with expected future contributions, sufficient to provide the benefits promised under the plan, if the plan continues to exist. It allows a plan to use a discount rate based on the expected long-term rate of return on its plan assets to determine its funding requirements. At the same time, it requires plans to keep reserves to buffer against adverse deviations in experience versus assumptions made in the valuation. Our research is an attempt to assess the potential impact of the new funding regime on the long-term funding level of a pension plan, and to inform the appropriate value of its parameters.

Key lessons learned from our exploration are as follows.

Amortization policy

One of the funding measures adopted in the new funding regime is to require that going concern deficits be amortized over 10 years on a fresh-start basis, instead of the 15-year closed amortization prescribed in previous regulations. The 10-year open amortization appears to be a compromise between the need for contribution stability and the adequate provision for plan liabilities, noting that the choice is not based on any established principles.

We have estimated the impact of different amortization periods on the plan’s long-term funding risk. A long amortization period adopted by a plan would help to stabilize the plan’s annual funding requirements, but could expose it to the risk of both significant overfunding and underfunding.

Investment policy

Under the new funding regime, funding requirements will be primarily driven by going concern valuations. The most important assumption in going concern valuations is the discount rate, which is generally determined as the expected long-term rate of return on plan assets.

The choice of investment mix for a pension plan has the most important impact on the plan’s costs and risks. A higher allocation of plan assets to return-seeking investments (e.g., equities) could potentially lower the plan’s costs (through the realization of higher investment returns), but would entail a greater volatility in its funded ratio (which would lead to more unstable and unpredictable funding requirements) as well as a greater risk of underfunding (which would require additional contributions from the employer to make provision for the plan’s liabilities). In making a decision on investment mix, plan sponsors/trustees should consider the trade-off between the level of funding costs and the risk of the future funded ratio.

Another consideration is the employer’s financial capacity to meet unexpected increases in funding requirements. For example, if a plan has reached a mature state where the potential deficit amortization payments under a risky investment strategy could be sizable relative to the employer’s cash flow and operating budgets, it will be more appropriate for the plan to pursue a low-risk investment strategy.

Surplus policy

Given that the funding of deficits is spread out over a number of years, our modelling shows that unrestricted use of surplus could increase a plan’s exposure to the risk of underfunding over time. For the control of this risk, there should be some limits placed on the use of surplus.

Canadian jurisdictions adopting the new funding regime require pension plans to maintain a funding threshold above 100% (going concern or solvency basis) before any contribution holiday or refund of surplus is permitted. Typically, a plan is not allowed to use any amount of surplus assets that will result in the plan having a solvency shortfall. This solvency-based limit may be overly restrictive, as it could prevent a plan from taking a contribution holiday even if there is a considerable amount of funding surplus in the plan. It could potentially lead to a significant risk of overfunding, thereby undermining the principle of intergenerational equity.

Funding reserve policy

We have demonstrated that a PfAD would help improve the future funding level of a plan and reduce its risk of being underfunded in the long term. The higher the PfAD, the greater the funding improvement and the lower the risk of underfunding. However, a higher PfAD could lead to a significantly higher risk of overfunding.

We have also established a PfAD that aims to meet a plan’s long-term funding goal, which is to achieve full funding at the end of a chosen time horizon (e.g., 20 years), with a certain level of confidence (e.g., 75%). The PfAD reflects the investment risk being taken by a plan. Plans with a higher proportion of their pension fund invested in equities would require a higher level of PfAD than plans with a more conservative investment strategy, all else being equal.

Under the new funding regime, plans that adopt an investment strategy with a higher expected return (which generally involves a higher allocation of the pension fund to equity investments) would have lower funding requirements than plans that adopt a more conservative investment strategy. From a public policy perspective, the going concern discount rate, on its own, should not create incentives for plan sponsors/trustees to increase their plan's investment risk exposure so that they may benefit from a decrease in required contributions. Our PfAD has the effect of mitigating such potentials as it takes into account the degree of investment risk assumed by the plan.

Once a plan has adopted its investment policy, the plan's membership profile (stationary vs. non-stationary) is not a primary driver of long-term funding risk. It is not a significant factor for the determination of a PfAD.

Finally, we contend that the PfAD designed to achieve a plan's long-term funding goal should reflect the evolving funding position of the plan. The level of PfAD should be adjusted upward (or downward) when the plan's funding level declines (or improves) significantly.

Developing a sustainable funding strategy

Funding decisions for a DB plan should be aligned with the long-term nature of the plan and its funding objectives, which generally include the following:

- Funding adequacy – to ensure the accumulation of sufficient assets to make provision for the plan's liabilities.
- Cost stability – to limit the degree of volatility in funding requirements.
- Affordability – to keep the plan and benefit costs affordable for the plan sponsors and members.
- Intergenerational equity – to ensure that the plan's costs and risks are shared between different generations of stakeholders fairly.

For a DB plan that is not expected to cease operation in the foreseeable future, we can develop a long-term funding strategy to meet its funding objectives by combining the four policy measures discussed above in an appropriate manner. Where an investment policy and a deficit amortization rule have been established for a plan, the risk of the plan's future funding level, either upside or downside, could be controlled by setting appropriate values for the following two parameters: (1) the level of PfAD to be included in the plan's funding target and (2) the limit on the amount of surplus that is available for use as a contribution holiday or for other purposes.

As an illustration, consider the baseline pension plan with a "Balanced" investment strategy (50% in equities and 50% in bonds) and a 10-year amortization policy (for both deficits and surpluses). The risk tolerance band the plan sponsor is willing to accept in pursuit of its long-term funding objectives is set as a range of funded ratios with a lower bound of 0.85 and an upper bound of 1.15. Our modelling shows that there is a 21% probability that the funded ratio of the plan at the end of 20 years will fall below 0.85 and that there is a 37% probability that the funded ratio will exceed 1.15 (Table B1, Appendix B). These probabilities measure the risk of underfunding or overfunding that is beyond the sponsor's risk tolerance.

A PfAD could help reduce the plan's underfunding risk but would increase its overfunding risk. By imposing a 5% surplus threshold, our modelling shows that a PfAD of 7.7% would be required to fully fund the plan at the end of 20 years with 75% probability (Subsection 3.4). If both of these funding measures are applied, the plan's underfunding risk would decline to 10% but its overfunding risk would increase to 55% (Column "10%," Table B7, Appendix B).

We could reduce the above level of overfunding risk by increasing the limit on the amount of surplus to be used each year (i.e., the spendable surplus limit). For example, if the spendable surplus limit is increased from 10% of available actuarial surplus to 50%, the plan's overfunding risk would decrease to 30% while its underfunding risk would increase to 13%. The funded ratio that the plan will be able to attain with 75% probability would drop from a level of around 1.0 to 0.93 (Column "50%," Table B7, Appendix B).

It should be noted that plan sponsors/trustees are responsible for determining if the resulting levels of underfunding and overfunding risk are acceptable, based on their funding objectives as regards funding adequacy and intergenerational equity.

Outlined below is a proposed approach to developing a long-term funding strategy for a DB plan. Plan sponsors/trustees should review their plan's funding strategy periodically (say every 5 years), and revise it to reflect any material changes in plan circumstances and/or economic environments.

Steps	Actions/Measures/Considerations
1. Develop an investment mix policy	• Trade-off between affordability and cost stability • Employer's financial capacity to meet unexpected increases in required contributions
2. Establish an amortization policy	• Trade-off between cost stability and funding adequacy • Prescribed maximum amortization period for deficits (10 years in most jurisdictions), if any
3. Set up a risk management goal	• Long-term funding risk inherent in the plan under the investment and amortization policies established in Steps 1 and 2 – use of a stochastic modelling technique to identify possible ranges of future funded ratios • Setting the boundaries of funded ratio risk (i.e., the risk tolerance band) the plan sponsor[24] is willing to accept • Putting a constraint on the risk of underfunding or overfunding that is beyond the sponsor's risk tolerance • Risk management goal should be aligned with the plan's funding objectives in terms of funding adequacy and intergenerational fairness
4. Determine a risk-based PfAD	• Setting a long-term funding goal (e.g., achieving full funding at the end of 20 years with 75% certainty) for the determination of a risk-based PfAD • Determine a risk-based PfAD using a pension projection model,[25] based on the investment and amortization[26] policies established in Steps 1 and 2 as well as any applicable surplus threshold • The PfAD adopted by the plan must not be lower than the prescribed PfAD, if any
5. Find a combination(s) of funding policy measures to achieve the plan's long-term funding objectives	• Project the plan's future funding levels using a pension projection model, under different spendable surplus limits (10%, 30%, 50%, etc. of the available actuarial surplus or any other limits) together with the investment policy, amortization policy, and PfAD established in Steps 1, 2, and 4 • Model outputs include probability measures of underfunding and overfunding risks • Identify a spendable surplus limit(s), together with the pre-set investment policy, amortization policy, and PfAD, that best fit with the risk management goal set up in Step 3

[24] Or trustees, depending on the plan's governance structure.
[25] Along the lines of the one presented in this paper, with enhancements suitable to the plan's investment strategy.
[26] For the determination of a risk-based PfAD, both deficits and surpluses are subject to the same amortization rule.

8. Conclusion

Until the recent pension funding reforms, most DB plans in Canada had been required to fund the plan benefits on both a going concern and solvency basis. The continued decline in long-term bond yields in the last two decades had resulted in large and volatile funding requirements, making DB plans more expensive to fund. This has contributed to the shift away from DB plans in the private sector, toward defined contribution and other retirement savings arrangements.

To provide a more stable and accommodative environment for DB plans, some provincial governments have made significant changes to their regulatory funding frameworks. They have moved to a "going concern plus" funding regime that reduces the focus on solvency funding while enhancing the going concern funding requirements.

This paper presents a usable model for projecting the assets, liabilities, and funded ratio in respect of a plan that provides flat-dollar defined benefits. Through stochastic simulations, we have assessed how different funding-related policies under the new funding regime would impact the risks and costs of a plan.

It is possible to develop a pension projection model for final or career average earnings plans, along the lines of the one for flat-dollar benefit plans provided in Appendix A. The quantitative results generated from stochastic simulations on such a plan might well be different from those illustrated in this paper. However, the policy insights drawn from such results are not expected to differ from what we have set forth in the penultimate section.

Results of our analysis inform DB stakeholders (which include plan sponsors, administrators, government policy-makers, and regulators) on the trade-offs involved in setting policies that affect the funding of DB plans. For plan sponsors and/or trustees trying to deliver promised benefits to plan beneficiaries while managing the volatility of contribution requirements, the policy insights provided in this paper may help them put plans on a sustainable funding path.

Appendix A: Modelling methodology and assumptions

We apply a stochastic model to assess the impact of funding-related policies on the risks and costs of DB plans, in order to capture variability in outcomes with probabilities. The results set out in this paper are based on 1,000 stochastic simulations, using an economic scenario generator (ESG) and a pension projection model, described below. The ESG generates investment scenarios (paths) consisting of annual time series for long-term bond yields and equity returns, while the pension projection model enables us to project the funded ratios of a pension plan under various investment scenarios.

1. Economic scenario generator

Underlying all asset return forecasts is the Vasicek model, which describes the evolution of long-term bond yields over time. It is a continuous mean-reverting time series represented by the following stochastic differential equation:

$$dy_t = \theta(\mu - y_t)dt + \sigma_y dW_t$$

where y_t is the instantaneous bond yield at time t, θ is the rate of reversion to the mean, μ is the long-run mean, σ_y is the standard deviation of the process, and W_t is a standard Brownian motion.

In a discrete setting, the mean-reverting process can be expressed as

$$y_{t+1} = y_t + \theta\,(\mu - y_t) + \varepsilon_y(t+1) \tag{1}$$

where y_t and y_{t+1} are the bond yields at times t and $t+1$, and $\varepsilon_y(t+1)$ is the value of a random shock at time $t+1$. The random shocks $\{\varepsilon_y(t), t = 1, 2, \dots\}$ are independent, normally distributed with mean *zero* and standard deviation of σ_y, i.e., $\varepsilon_y(t) \sim i.i.d.\,N(0, \sigma_y^2)$.

For fixed income investments (e.g., bonds), the price process BP_t is modelled through the exponential rate of the evolution of long-term bond yields:

$$BP_{t+1} = BP_t \cdot e^{y_t + D(t+1)\cdot(y_t - y_{t+1})} \tag{2}$$

$D(t+1)$ in this formula is the duration of the fixed income portfolio of the pension fund at time $t+1$. It is set equal to $h(t+1) \cdot D^L(t+1)$, where $h(t+1)$ is the hedge ratio and $D^L(t+1)$ is the duration of the plan liabilities at time $t+1$.

For return-seeking investments (e.g., equities), the price process EP_t is modelled through the exponential rate of the evolution of long-term bond yields, a fixed equity risk premium (ERP), and a random shock:

$$EP_{t+1} = EP_t \cdot e^{y_t + ERP + \varepsilon_e(t+1)} \tag{3}$$

where $\varepsilon_e(t) \sim i.i.d.\,N(0, \sigma_e^2)$ and σ_e is the standard deviation of the noise.

The normal random variables $\varepsilon_y(t)$ and $\varepsilon_e(t)$ are assumed to be independent.

We develop the parameters for the bond yield model (Equation (1)) based on the historical federal bond yields provided in the *Report on Canadian Economic Statistics 1924–2020*, published by the Canadian Institute of Actuaries (Canadian Institute of Actuaries, 2021): $\mu = 0.0493, \theta = 0.0194, \sigma_y = 0.0076$.[27] To reflect the current low-rate environment, the initial bond yield y_0 is set as 2%.

The ERP and standard deviation in the equity price model (Equation (3)) are as follows: $ERP = 0.05$[28], $\sigma_e = 0.15$[29].

Figures A1 and A2 show the outputs of the ESG based on the above parameters. The distribution of the fund return in Figure A2 is for a pension fund with 50% invested in equities and 50% in bonds. The distributions are shown for the median of all paths as well as the 5th, 25th, 75th, and 95th percentiles.

[27] These parameters are estimated from the nominal yields of federal bonds (V122487) over the period 1948–2020, based on Maximum Likelihood Estimation (MLE).
[28] Equity risk premium is generally accepted to be in the range of 3–7% in the long run, but the process of its calculation, and selection of data used, are highly subjective. According to a source from Investopedia, a survey of academic economists gives an estimate of equity risk premium of 3–3.5% for a one-year horizon, and 5–5.5% for a 30-year horizon.
[29] It is assumed that each 1% of risk premium adds 3% to the standard deviation.

Figure A1: Evolution of distribution of long-term bond yields

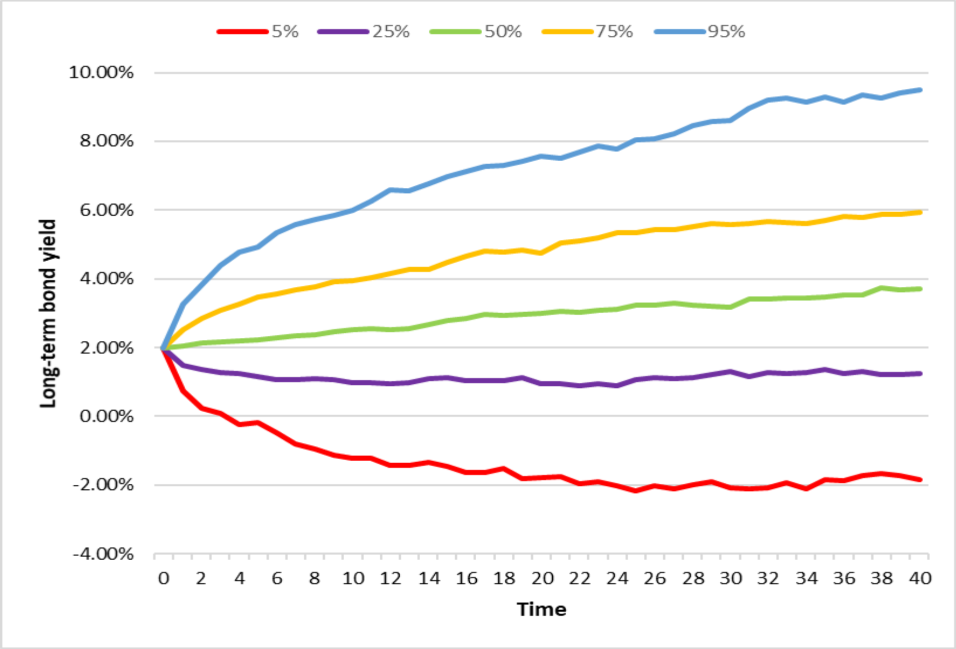
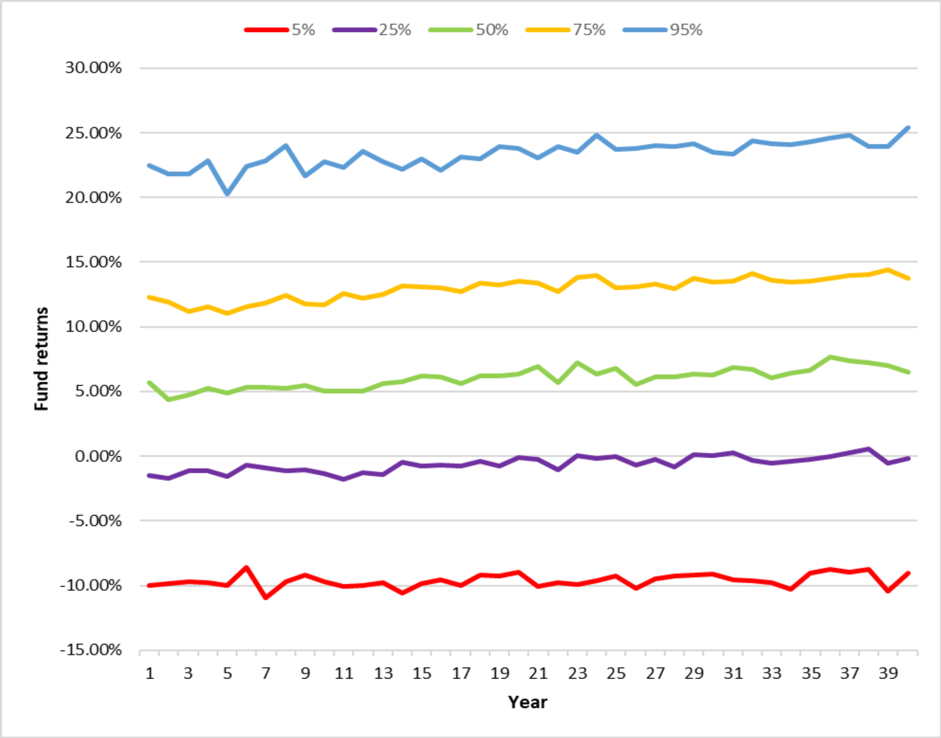
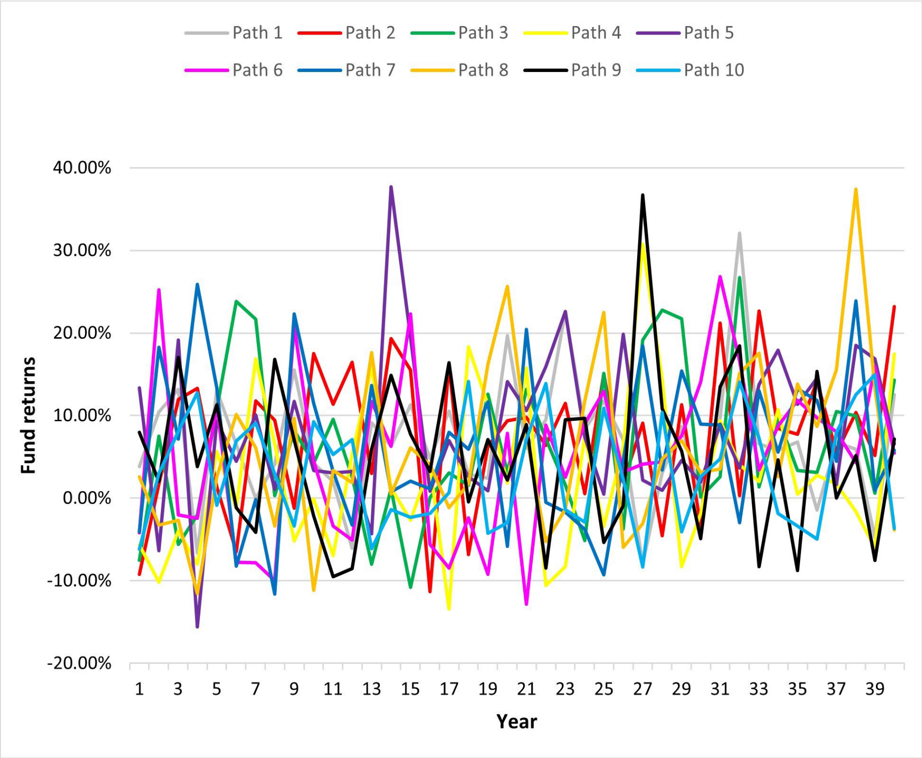


Figure A2: Evolution of distribution of fund returns under 50/50 asset mix



The projected returns along any given scenario (path) are very volatile, as Figure A3 shows.

Figure A3: Fund returns under 10 economic scenarios



2. Pension projection model

We use a mathematical model to project the assets and liabilities in respect of a DB plan, under an economic scenario generated from the model described in Part 1. The plan is assumed to have the following characteristics:

- Membership is either stationary,[30] growing, or declining.
- Members join the plan at plan anniversaries only.
- The plan is non-contributory; i.e., no employee contributions are required.
- Benefits are independent of members' salaries and are not subject to indexation before and after retirement.
- None of the benefits are contingent on the funded status of the plan or other factors.

The plan invests in two types of financial assets: *fixed income assets* and *return-seeking assets*.

Notation

Our model assumes annual valuations. An individual cost method that is in accordance with accepted actuarial practice[31] is used to calculate the liabilities and normal cost of the plan.

To develop the projection formulas of the model, we introduce the following notation for a valuation period $(t, t+1)$.

[30] Membership is said to be stationary if it has a static distribution of age, service, earnings (if benefits are earnings-related), and pension payments.
[31] Such as the *unit credit* or *accrued benefit* cost method, the *entry age normal* cost method, or the *individual level premium* cost method.

Financial model parameters	
y_t	Long-term bond yield at time t
$D^L(t)$	Duration of plan liabilities at time t
$D(t)$	Duration of fixed income portfolio at time t
ERP	Assumed equity risk premium for return-seeking investments
DR	Full diversification return for balanced investment portfolio
Pension plan parameters	
w	Proportion of fund assets invested in fixed income assets; $1 - w$ is the proportion invested in return-seeking assets
n	Amortization period with respect to unfunded liability or available actuarial surplus
τ	Surplus threshold as a percentage of plan liabilities; it is the level of funding excess beyond which surplus assets can be used for contribution holidays or refunded to the employer
$\bar{r}_t$	(Continuous compounding) best-estimate discount rate applicable at time t
δ_t	Liability growth rate (> 0) at time t
$PfAD_t$	Provision for adverse deviation applicable at time t; it is a percentage applied to $L_{t,t}$ (defined below) for determining additional liabilities to be funded at time t
Funding valuation variables	
$L_{t,t}$	Accrued liabilities as of time t calculated using discount rate $\bar{r}_t$
$L_{t,t-1}$	Accrued liabilities as of time t calculated using discount rate $\bar{r}_{t-1}$
ρ_t	Adjustment factor applied to $L_{t,t-1}$ such that $L_{t,t} = \rho_t L_{t,t-1}$
F_t	Value of fund assets at time t
UL_t	Unfunded liability at time t
AAS_t	Available actuarial surplus[32] at time t
NC_t	Normal cost, assumed to be payable at time $t + 1/2$
B_t	Benefit payments, assumed to be payable at time $t + 1/2$
SP_t	Special payment made with respect to unfunded liability or the appropriation of available actuarial surplus, payable at time $t + 1/2$

[32] In this paper, we define "available actuarial surplus" as the excess of the value of fund assets over (1+surplus threshold (τ)) x going concern liabilities (including PfAD if applicable). The excess amount can be used for contribution holidays or refunds of surplus. This definition of AAS may not be identical to the actual provisions of pension legislation. For example, in Ontario regulations, a plan has available actuarial surplus to be used for contribution holidays if (i) the plan is fully funded on a going concern basis (including the PfAD) and (ii) the plan's transfer ratio (which is calculated based on solvency liabilities) is not less than 105% after the surplus is applied.

α_t	Funded ratio in relation to liabilities $L_{t,t}$ at time t
φ_t	Change in the value of fund assets at time t due to the change in the long-term bond yield and/or the random shock of the return-seeking investments at time t
C_t	Annual required contribution payable at time $t + 1/2$

Discount rate assumption

For the funding valuation of a pension plan, it is a customary practice in Canada to select the going concern discount rate based on the long-term expected rate of return on plan assets. One approach for determining the best-estimate discount rate is the building block approach described in the CIA revised educational note, published in 2015 by the Canadian Institute of Actuaries (Canadian Institute of Actuaries, 2015).

Ontario regulations (Ontario, 2022) prescribes a benchmark discount rate (BDR)[33] for a pension plan as the amount calculated using the formula

$$0.005 + H + (0.015 \times J) + (0.05 \times K)$$

in which:

- "H" is the benchmark yield on long-term bonds issued by the Government of Canada for the valuation date,
- "J" is the combined target asset allocation for fixed income assets, and
- "K" is the combined target asset allocation for non-fixed income assets.

The value of 0.005 in this formula represents an excess return from diversification and rebalancing of the pension fund (the "diversification return"). The value of 0.015 represents the risk premium for fixed income investments and the value of 0.05 the risk premium for non-fixed income investments.

The Ontario BDR formula appears to have been based on the building block approach used by actuaries in practice. We have adopted a modified version of it as the basis for determining the valuation discount rate at time t:

$$\overline{r_t} = y_t + (1-w) \cdot ERP + \left(1 - \frac{abs(50 - w \cdot 100)}{50}\right) \cdot DR^{\,34} \tag{4}$$

This discount rate formula is consistent with the models underlying the ESG. At any valuation date, the expected returns on the bond and equity portfolios of the pension fund are both dependent on the prevailing long-term bond yield at that date.

For the analysis in this paper, we assume *zero* risk premium for the fixed income assets and make no provision for expected future investment and administrative expenses (which are assumed to be paid outside the plan).

Development of projection formulas

The liabilities at time $t + 1$ can be derived using the following formula:

$$L_{t+1,t} = L_{t,t} e^{\overline{r_t}} + (NC_t - B_t) e^{(1/2)\overline{r_t}}$$

The liabilities are assumed to change in the following manner:

$$L_{t+1,t} = (1 + \delta_t)^{\emptyset} L_{t,t}, \text{ for } t = 0, 1, 2, \dots \tag{5}$$

[33] The BDR is an element for determining the PfAD to be included in the going concern funding requirements for pension plans registered in Ontario.

[34] Full diversification return is allowed for portfolios with a 50/50 mix. For a target asset mix of u/(100 − u), where u is not equal to 50 and not more than 100, only a fraction of the full diversification return is allowed. The fraction is calculated as $1 - abs(50 - u)/50$.

where $\emptyset = -1, 0, 1$ depending on whether the membership is declining, stationary, or growing.

Thus,

$$NC_t - B_t = \frac{L_{t,t}((1+\delta_t)^{\emptyset} - e^{\overline{r}_t})}{e^{(1/2)\overline{r}_t}} \tag{6}$$

By definition, we have the following relationships:

$$F_t = \alpha_t L_{t,t} \tag{7}$$

$$UL_t = max\big(0, (1 + PfAD_t)L_{t,t} - F_t\big) = max(0{,}1 + PfAD_t - \alpha_t)\, L_{t,t} \tag{8}$$

$$AAS_t = max\big(0, F_t - (1+\tau)(1 + PfAD_t)L_{t,t}\big) = max\ (0, \alpha_t - (1+\tau)(1 + PfAD_t))L_{t,t} \tag{9}$$

$$SP_t = (UL_t - AAS_t)/n \tag{10}$$

A negative amount of SP_t means that a contribution holiday and/or a surplus refund occurs. The annual required contribution, payable at time $t + 1/2$, is equal to the sum of normal cost and special payment:[35]

$$C_t = NC_t + SP_t$$

A negative amount of C_t means that a refund of surplus occurs at time $t + 1/2$. The expected and actual values of fund assets at time $t + 1$, denoted as $\widetilde{F_{t+1}}$ and F_{t+1}, respectively, are derived as follows:

$$\widetilde{F_{t+1}} = F_t e^{\overline{r}_t} + (NC_t - B_t)e^{(1/2)\overline{r}_t} + SP_t e^{(1/2)\overline{r}_t} \tag{11}$$

$$F_{t+1} = \widetilde{F_{t+1}}\varphi_{t+1} \tag{12}$$

where

$$\varphi_{t+1} = exp\left(w \cdot \left(-h(t+1)D^L(t+1)(y_{t+1} - y_t)\right) + (1-w)\cdot \varepsilon_e(t+1)\right)$$

is the experience adjustment due to the change in bond yields and the random shock in equity prices, and

$$y_{t+1} - y_t = \theta\ (\mu - y_t) + \varepsilon_y(t+1)$$

is the change in bond yields according to our bond yield model.

Substituting Equations (6) to (11) into Equation (12), we obtain

$$\begin{aligned} F_{t+1} &= \left[\alpha_t L_{t,t} e^{\overline{r}_t} + (NC_t - B_t)e^{(1/2)\overline{r}_t} + SP_t e^{(1/2)\overline{r}_t}\right]\varphi_{t+1} \\ &= \left(\alpha_t e^{\overline{r}_t} + ((1+\delta_t)^{\emptyset} - e^{\overline{r}_t}) \right. \\ &\quad \left. + \left(\frac{max(0{,}1 + PfAD_t - \alpha_t) - \max\ (0, \alpha_t - (1+\tau)(1 + PfAD_t))}{n}\right) e^{(1/2)\overline{r}_t}\right)\varphi_{t+1}L_{t,t} \end{aligned} \tag{13}$$

Note that $L_{t+1,t+1} = \rho_{t+1} L_{t+1,t}$. The adjustment factor ρ_{t+1} can be estimated using the following formula developed in a paper on discount rate sensitivities, published by the Canadian Institute of Actuaries and the Society of Actuaries in 2017 (Chandler, 2017):

$$\rho_{t+1} = \frac{L_{t+1,t+1}}{L_{t+1,t}} \tag{14}$$

[35] Our model assumes that there is no provision for adverse deviations in respect of the normal cost to be funded.

$$= \exp\left[-(18 - 10.5p(t+1)) \times (y_{t+1} - y_t) \times \left(1 - 8\left(\frac{y_{t+1} + y_t}{2} + (1-w)ERP - 5.25\%\right)\right)\right]$$

where $p(t+1)$ is the proportion of the plan liabilities that relate to pensions in pay at time $t+1$.

Equation (14) gives an estimate of the duration of the plan liabilities at time $t+1$, $D^L(t+1)$, as follows:

$$D^L(t+1) = (18 - 10.5p(t+1)) \times \left(1 - 8\left(\frac{y_{t+1} + y_t}{2} + (1-w) \cdot ERP - 5.25\%\right)\right)$$

The adjustment factor ρ_{t+1} can thus be simplified to $\exp\left(-D^L(t+1)(y_{t+1} - y_t)\right)$.

Since $L_{t+1,t} = (1+\delta_t)^{\emptyset} L_{t,t}$, we have

$$L_{t+1,t+1} = \rho_{t+1}(1+\delta_t)^{\emptyset} L_{t,t} \qquad (15)$$

From Equations (13) and (15), we obtain the following recursive formula:

$$\begin{aligned} \alpha_{t+1} &= \frac{F_{t+1}}{L_{t+1,t+1}} \qquad (16) \\ &= \left(\alpha_t e^{\bar{r}_t} + \left((1+\delta_t)^{\emptyset} - e^{\bar{r}_t}\right)\right. \\ &\left. + \left(\frac{max(0,1 + PfAD_t - \alpha_t) - \max\left(0, \alpha_t - (1+\tau)(1 + PfAD_t)\right)}{n}\right) e^{(1/2)\bar{r}_t}\right)\left(\frac{\varphi_{t+1}}{\rho_{t+1}(1+\delta_t)^{\emptyset}}\right) \end{aligned}$$

Starting with an initial funded ratio α_0 at time 0, we can apply the above formula to find future funded ratios α_t , $t = 1, 2, \dots$

Appendix B: Summary of model outputs

The tables below were generated from the simulations related to the policy impacts analysis discussed in Sections 3 to 6.

Amortization policy

Table B1: Statistics on distribution of funded ratio by amortization policy

Funded ratio (α_{20})	Amortization period				
	1 year	5 years	7 years	10 years	15 years
Median	1.00	1.03	1.04	1.05	1.05
Mean	1.01	1.05	1.08	1.11	1.15
Standard deviation	0.09	0.19	0.24	0.32	0.44
Pr($\alpha_{20} < 0.85$)	0.03	0.13	0.16	0.21	0.25
Pr($\alpha_{20} > 1.15$)	0.06	0.28	0.33	0.37	0.40

Notes: Pr(α_{20} < 0.85) in the above table stands for the probability that the funded ratio at time 20 is less than 0.85; Pr(α_{20} > 1.15) stands for the probability that the funded ratio at time 20 is more than 1.15.

Investment policy

Table B2: Statistics on distribution of funded ratio by investment policy

Funded ratio (α_{20})	Investment policy (hedge ratio = 1.0)				
	All bonds	Conservative	Balanced	Risky	All equities
Median	1.00	1.03	1.05	1.11	1.11
Mean	1.00	1.04	1.11	1.23	1.42
Standard deviation	0.00	0.14	0.32	0.61	1.20
Pr($\alpha_{20} < 0.85$)	0.00	0.05	0.21	0.29	0.32
Pr($\alpha_{20} > 1.15$)	0.00	0.20	0.37	0.46	0.47

Surplus policy

Table B3: Statistics on distribution of funded ratio by surplus policy

Funded ratio (α_{20})	Surplus threshold				
	A 0%	B 5%	C 10%	D 5%	E 0%
	1/10th of AAS applied			AAS fully applied	None of AAS applied
Median	1.05	1.10	1.13	0.93	1.26
Mean	1.11	1.17	1.22	0.92	1.59
Standard deviation	0.32	0.37	0.42	0.15	1.02
Pr($\alpha_{20} < 0.85$)	0.21	0.17	0.18	0.32	0.16
Pr($\alpha_{20} > 1.15$)	0.37	0.46	0.49	0.04	0.58

Funding reserve policy

Table B4: Statistics on distribution of funded ratio by PfAD

Funded ratio (α_{20})	PfAD					
	0%	5%	10%	15%	20%	25%
Median	1.05	1.14	1.20	1.28	1.36	1.42
Mean	1.11	1.19	1.27	1.34	1.42	1.50
Standard deviation	0.32	0.34	0.39	0.38	0.41	0.45
Pr($\alpha_{20} < 0.85$)	0.21	0.13	0.10	0.06	0.03	0.02
Pr($\alpha_{20} > 1.15$)	0.37	0.49	0.56	0.65	0.72	0.78

Membership profile

Table B5: Statistics on distribution of funded ratio by membership profile

Funded ratio (α_{20})	Membership profile		
	Growing	Stationary (baseline plan)	Declining
Median	1.07	1.05	1.05
Mean	1.11	1.11	1.13
Standard deviation	0.29	0.32	0.37
Pr($\alpha_{20} < 0.85$)	0.18	0.21	0.22
Pr($\alpha_{20} > 1.15$)	0.38	0.37	0.40

Initial funding position

Table B6: Statistics on distribution of funded ratio by starting funded ratio

Funded ratio (α_{20})	Starting funded ratio		
	0.9	**1.0**	**1.1**
Median	1.01	1.05	1.10
Mean	1.06	1.11	1.17
Standard deviation	0.30	0.32	0.36
Pr($\alpha_{20} < 0.85$)	0.25	0.21	0.16
Pr($\alpha_{20} > 1.15$)	0.31	0.37	0.43

Alternative funding strategies

Table B7: Statistics on distribution of funded ratio by spendable surplus limit

Funded ratio (α_{20})	Spendable surplus limit				
	10%	**30%**	**50%**	**70%**	**100%**
Median	1.19	1.10	1.06	1.05	1.02
Mean	1.26	1.10	1.06	1.04	1.01
Standard deviation	0.40	0.21	0.18	0.16	0.16
Pr($\alpha_{20} < 0.85$)	0.10	0.13	0.13	0.13	0.16
Pr($\alpha_{20} > 1.15$)	0.55	0.41	0.30	0.26	0.20
25th percentile	0.99	0.96	0.93	0.92	0.90

Notes:

“Spendable surplus limit” is the percentage of surplus that is permitted to be used for contribution holidays or other purposes in any year before the next valuation date.

Each column corresponds to a funding strategy that comprises a “Balanced” investment policy (50/50 target asset mix), 10-year deficit amortization, 5% surplus threshold, 7.7% PfAD, and the indicated spendable surplus limit.

“25th percentile” is the funded ratio that the plan will be able to attain with 75% probability under the specific funding strategy.

References

BCFSA. (2020, November). *Pension Benefits Standards Regulation*.
Retrieved from https://www.bclaws.ca/civix/document/id/complete/statreg/71_2015#section1

Canadian Institute of Actuaries. (2015, December). *Revised Educational Note – Determination of Best Estimate Discount Rates for Going Concern Funding Valuations*.
Retrieved from https://www.cia-ica.ca/publications/publication-details/215106

Canadian Institute of Actuaries. (2021, July). *Report on Canadian Economic Statistics 1924-2020*.
Retrieved from https://www.cia-ica.ca/publications/publication-details/rp221081

Chandler, D. (2017, March). *Discount Rate Sensitivities in Pension Plans*.
Retrieved from https://www.cia-ica.ca/publications/publication-details/217034

Ma, C.-M. (2021, June). *A Mathematical Model for Assessing Impacts of Policies Related to Funding of Pension Plans*.
Retrieved 2022, from CIA: https://www.cia-ica.ca/publications/publication-details/rp221085

Ontario. (2022). *R.R.O. 1990, Reg. 909: GENERAL*.
Retrieved from https://www.ontario.ca/laws/regulation/900909

Québec. (2015). *Bill 57 - An Act to amend the Supplemental Pension Plans Act mainly with respect to the funding of defined benefit pension plans.*

Appendix C: Peer review of *A Stochastic Analysis of Policies Related to Funding of Defined Benefit Pension Plans*

By Doug Chandler, FCIA, FSA

November 2022

George Ma's paper *A Stochastic Analysis of Policies Related to Funding of Defined Benefit Pension Plans* probes an area that has not been addressed until now – how funding rules interact. Other recent Canadian Institute of Actuaries (CIA) research reports on defined benefit pension plan funding have investigated provisions for adverse deviations (PfADs) but not their interaction with amortization or surplus policies. Ma's paper is a valuable addition.

Ma contemplates a PfAD that is expressed as a percentage of the initial going concern funding target, referred to in his paper as the "liabilities." Liabilities are measured using a discount rate that anticipates equity risk premiums to be earned in the future but does not anticipate expected gains due to rising interest rates. That is, the measure of liabilities used by Ma as a going concern funding target is like a best estimate of the assets that would fund the accrued benefit obligations except that it includes an element of conservatism. Initial assets in the simulations are set at 100% or some other percentage of this funding target (before addition of the PfAD). Any amortization is applied on a declining balance basis by dividing the projected deficit or available surplus by a fixed number of years, without adjustment for interest. This perspective explains several of the conclusions concerning the sensitivity of required PfADs to variables such as plan maturity and an initial funding level different from 100%. If PfADs were determined as a percentage of a projected (rather than initial) best-estimate funding target and amortization were adjusted to reflect projected interest on unfunded or surplus assets, they would be less sensitive to funding levels, market conditions, and plan demographics.

Context

In the mid-1980s, Canada embarked on a 30-year experiment with solvency funding. In addition to the pre-existing requirement for going concern funding, pension plan contributions were required to target the cost of a hypothetical plan wind-up. As interest rates declined and pension plans matured, these new solvency contributions overtook going concern contributions, and the precise details of going concern funding became less important. Over the past decade, the experiment has come to an end. Plan members were still losing benefits in actual plan wind-ups, and the intolerable volatility of solvency contributions was contributing to the demise of defined benefit pension plans. The outcome of recent changes to de-emphasize or eliminate solvency funding is that going concern funding is re-emerging as the primary determinant of pension plan contributions. Research into going concern funding of Canadian pension plans can no longer be neglected.

Ma's research into the key features of a going concern funding regime is timely. It fills a gap not addressed by similar studies sponsored or co-sponsored by the CIA over the past decade. Although the focus of the research is on the shrinking class of traditional employer-sponsored defined benefit pension plans, the insights are also relevant to the risk-sharing pension deals that are emerging to fill the middle ground between pure defined contribution retirement savings plans and pure defined benefit pension plans. Indeed, getting the going concern funding regime just right is even more important for the contribution partnerships and target benefit pension plans in the middle of the spectrum. Accounting disclosures can provide an element of intergenerational equity to shareholders who bear the funding risks in a traditional corporate pension plan, but the same cannot be said when the risk is borne by pensioners and contributors in other classes of risk-sharing deals (Chandler 2020).

Comparison to past CIA research

1. **The 2013 report of the Task Force on the Determination of Provision for Adverse Deviations in Going Concern Valuations (Bonnar 2013)**

   This task force was mandated to respond to a 2009 request from Alberta Finance for assistance in the development of PfAD guidance. The focus was on flat benefits of the type typically offered by multi-employer pension plans, without solvency funding. The task force developed risk-based PfADs with 75% and 90% confidence levels, four time horizons ranging from 3 to 15 years, four equity allocations ranging from 20% to 80%, and three maturity levels ranging from 25% retirees to 75% retirees. There was an explicit new entrant profile and demographic projection. PfADs were expressed as percentages of the initial obligation, with no allowance for variations in contributions to reflecting emerging experience during the projection period.

2. **A 2017 research report on PfADs in going concern valuations co-sponsored by the Society of Actuaries and the Canadian Institute of Actuaries (Chandler 2017)**

   This report expanded on the 2013 results by including information on variations in hypothetical wind-up funding levels and by examining differences in required PfADs attributable to plan design. It developed risk-based PfADs with three confidence levels, four situations (maturity, inflation sensitivity), and seven asset strategies. Sensitivities to discount rate changes were determined in aggregate, without explicit demographic projections. PfADs were expressed as a percentage of a projected best-estimate funding target at the end of the fixed three-year projection period.

3. **The 2018 report of a task force jointly sponsored by the CIA and Retraite Québec on the Stabilization Provision Scale (Couture et al. 2018)**

   This task force conducted a review of stabilization provisions (PfADs) adopted in Quebec regulations in 2016 and recommended adjustments. The stabilization provision is expressed as the percentage of initial going concern liabilities. The adopted target was full funding at the end of a three-year time horizon in at least 85% of the trials. Stabilization factors in the regulations vary according to the allocation to variable income investments (equities) and duration coverage. Detailed methodology choices were the same as or similar to those in the second report (Chandler 2017). The final recommendations of the task force do not simply reflect simulation results, but also reflect judgment as to the appropriateness of the outcomes and considerations not reflected in the model.

The methodologies in all three of these reports have elements in common with the methodology adopted by Ma. Discount rates are linked to long bond yields, and investment returns are stochastic, with variability and an equity risk premium linked to the asset allocation. However, there are important differences in approach, the questions being answered, and the underlying economic scenario simulations.

The PfADs required in Ma's paper for comparable confidence levels are remarkably close to the past research. A comparison of Table 5 to results for the "traditional" asset mix and "average" plan circumstances in Appendix C of the second report or results with 25% duration coverage in Table 4.3 of the third report reveals similar absolute levels of PfADs and patterns by confidence level and equity allocation. This similarity must be regarded as a coincidence, given the important differences:

- Ma uses a 20-year time horizon rather than a 3-year time horizon. However, this does not mean that gains and losses are allowed to accumulate over the entire 20-year projection period. The baseline 10% amortization method means that unamortized gains and losses at the end of a 20-year time horizon represent unamortized balances of an average of approximately nine years' worth of accumulated experience. Considering the independent lognormal distribution of annual random shocks to equity returns, the standard deviation of the product of N independent identically distributed lognormal random variables will be the standard deviation for one year multiplied by the square root of N. While this pattern holds in modest extensions of a 3-year time horizon, both in results from the first report noted above (Bonnar 2013) and in unpublished calculations in conjunction with the third report above (Couture 2018), it does not hold for longer extensions of the time horizon because normal contributions become a more important element of the total obligation and benefits are paid in respect of the initial obligation. Based on results in Appendix B of the first report, PfADs under Ma's approach (with amortization) could be expected to be roughly 40% higher than PfADs required with a 3-year time horizon (without amortization).
- Overall, the scenario simulators used in the earlier studies generated slightly more funding risk, due to higher standard deviation and jump diffusion of equity returns and other esoteric aspects of model construction. The effect would be PfADs roughly 20% lower under Ma's approach.
- Ma uses the Ontario approach to going concern discount rates in combination together with an expectation that market bond yields will increase from an initial yield of 2% to a mean reversion level of 4.73%. Consequently, median funding levels at the end of the projection period are 105% rather than 100% in the baseline scenario. This expected experience gain shifts the distribution of unexpected gains and losses and reduces required PfADs at all confidence levels. This sort of bias can skew tail probabilities quite significantly (Chandler 2017, pages 11−12) and could account for most of the remaining differences between Ma's results and the results of prior research.

There may be other subtle differences attributable to details such as the skewness of compound investment returns, the rebalancing premium, the amortization approach, and timing of contributions, benefits, and accruals.

Model construction

It is often said that models should be as complicated as necessary, but no more so.

- The stochastic model adopted by Ma is somewhat more complicated than used in his previous research into target benefit plans (Ma 2018), in that it links bond returns and going concern discount rates to stochastic long bond yields.
- The approach to investments is considerably less complicated than the models used in past CIA-sponsored research and economic scenario simulators typically used by Canadian pension actuaries for asset/liability studies.
- The approach to redetermination of contributions from year to year is more complicated than the models used in past CIA-sponsored research, although it does not fully implement all the nuances of current pension regulations.

The choices concerning model construction and complexity are appropriate to the purpose of the research. They reflect the most important determinants of pension funding risk, while maintaining transparency and simplicity. However, it must be stressed that a model designed for one purpose is not necessarily appropriate for another purpose. The model used here

- does not allow for corporate bonds, real estate, or other types and sub-types of direct and derivative investments that can combine (i) long-term correlation with pension obligations and (ii) added value due to illiquidity and/or uncorrelated risks;
- does not take account of the term structure of interest rates;
- does not incorporate important correlations between investment classes or pension funding risks driven by economic variables such as inflation, unemployment, and GDP growth;
- does not allow for variations in net cash flow (the difference between normal contributions and benefit payments) due to variations in market yields and inflation; and
- does not reflect explicit new entrant assumptions, plan provisions, or other details affecting the projected evolution of benefit payments.

Thus, it would be inappropriate to use this model to guide investment, funding, and benefit strategy for a specific pension plan.

The input assumptions produce a plausible distribution of interest rates and investment returns, relative to historical benchmarks and results seen elsewhere. Of the input assumptions, by far the most important to the calculation of surplus variability and PfADs are the variance in equity returns and the linkage between bond yields and pension discount rates. Clearly, different choices would also be reasonable and would lead to significantly different absolute levels of PfADs. However, except as discussed below, I am satisfied that the conclusions of the research regarding the relationships between plan circumstances, funding rules, and outcomes are instructive and not merely accidental products of choices concerning model construction.

PfAD and funded ratio

Ma concludes that “the level of a PfAD will need to change with the funded ratio of a plan.” A larger PfAD is required when a plan is underfunded. This is a consequence of the approach to amortizing deficits. Funding the deficit and the PfAD on a declining balance amortization schedule will not be as likely to catch the deficit as immediately funding the deficit. Once a PfAD is funded, investment returns on that deficit help to maintain and increase it. On the other hand, a schedule of straight-line principal and interest special payments (with any future gains applied to shorten rather than reduce the schedule) operates like a guaranteed investment with a rate of return equal to the interest rate used to determine the schedule.

In any event, varying PfADs according to the funded ratio will likely prove unpopular. Increasing the PfAD as a result of an experience loss will be perceived as two penalties for the same failure. Funding for a target future PfAD should take account of the investment returns that will be earned once that PfAD begins to be established.

Bias in discount rate

The use of the Ontario approach to setting the discount rate together with a Vasicek model for interest rates (with no term structure) leads to an expected trend in long-term yields that is not reflected in the discount rate used to determine initial assets. Interest rates tend to rise toward the long-run mean, producing reductions in the funding target that outweigh the corresponding reductions in the market value of bonds. Table B2 shows this is a non-issue when the investment policy is all bonds with a hedge ratio of 1.0 but can lead to gains of as much as 11% over the 20-year time horizon with a hedging ratio less than 1.0.

The problem with this approach is that it makes the required PfAD dependent on market expectations for interest rates. PfADs will need to be redetermined at each valuation, with higher PfADs in an inverted yield curve environment and lower PfADs in a normal or steep yield curve environment. While this sort of routine reassessment may be desirable in any event, the bias and market-related variability in required PfADs could be eliminated by

- setting the funding target using a discount rate that reflects the median rate of return from the stochastic model used to assess the PfAD; and
- expressing the PfAD as a percentage of the projected obligation at the end of the time horizon, after taking account of the shorter time horizon for benefit payments and normal costs in the intervening years (Chandler 2017).

This approach is more consistent with the Educational Note that suggests a “discount rate based on fixed income yields typically would reflect the yields on … bonds that would ... have a duration comparable to that of the projected benefit cash flows" (CIA 2015, page 12).

Membership profile and risk of overfunding

Ma finds funding policies that include a PfAD tend to give rise to overfunding. The challenge for closed or mature employer-backed defined benefit pension plans is particularly acute since application of surplus is often restricted to contribution holidays, either by regulations or by legal precedent and plan terms. Even a modest surplus can exceed the present value of all future contributions. Table B4 shows that any PfAD greater than 5%, when combined with a biased discount rate, is likely to eventually give rise to overfunding.

This is one of the problems with a solvency funding target and one of the reasons that provinces have relaxed minimum solvency funding requirements. One innovation that has emerged in response to the unwillingness of corporate pension plan sponsors to fully fund solvency liabilities in ongoing pension plans is the introduction of letters of credit and refundable solvency reserve accounts. These approaches require employers or their bankers to post security to improve security for pension obligations in the event of a corporate bankruptcy, without increasing the non-refundable pension trust fund beyond the employer’s tolerance for overfunding. A similar approach can be taken to avoid the overfunding that results from a going concern PfAD.

Ma observes that plans with declining membership require smaller PfADs and plans with growing membership require larger PfADs. This is a direct consequence of expressing PfADs as a percentage of the initial obligation rather than as a percentage of the projected obligation. In the case of a closed or mature pension plan (either a traditional employer-backed plan or a plan that incorporates risk-sharing), special attention must be paid to avoiding overfunding or underfunding as the plan shrinks. However, the solution lies in accelerated amortization, rather than a lower PfAD.

It is worthwhile to observe that mean funded ratios are always higher than median funded ratios at the end of a 20-year simulation. This skewed distribution of funded ratios is a consequence of the skewed distribution of compound rates of investment returns.

The use of a level percentage amortization of the declining balance of deficits and surpluses also contributes to the imbalance between overfunding and underfunding that would exist even with no PfAD and no bias in the discount rate. The expected change in funded status as a result of the fresh-start 10-year amortization used by Ma is not 10% but is the difference between 10% and the current discount rate. When bond yields are high, discount rates and expected returns are also high and amortization is slow. For example, if bond yields and the asset mix at a particular point in time in the projection produce a discount rate of 9%, this means that liabilities are expected to increase in the following year by 9% and, in the absence of amortization, assets and the funding surplus or deficit are expected to increase by about 9%. Amortization at a rate of 10% will translate into an expected 1% reduction in the funding surplus or deficit. An increase in bond yields is favourable to a pension plan with a hedge ratio less than 100%, while a decrease in bond yields is unfavourable. Surpluses caused by increases in bond yields will be amortized more slowly than deficits caused by decreases in bond yields.

References

Bonnar S. (2013, June). *Provisions for Adverse Deviations in Going Concern Actuarial Valuations of Defined Benefit Pension Plans*. Canadian Institute of Actuaries.
https://www.cia-ica.ca/publications/publication-details/213002

Canadian Institute of Actuaries. (2015, December). *Revised Educational Note – Determination of Best Estimate Discount Rates for Going Concern Funding Valuations*.
https://www.cia-ica.ca/publications/publication-details/215106

Chandler D. (2017, March). *Provisions for Adverse Deviations in Going Concern Actuarial Valuations.* Canadian Institute of Actuaries and Society of Actuaries.
https://www.cia-ica.ca/publications/publication-details/217035

Chandler D. (2020, March). *Classification of Risk Sharing In Pension Plans – Canadian Practices and Possibilities*. Canadian Institute of Actuaries and Society of Actuaries.
https://www.cia-ica.ca/publications/publication-details/220040

Couture G., Provost P., Lemieux C. and Provost M. (2018, September). *Report of the Task Force on the Stabilization Provision Scale*. Canadian Institute of Actuaries.
https://www.cia-ica.ca/publications/publication-details/rapport-ica-ps

Ma G. (2018, October). *A Practical Approach to Establishing MfADs in Going Concern Funding Valuations*.
https://www.cia-ica.ca/publications/publication-details/218119

Canadian Institute of Actuaries
360 Albert Street, Suite 1740
Ottawa, ON K1R 7X7
613-236-8196
head.office@cia-ica.ca
cia-ica.ca
seeingbeyondrisk.ca

The Canadian Institute of Actuaries (CIA) is the qualifying and governing body of the actuarial profession in Canada. We develop and uphold rigorous standards, share our risk management expertise, and advance actuarial science to improve lives in Canada and around the world. Our more than 6,000 members apply their knowledge of math, statistics, data analytics, and business in providing services and advice of the highest quality to help Canadian people and organizations face the future with confidence.