Member's Paper

A Mathematical Model for Assessing the Impacts of Policies Related to the Funding of Pension Plans

By

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Abstract	**Résumé**
In recent years some Canadian jurisdictions have reformed their pension legislation that governs the funding of defined benefit (DB) pension plans. Their legislative intent was to ease and stabilize the funding requirements for plan sponsors while maintaining a certain level of benefit security for plan members. However, the funding rules ultimately adopted by various jurisdictions appear to be diverging. The long-term impacts of the different sets of reform measures on DB plans are not fully transparent to stakeholders. Modeling and analytics that have been employed to assess the long-term impacts of the new funding requirements typically involve a specific, or an abstract, pension plan design and demographic model, together with economic time series for equity prices and long-term bond yields. They are fairly complex and are not readily accessible to government or pension plan policy makers. This has motivated the author to explore new ways to assess the impacts of policies related to the funding of pension plans. The paper introduces a relatively simple model for the assessment of funding reform impacts. It involves making an assumption regarding the membership profile	Ces dernières années, certaines juridictions canadiennes ont modifié les lois encadrant les régimes de retraite à prestations déterminées (PD). L'intention des législateurs était d'alléger et de stabiliser les exigences de provisionnement pour les promoteurs des régimes, tout en conservant un certain niveau de sécurité pour les prestations des membres du régime. Cependant, les règles de provisionnement adoptées par les différentes juridictions semblent diverger. Les répercussions à long terme des différentes mesures de réforme sur les régimes PD ne sont pas parfaitement claires pour les parties prenantes. La modélisation qui a été utilisé pour analyser les répercussions à long terme des nouvelles exigences de provisionnement comporte typiquement une conception d'un régime de retraite et un modèle démographique, spécifiques ou théoriques, avec des séries chronologiques économiques pour les cours boursiers et les rendements des obligations à long terme. Elles sont assez complexes et difficiles à obtenir pour les décideurs du gouvernement ou des régimes de retraite. C'est ce qui a motivé l'auteur à explorer d'autres méthodes d'évaluation des répercussions des politiques liées au provisionnement des régimes de retraite.

underlying a pension plan: *stationary*, *growing* or *declining*. The model comprises several recursive formulas which can be used to project the assets, liabilities, special payments and funded ratios of a pension plan over an extended time period. The projection results may support government policy makers in their development of a regulatory funding framework, to ensure pension plans will operate effectively over the long term. The paper demonstrates how the model can be applied to assessing the long-term impacts of the following three funding-related measures on a pension plan: (1) an amortization rule, (2) the design of provision for adverse deviations, and (3) an investment policy. Some insights into the trade-offs between costs and benefit security level as well as the potential for adjustments to contributions are provided.	Le document présente un modèle relativement simple pour évaluer les répercussions de la réforme du provisionnement. Il consiste à formuler une hypothèse quant au profil général des participants d'un régime de retraite : *stable, croissant* ou *décroissant*. Le modèle comprend plusieurs formules récurrentes qui peuvent être utilisées pour projeter les actifs, le passif, les paiements spéciaux et les ratios de provisionnement d'un régime de retraite sur une période de temps prolongée. Ces projections pourraient être utiles aux décideurs gouvernementaux dans la conception du cadre de réglementation du provisionnement pour garantir que les régimes de retraite demeureront efficaces à long terme. Le document démontre comment le modèle peut être appliqué pour évaluer les répercussions à long terme de ces trois mesures liées au provisionnement sur un régime de retraite : (1) une règle d'amortissement, (2) la conception d'une provision pour écarts défavorables et (3) une politique de placement. On offre également quelques observations sur les compromis entre les coûts et le niveau de sécurité des prestations pour l'ajustement des cotisations.

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1. Introduction

Defined benefit (DB) pension plans registered under Canadian pension standards legislation are required to report their funding level on both a going-concern basis and a solvency basis. For the first basis, the liabilities are typically measured using the expected return on plan assets, whereas for the second the plans are assumed to terminate immediately and the liabilities are measured using long-term bond yields. Until recently, a deficiency under either of these two bases was required to be funded by the employer, over a period of not more than 15 years for a going-concern unfunded liability and five years for a solvency deficiency.

DB pension plans have been under significant financial strain for a number of years, one contributing factor being the legislative funding requirements. The trend of declining long-term bond yields has led to a significant increase in the amount of solvency liabilities, which is sensitive to the level of bond yields. Meanwhile, equity markets have continued to be volatile. This has resulted in substantial and volatile increases in solvency funding requirements.

Several Canadian jurisdictions have implemented funding reform amendments in respect of pension plans in the private sector. Through adoption of Bill 57 in 2015 (Québec, 2015), Québec amended the funding rules for DB plans by eliminating the funding of solvency deficiencies. At the same time, it strengthened the going-concern funding requirements by adding a stabilization provision. In 2018, Ontario implemented a new funding framework to ease the solvency funding requirements; employers now only need to fund a solvency deficiency up to an 85% solvency level. It also added a provision for adverse deviations (PfAD; similar to Québec's stabilization provision) that must be funded on a going-concern basis. Other jurisdictions that have reformed their legislative funding requirements include British Columbia and Nova Scotia.

While the objective of the regulatory reforms was to stabilize the funding for DB plans, the requirements ultimately adopted by various jurisdictions appear to be diverging. The long-term impacts of the different sets of reform measures on DB plans are not fully transparent to stakeholders. There is a need for more robust modeling and analytics to examine the trade-offs between costs[1] and the security of benefits as well as the potential for adjustments to contributions and/or benefits.

This paper presents a mathematical model for assessing the impacts of policy changes related to the funding of pension plans other than defined contribution plans.[2] The model may be useful to government policy makers in their development of a regulatory funding framework, to ensure that pension plans will operate effectively over the long term.

In May 2021, the Canadian Association of Pension Supervisory Authorities (CAPSA) released the revised *CAPSA Guideline No. 7 Pension Plan Funding Policy* (CAPSA, 2021). The Guideline identifies best practices that, in CAPSA's view, plan administrators and sponsors should consider and incorporate into a plan's funding policy. It is believed that the model presented in this paper may also assist plan decision makers in developing a funding policy, in line with the guidance provided by CAPSA.

The balance of the paper is organized as follows. In Section 2, we describe some objectives of funding a pension plan that provides defined benefits or target benefits, and identify the funding reform measures recommended by CAPSA for consideration by government policy makers. In Section 3, we analytically

[1] Costs comprise the normal cost and contributions toward unfunded liabilities plus any additional provision, if applicable.

[2] These include both traditional DB plans and shared-risk plans (such as target benefit plans or collective defined contribution plans).

develop a model for projecting the assets, liabilities and funded ratios of a pension plan with a stationary membership, based on a fundamental actuarial concept that links the liabilities between two valuation dates. In Section 4, we apply the model developed in Section 3 to assess the impacts of three funding-related measures on the long-term funded status of a DB plan, and make some observations from the modeling results. Section 5 extends the application of the model to pension plans with a non-stationary membership. Section 6 identifies some additional areas of interest for further study with the use of the model.

2. Funding Objectives

Generally, the objectives of funding a pension plan include:

- **Benefit security** – to ensure that the plan accumulates sufficient assets to provide the pension benefits promised or targeted by the plan.
- **Contribution/benefit stability** – to achieve stable and predictable funding costs/benefits, recognizing that a certain amount of inherent volatility is unavoidable.
- **Affordability** – to keep the plan costs affordable for the sponsors and/or members, while ensuring that the plan remains sustainable over the long term.
- **Intergenerational equity** – to support inter-cohort or intergenerational fairness by recognizing that the plan's costs and risks may be shared between current and future plan members, as well as current and future shareholders/taxpayers.

These objectives do not always align with each other. The challenge for government policy makers is to create a funding framework that strikes a proper balance among these objectives.

Up to this point in time, Canadian jurisdictions appear to have taken different approaches to reforming their regulatory funding framework. In keeping with its mandate to promote harmonization of pension regulation across Canada, CAPSA published a position paper in February 2019 recommending a common set of funding measures to be considered by policy makers when making reform changes to the legislation (CAPSA, 2019). The recommended measures are:

- Ease the solvency funding requirement but enforce a stronger going-concern funding requirement than that under the previous regulations.
- Include a buffer in the going-concern funding requirement, through the addition of a PfAD that takes into account certain economic and demographic factors; e.g., investment risk, interest rate risk, plan maturity.
- Shorten the amortization period for funding going-concern deficits from 15 years to 10 years; allow all unfunded liabilities to be consolidated into a single amount and re-amortize over a period of not more than 10 years from the valuation date – the so-called "fresh-start" approach.
- Establish a "side-car" fund to enable plan sponsors to recover assets that exceed the required funding level.
- Impose restrictions on the use of excess assets (i.e., surplus) for contribution holidays or benefit improvements.
- Allow plan sponsors to use letters of credit for plan funding.

The government is responsible for establishing the acceptable levels of funding target and for setting the rules of deficit funding and surplus utilization in the pension regulations. The mathematical model presented in the next section is intended to help policy makers assess the impacts of funding policy measures, including the ones recommended by CAPSA, as they attempt to find a balance between two funding objectives that are of primary concern to plan stakeholders: (a) contribution/benefit stability, and (b) long-term benefit security[3] which affects the sustainability of the plan.

3. The Model

This section presents a mathematical model for projecting the funded status and deficit contribution payments in respect of a collectively funded pension plan, under a given economic and investment scenario. The plan is assumed to have the following characteristics:

- Plan membership is stationary, with a static distribution of age, service, earnings (if benefits are earnings-related) and pension payments.
- Members would join the plan at plan anniversaries only.
- Fifty percent of the plan's liabilities are related to pensions in pay.

The model can be applied to an indexed plan, but only if both the accrued benefits of active and deferred vested members and the pensions in pay are increased annually by the same indexing rate (e.g., rate of Consumer Price Index increase).

With the above assumptions, it is not necessary to stipulate the benefit provisions of a plan for the model to apply.

3.1 Defined Benefit Pension Plan

We derive the formulas for projecting assets, liabilities, special payments and funded ratios for the plan scenario described above. The plan is funded on a going-concern basis only, pursuant to the following policy:

- Actuarial valuations to determine the funding requirements of the plan are performed at each plan anniversary.
- A cost method that is in accordance with accepted actuarial practice is used to calculate the accrued liabilities and normal cost of the plan.
- A discount rate based on the best estimate of expected return on plan assets is used to measure the benefit obligations.
- Contributions are made toward a PfAD with respect to accrued liabilities but not normal cost.
- Any unfunded liability is amortized over a fixed number of years on a linear, fresh-start basis.
- Funding excess (i.e., surplus), if any, is retained in the plan as a cushion against future contingencies.[4]
- Fund assets are invested in accordance with an adopted investment policy.

We first introduce the following notation for a valuation period $(t, t+1)$.

i_t Applicable best-estimate discount rate at time t

[3] As measured by the going-concern funded ratio.

[4] Alternatively, all or part of the surplus may be applied to reduce the funding contributions that may otherwise be required.

f_t Assumed indexing rate used for valuation at time t; it is set as the actual indexing rate j_{t-1} granted in respect of the preceding valuation period

k_t Effective *net* discount rate at time t, after factoring in the assumed indexing rate:

$$k_t = \left(\frac{1+i_t}{1+f_t}\right) - 1$$

$L_{t,t}$ Accrued liabilities as of time t calculated using net discount rate k_t

$L_{t,t-1}$ Accrued liabilities as of time t calculated using net discount rate k_{t-1}

$PfAD_t$ Provision for adverse deviation applicable at time t; it is a percentage applied to $L_{t,t}$ for determining additional liabilities to be funded at time t

δ_t Adjustment factor applied to $L_{t,t-1}$ such that $L_{t,t} = (1+\delta_t)L_{t,t-1}$

F_t Value of fund assets at time t

UL_t Unfunded liability at time t

NC_t Normal cost, assumed to be payable at time $t+1/2$

B_t Benefit payments, assumed to be payable at time $t+1/2$

SP_t Special payment made with respect to unfunded liability, payable at time $t+1/2$

α_t Funded ratio in relation to liabilities $L_{t,t}$ at time t

r_t Annual rate of fund return over the valuation period, compounded continuously

j_t Actual indexing rate granted in respect of the valuation period

C_t Annual required contribution payable at time $t+1/2$

n Amortization period with respect to unfunded liability

The expected liabilities at time $t+1$ (denoted as $\widetilde{L_{t+1,t}}$) can be derived using the following formula:

$$\widetilde{L_{t+1,t}} = L_{t,t}(1+i_t) + (NC_t - B_t)(1+i_t)^{1/2}$$

Since the plan membership is stationary, we have

$$\widetilde{L_{t+1,t}} = (1+f_t)L_{t,t}, \text{ for } t = 0, 1, 2, \ldots$$

Thus,

$$NC_t - B_t = \frac{L_{t,t}(f_t - i_t)}{(1+i_t)^{1/2}} \tag{1}$$

By definition, we have the following relationships:

$$F_t = \alpha_t L_{t,t} \tag{2}$$

$$UL_t = max\ (0, (1 + PfAD_t)L_{t,t} - F_t) = max(0,1 + PfAD_t - \alpha_t)L_{t,t} \tag{3}$$

$$SP_t = \frac{UL_t}{n} = \frac{max(0,1 + PfAD_t - \alpha_t)\ L_{t,t}}{n} \tag{4}$$

The annual required contribution, payable at time $t + 1/2$, is equal to the sum of normal cost and special payment

$$C_t = NC_t + SP_t$$

Thus,

$$F_{t+1} = F_t(1 + r_t) + (NC_t - B_t)(1 + r_t)^{1/2} + SP_t(1 + r_t)^{1/2} \tag{5}$$

Substituting Equations (1) to (4) into Equation (5), we obtain

$$\begin{aligned} F_{t+1} &= \alpha_t L_{t,t}(1 + r_t) + \left(\frac{L_{t,t}(f_t - i_t)}{(1 + i_t)^{1/2}}\right)(1 + r_t)^{\frac{1}{2}} \\ &\quad + \left(\frac{max(0,1 + PfAD_t - \alpha_t)L_{t,t}}{n}\right)(1 + r_t)^{\frac{1}{2}} \\ &= \left(\alpha_t(1 + r_t) + (f_t - i_t)\left(\frac{1 + r_t}{1 + i_t}\right)^{\frac{1}{2}} \right. \\ &\quad \left. + \left(\frac{max(0,1 + PfAD_t - \alpha_t)}{n}\right)(1 + r_t)^{\frac{1}{2}}\right)L_{t,t} \end{aligned} \tag{6}$$

Due to the deviation between the actual and assumed indexing rates, we must adjust $\widetilde{L_{t+1,t}}$ by a factor of $\left(\frac{1+j_t}{1+f_t}\right)$ to calculate the liabilities $L_{t+1,t}$, i.e., $L_{t+1,t} = \left(\frac{1+j_t}{1+f_t}\right)\widetilde{L_{t+1,t}}$.

Note that $L_{t+1,t+1} = (1 + \delta_{t+1})L_{t+1,t}$. It follows that

$$L_{t+1,t+1} = (1 + \delta_{t+1})\left(\frac{1 + j_t}{1 + f_t}\right)\widetilde{L_{t+1,t}} \tag{7}$$

The adjustment factor δ_{t+1} in the above formula can be estimated using the following formula developed in a paper on discount rate sensitivities published by the Society of Actuaries and Canadian Institute of Actuaries (Society of Actuaries/CIA, 2017):

$$\delta_{t+1} = \frac{L_{t+1,t+1}}{L_{t+1,t}} - 1 \tag{8}$$

$$= exp\left[-(18-10.5p)\times(k_{t+1}-k_t)\times\left(1-8\left(\frac{k_t+k_{t+1}}{2}-5.25\%\right)\right)\right]-1$$

where p is the proportion of the plan liabilities that relate to pensions in pay. For the model plan, $p = 0.5$.

Since $\widetilde{L_{t+1,t}} = (1+f_t)L_{t,t}$, Equation (7) can be written as

$$L_{t+1,t+1} = (1+\delta_{t+1})(1+j_t)L_{t,t} \tag{9}$$

From Equations (6) and (9), we obtain the following recursive formula:

$$\alpha_{t+1} = \frac{F_{t+1}}{L_{t+1,t+1}} = \left(\alpha_t(1+r_t) + (f_t - i_t)\left(\frac{1+r_t}{1+i_t}\right)^{\frac{1}{2}} + \left(\frac{max(0,1+PfAD_t-\alpha_t)}{n}\right)(1+r_t)^{\frac{1}{2}}\right)\left(\frac{1}{(1+\delta_{t+1})(1+j_t)}\right) \tag{10}$$

Starting with an initial funded ratio α_0 at time 0, we can apply the above formula to find future funded ratios α_t , $t = 1, 2, \ldots$

If the plan is non-indexed, the two parameters f_t and j_t in the above formulas are both set to zero.

3.2 Target Benefit Plan

The model described in Subsection 3.1 can be modified for application to a special type of target benefit plan that adjusts benefit payments in the following manner:

(a) A proportion of the unfunded liability (or surplus) is applied to adjust the amount of benefits payable in the year following the valuation date.

(b) Any difference between the predefined contributions and the normal cost is also applied as an adjustment to the benefits payable.

(c) The total adjustment to the benefits payable, either upside or downside, is limited to a specified threshold. The remainder of the adjustment is applied as an increase or decrease to the contributions payable, or as a refund to the sponsors.

If both the unfunded liability and surplus are amortized, on a linear basis, over n years, the benefits payable at time $t+1/2$ would be reduced by the following two amounts:[5] $NC_t - C_t$ and $AB_t = \frac{(1+PfAD_t-\alpha_t)\,L_{t,t}}{n}$, subject to a maximum adjustment (upside or downside) of $\theta_t B_t$ where θ_t is a fraction between 0 and 1 (e.g., 0.5).

Denote the actual contributions payable and benefits payable as C'_t and B'_t, respectively. These two amounts are calculated as follows:

(a) If $NC_t - C_t + AB_t$ falls between $-\theta_t B_t$ and $\theta_t B_t$, then

$$C'_t = C_t;\ B'_t = B_t - (NC_t - C_t + AB_t)$$

[5] Both amounts can be either positive or negative.

$$C_t' - B_t' = C_t - \big(B_t - (NC_t - C_t + AB_t)\big) = NC_t - B_t + AB_t$$

(b) If $NC_t - C_t + AB_t > \theta_t B_t$, then

$$C_t' = C_t + \big((NC_t - C_t + AB_t) - \theta_t B_t\big);\ B_t' = (1 - \theta_t)B_t$$

$$C_t' - B_t' = C_t + \big((NC_t - C_t + AB_t) - \theta_t B_t\big) - (1 - \theta_t)B_t = NC_t - B_t + AB_t$$

(c) If $NC_t - C_t + AB_t < -\theta_t B_t$, then

$$C_t' = C_t + \big((NC_t - C_t + AB_t) + \theta_t B_t\big);\ B_t' = (1 + \theta_t)B_t$$

$$C_t' - B_t' = C_t + \big((NC_t - C_t + AB_t) + \theta_t B_t\big) - (1 + \theta_t)B_t = NC_t - B_t + AB_t$$

Note that a refund of surplus would occur if $C_t' < 0$.

To project the fund assets and the funded ratios of the plan, we can use the following modified version of Equations (5) and (10):

$$\begin{aligned} F_{t+1} &= F_t(1 + r_t) + (C_t' - B_t')(1 + r_t)^{1/2} \\ &= F_t(1 + r_t) + (NC_t - B_t)(1 + r_t)^{1/2} + AB_t(1 + r_t)^{1/2} \end{aligned} \tag{11}$$

$$\begin{aligned} \alpha_{t+1} = \frac{F_{t+1}}{L_{t+1,t+1}} = \Bigg(& \alpha_t(1 + r_t) + (f_t - i_t)\left(\frac{1 + r_t}{1 + i_t}\right)^{\frac{1}{2}} \\ &+ \left(\frac{1 + PfAD_t - \alpha_t}{n}\right)(1 + r_t)^{\frac{1}{2}}\Bigg)\left(\frac{1}{(1 + \delta_{t+1})(1 + j_t)}\right) \end{aligned} \tag{12}$$

4. Impacts Analysis

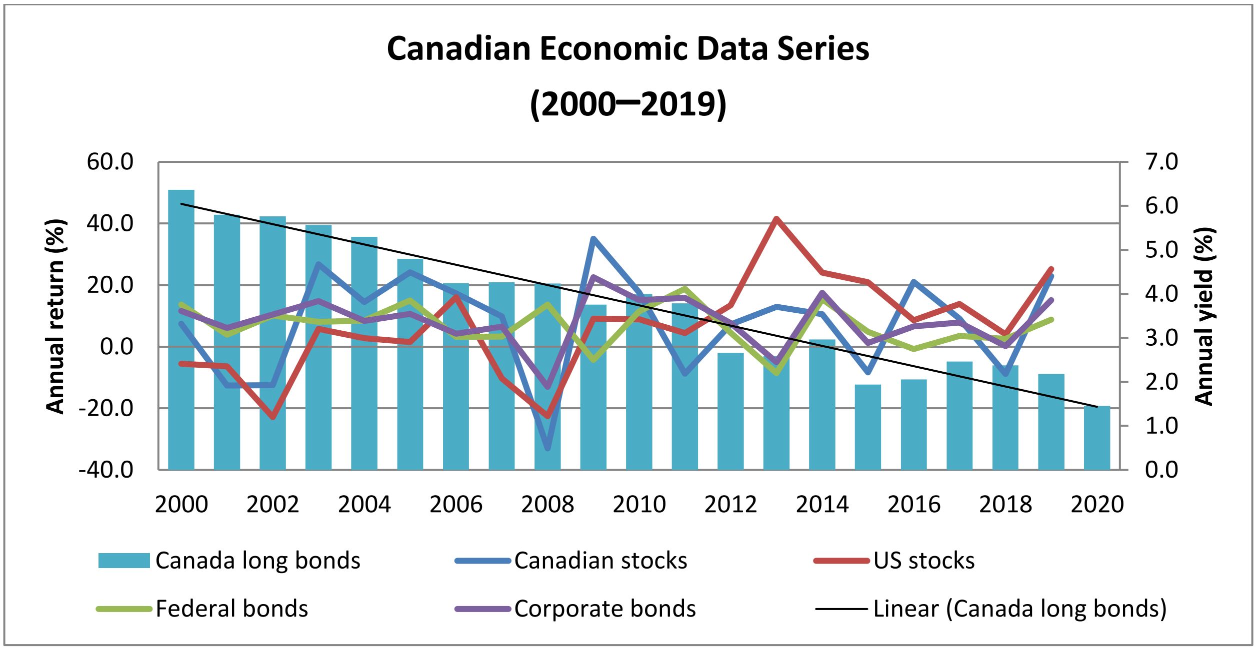
The following are two key funding measures recommended by CAPSA:

1. Amortization of unfunded liability over a period of not more than 10 years, on a fresh-start basis
2. Inclusion of a PfAD in the going-concern funding requirement

This section provides a demonstration of how to apply the model developed in Section 3 to assess the funding impacts of different policy decisions, including the ones above, on a non-indexed DB plan. We apply the model to the 20-year period from 2000 to 2019 inclusive. The relevant economic and investment data is extracted from the Report on Canadian Economic Statistics, 1924–2019, published by the Canadian Institute of Actuaries (CIA, 2020), and the Bank of Canada website (Bank of Canada); see Appendix A.

The economic environment of the above period was characterized by a trend of declining long-term bond yields (the yields dropped by approximately 4.5% over the 20-year period) and volatile equity returns, as displayed on the following chart. The lines in the chart pertain to the annual rates of returns of various asset classes. The bars are the beginning-of-year annual yields of long Canada bonds, and the straight line shows the prevailing direction of yields. The returns of US common stocks are in Canadian dollar terms.

Bond portfolios had earned fairly decent returns over this period due to the continued decline in interest rates. Some academics and professionals in finance have opined that this is not the norm, and they do not expect the downward trend to persist. We have chosen this period for our analysis as it does manifest the potential economic risk to which pension plans could be exposed in the long term.



In each of the subsections that follow, we will vary one policy decision by the government policy makers or plan decision makers, and keep the other decisions constant. The following table summarizes the policy decisions under consideration.

Policy decision	**Target asset mix**	**Amortization period**	**PfAD design/level**
1. Alternative amortization rules	60% equity/40% fixed income ("60/40 mix")	1, 5, 10 and 15 years	No PfAD
2. Alternative PfAD designs	60/40 mix	10 years	Baseline (no PfAD), Ontario, Alternative and British Columbia (BC)
3. Alternative investment policies	20/80, 40/60, 60/40 and 80/20 mixes	10 years	Level required to achieve target funded ratio

The measurement of liabilities for calculating the plan's funding level each year is based on the benchmark discount rate (BDR) prescribed in the Ontario regulations, which is used as a proxy for best-estimate discount rate. The BDR for a plan with a target asset mix of 60% equities and 40% fixed income is calculated using the following prescribed formula:

$$0.005 + H + (0.015 \times 0.4) + (0.05 \times 0.6)$$

where H is the yield on long-term Government of Canada (GoC) bonds (CANSIM series 39056) for the valuation date. The value of 0.005 in this formula represents an excess return from diversification and rebalancing of the pension fund portfolio. For our analysis, we assume that this value is only valid for plans with a target asset mix of 50% equities and 50% fixed income, and use instead an adjusted value of 0.004 to reflect the plan's asset mix.[6] As at January 1, 2020, the long-term GoC bond yield (annualized) is 1.46%. If the target asset mix is 60% equities and 40% fixed income, then the BDR for the valuation at this date is equal to: 0.004+0.0146+0.006+0.03 = 0.0546 or 5.46%.

If the plan's target asset mix is maintained in future, the BDRs would move in tandem with the long-term GoC bond yields.

For equity investments of the fund, it is assumed that half of the assets are invested in Canadian common stocks and half in US common stocks. For fixed-income investments, half of the assets are invested in Canada federal long bonds and half in corporate long bonds.

The initial valuation is performed as of January 1, 2000 (time 0). The plan is assumed to be fully funded at the start of the projection.

We use the model's recursive formulas to project the assets, liabilities, deficit funding payments and funded ratios of the plan over the 20-year period. A summary of the simulation results under each policy option is shown below.

For ease of checking, Appendix B provides the detailed results for one policy option: *Option 2 with Ontario PfAD level*.

4.1 Alternative Amortization Rules

We use a straight-line, fresh-start method to amortize any unfunded liability. Four (4) amortization periods – 1, 5, 10 and 15 years – are considered.

Both assets and liabilities have a starting value of 100; the initial funded ratio (FR) α_0 is thus equal to 1.0. It is assumed that a 60/40 target asset mix is adopted by the plan. Over the 20-year period, the (arithmetic) average BDR used in the modeling is 7.85% per annum, whereas the average rate of fund return is 7.27% per annum. The plan would have experienced losses more often than gains during the modeling period. Where losses are funded within a short period of time (say one year), the plan would be able to maintain its funded status but the deficit contribution payments would be rather unstable. If a longer amortization period is used, the risk of losses would be spread out and funding would be more stable. However, the plan may become severely underfunded if experience losses persist. This is exemplified by the case where the unfunded liability is amortized over 15 years. The economic losses (from investment performance and/or interest rate decline) incurred during the first decade of the projection period were not fully offset by the gains in subsequent years. As losses were not paid off within a short period of time, a large amount of unfunded liability would remain outstanding at the end of the projection period.

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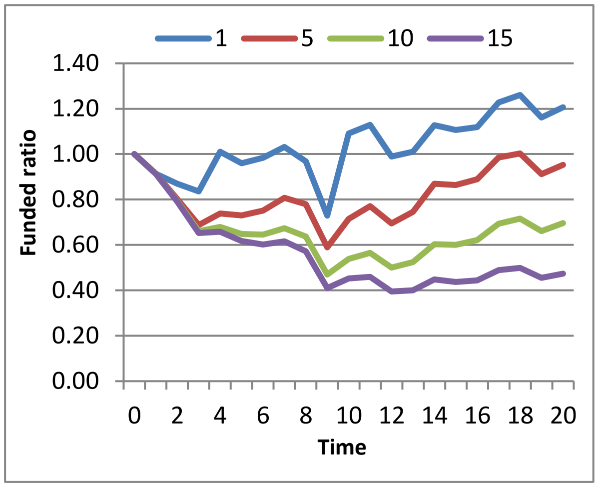
[6] Full diversification return is allowed for portfolios with a 50/50 mix. For a target asset mix of $x/(100-x)$, where x is not equal to 50 and not more than 100, only a fraction of the full diversification return is allowed. The fraction is calculated as: $1-\text{abs}(50-x)/50$.

Funded ratios

Table 1: Statistics on funded ratios

	Amortization period			
	1	**5**	**10**	**15**
FR @ 1/1/2020	1.21	0.95	0.70	0.47
Mean	1.03	0.82	0.66	0.56
Standard deviation	0.13	0.11	0.13	0.17
Minimum	0.73	0.59	0.47	0.39
Maximum	1.26	1.00	1.00	1.00

Figure 1: Evolution of funded ratios

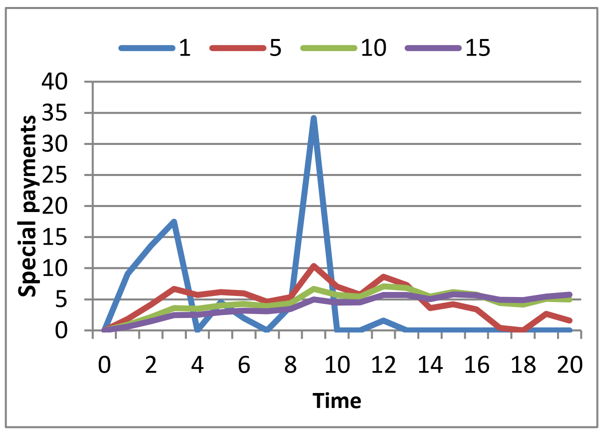


Deficit funding payments

Table 2: Statistics on special payments

	Amortization period			
	1	**5**	**10**	**15**
Mean	4.11	4.54	4.50	3.93
Standard deviation	8.45	2.81	1.82	1.77
Minimum	0.00	0.00	0.00	0.00
Maximum	34.16	10.35	7.09	5.79

Figure 2: Evolution of special payments



Key takeaway

If funding deficits of a plan are paid off over a long period of time, contribution payments would be relatively more stable but there is a risk that the cost of any funding deficit may be pushed indefinitely to the future. This may potentially lead to severe underfunding, which would weaken the plan's ability to deliver the promised or target benefits on an ongoing basis. In setting an appropriate amortization period to ensure the continuity and sustainability of pension plans, government policy makers need to consider the trade-off between contribution stability and long-term benefit security (measured by the going-concern funded ratio).

Where a plan is funded by public resources (e.g., a government-sponsored pension plan), the potential risk and cost transfers under an amortization rule should also be considered in the context of intergenerational fairness.

4.2 Alternative PfAD Designs

In a recent paper published by the CIA (Ma, 2021), the author presents a comparative analysis of three known PfAD design alternatives, namely the Ontario and BC PfAD rules[7,8] and an alternative design proposed in the author's earlier paper (Ma, 2018),[9] with a focus on their effectiveness at stabilizing funding for pension plans under a changing-interest-rate environment. The paper concludes that Ontario PfAD design is the least capable of stabilizing funding requirements in the face of long-term interest rate changes, while the BC design is the most effective. However, the BC PfAD does not reflect a plan's investment risk exposure or maturity. The alternative design overcomes the shortcomings of the BC approach and has the potential to achieve more stable funding than the Ontario approach.

We use the model developed in Section 3 to examine the funding dynamics of a DB plan, under an environment of both changing interest rates and investment returns, in respect of the above three PfAD designs. Also included is a case where there is no funding for a PfAD:

(a) Baseline – no PfAD included

(b) Ontario

(c) Alternative

(d) BC

Any unfunded liability is assumed to be amortized over a period of 10 years, on a straight line and a fresh-start basis. An investment policy of 60/40 target asset mix is adopted by the plan.

A summary of the projection results is given in the tables and graphs below.

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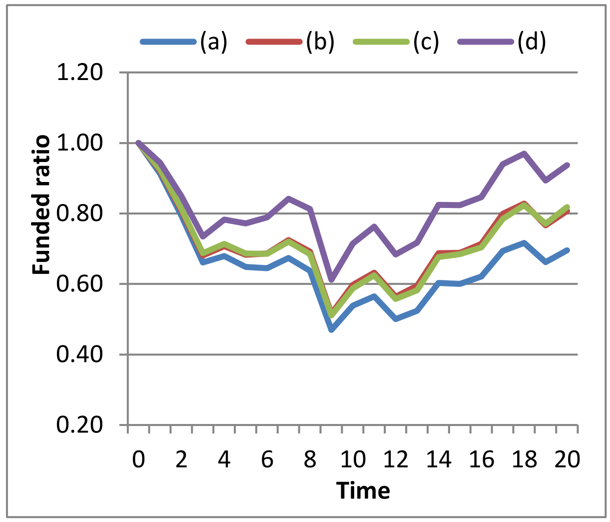
[7] The PfAD under Ontario regulations depends on whether a plan is open or closed to new members, and the plan's target asset mix. A higher PfAD is required if a plan adopts a riskier investment strategy (Ontario, 2017).
[8] The BC PfAD is calculated as the greater of 5% or five times the long-term GoC bond yields, as long as the plan's non-fixed-income allocation is 30% or more. Where the non-fixed-income allocation is less than 30%, the PfAD is proportionately reduced but still subject to a floor of 5% (BCFSA, 2020).
[9] The alternative PfAD proposed by the author is developed through a dynamic margin incorporated in the going-concern discount rate. The margin reflects a plan's investment policy, its level of maturity and the current level of long-term interest rates.

Funded ratios

Table 3: Statistics on funded ratios

	PfAD design			
	(a)	**(b)**	**(c)**	**(d)**
Average PfAD	0.00%	8.00%	8.35%	18.70%
FR @ 1/1/2020	0.70	0.81	0.82	0.94
Mean	0.66	0.72	0.72	0.82
Standard deviation	0.13	0.12	0.12	0.10
Minimum	0.47	0.52	0.51	0.61
Maximum	1.00	1.00	1.00	1.00

Figure 3: Evolution of funded ratios



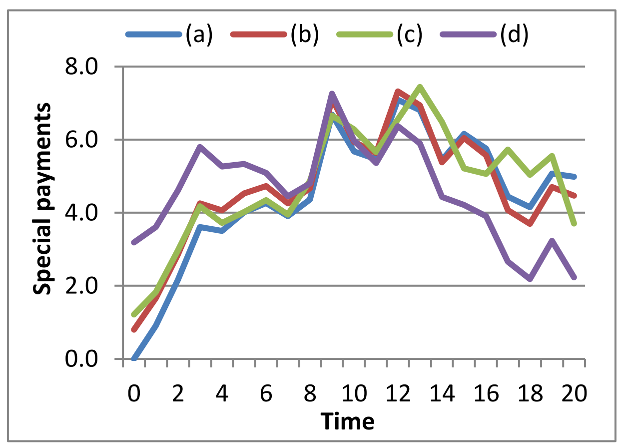
PfAD designs: (a) baseline, (b) Ontario, (c) Alternative, and (d) BC.

Deficit funding payments

Table 4: Statistics on special payments

	PfAD design			
	(a)	**(b)**	**(c)**	**(d)**
Mean	4.50	4.70	4.78	4.56
Standard deviation	1.82	1.63	1.59	1.37
Minimum	0.00	0.80	1.21	2.19
Maximum	7.09	7.32	7.44	7.26

Figure 4: Evolution of special payments



The following observations can be drawn from the results:

- Inclusion of a PfAD would improve the funded status of a plan.
- The Ontario and Alternative PfAD designs would produce similar funding outcomes, as their PfADs are in the same order of magnitude.
- The BC PfAD level would be much higher during the first decade of the projection period when interest rates were in the 4–5% range.

- The overall PfAD under the BC design would be significantly higher than that under either the Ontario or Alternative design. It therefore results in a better funding position at the end of the projection period.
- Paradoxically, the mean value of special payments under the BC approach is lower than that under either the Ontario or Alternative approach. This is because the PfAD level under the former approach was substantially higher during the first decade of the projection period. As such, the plan was able to build up a larger funding buffer in early years, which would serve to reduce the deficits and special payments in later years.

Key takeaway

A PfAD is a funding buffer designed to cover the risk of adverse deviations in funding levels. A higher level of PfAD will provide more improvement to the funding level of a plan, and hence a greater protection of members' benefits. In designing a PfAD, government policy makers need to consider the following issues: (1) What risks (e.g., interest risk, investment risk) are to be covered by a PfAD? (2) What level of benefit security is to be achieved with a PfAD, and over what time horizon? (3) What size of PfAD is considered to be appropriate? The size of a PfAD should not be so large as to put an undue strain on the plan sponsor's cash resources.

4.3 Alternative Investment Policies

In the preceding analysis, we assume the plan adopts a target asset mix of 60% equities and 40% fixed income (60/40 mix) as its investment policy. This subsection explores the impacts that different investment strategies have on the funding level of a plan, and shows how a PfAD could be used to mitigate the adverse funding impact from risky investments.

Four (4) target asset mixes are considered, for illustrative purposes:

(a) 20/80 mix

(b) 40/60 mix

(c) 60/40 mix

(d) 80/20 mix

For deficits funding, the plan adopts a method of straight line amortization over a period of 10 years, on a fresh-start basis.

We first consider the case where there is no PfAD included in the funding requirements. Then, we estimate the amount of PfAD required, under each investment policy, to achieve a target funded ratio at the end of the projection period (1/1/2020).

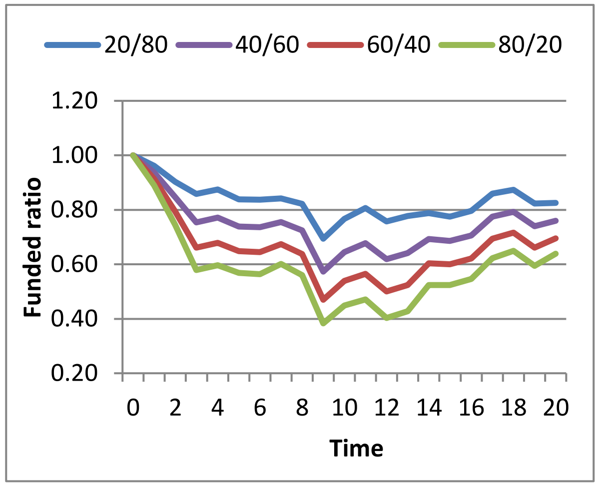
Funding without a PfAD

A summary of the modeling results is given below.

Table 5: Statistics on funded ratios

	Asset mix			
	20/80	**40/60**	**60/40**	**80/20**
FR @ 1/1/2020	0.83	0.76	0.70	0.64
Mean	0.83	0.74	0.66	0.59
Standard deviation	0.07	0.10	0.13	0.15
Minimum	0.69	0.57	0.47	0.38
Maximum	1.00	1.00	1.00	1.00

Figure 5: Evolution of funded ratios



Over the modeling period, the (arithmetic) average annual return on equity investments was 7.12%, whereas the average annual return on fixed-income investments was 7.51%. The average returns on plan assets and average BDRs used for liability measurement under various asset mixes are as follows:

Target asset mix	Average annual return	Average BDR
20/80 mix	7.43%	6.25%
40/60 mix	7.35%	7.15%
60/40 mix	7.27%	7.85%
80/20 mix	7.20%	8.35%

With a 60/40 or 80/20 investment strategy, the plan would be more likely to experience losses as a result of the shortfall in investment returns (relative to the discount rates used for liability measurement). Furthermore, the liabilities required to be funded would be increasing progressively due to the decline in long-term bond yields over the period. For the modeling period, the projected funded ratios at 1/1/2020 under the four investment policies would all be lower than 1.0. The riskier the investment strategy, the lower the funding level. The funding level under a riskier investment strategy is also more volatile, as indicated by the standard deviations with respect to the distribution of funded ratios.

Funding with a PfAD

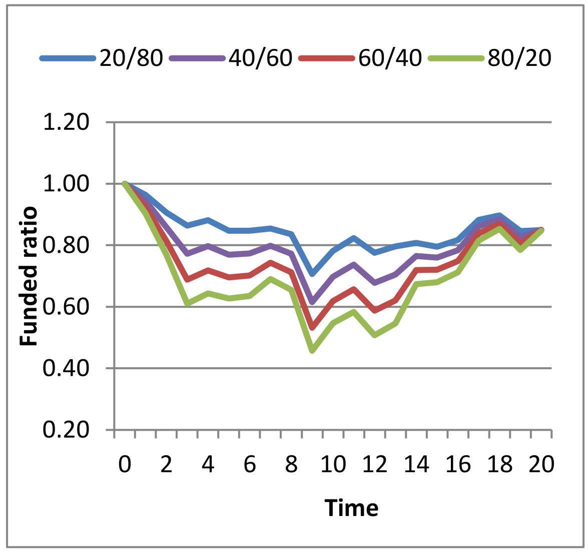
Suppose the plan were required to fund for a PfAD in order to maintain a minimum funded ratio of 0.85 at 1/1/2020. The required levels of PfAD would be as follows: 2% for a 20/80 mix, 7% for a 40/60 mix, 11% for a 60/40 mix and 14% for an 80/20 mix.[10] The riskier the investment strategy, the higher the PfAD level required to achieve the same target funded ratio.

Below is a summary of the results for various asset mixes with PfADs at the indicated levels.

Table 6: Statistics on funded ratios

	Asset mix			
	20/80	**40/60**	**60/40**	**80/20**
PfAD	2%	7%	11%	14%
FR @ 1/1/2020	0.85	0.85	0.85	0.85
Mean	0.85	0.79	0.74	0.69
Standard deviation	0.06	0.09	0.11	0.14
Minimum	0.71	0.62	0.53	0.46
Maximum	1.00	1.00	1.00	1.00

Figure 6: Evolution of funded ratios



Discussion

Funding requirements should be aligned with the nature of the benefit promises. A regulatory funding framework should require stronger funding for pension plans that offer firm guarantees for benefits. In general, the framework should have some security mechanism in place, either through the inclusion of a margin in the funding requirements (e.g., PfAD) or the operation of a pension protection fund (e.g., the United States' Pension Benefit Guaranty Corporation or Ontario's Pension Benefits Guarantee Fund), designed to offer the level of security required by the strength of the benefit guarantees and the regulatory objectives intended by the legislation.

If a PfAD is included in the funding requirements, the size of the PfAD should reflect the level of risks inherent in the investment policy adopted by a plan. A higher level of PfAD should be required if a plan pursues a riskier investment strategy. Note that this is not a countermeasure that fully compensates for the additional risk taken by the plan. Rather, it helps to alleviate any adverse funding impact caused by risky investments; see Table 6 and Figure 6 above. It is also worthy of note that a PfAD does not necessarily prevent a plan's funded ratio from falling to very low levels under adverse conditions.

Key takeaway

The going-concern funding requirement is generally determined using a discount rate based on the expected return on plan assets. Increasing the riskiness of the investment strategy would immediately improve the funding position of the plan and reduce the contribution requirements in the short term.

[10] The PfAD percentages would be held constant throughout the projection period and would be applied to the liabilities calculated using the BDRs. For comparison, the required PfADs under Ontario regulations are 5%, 6%, 8% and 12%, respectively.

However, the risk that the plan might not be able to accumulate sufficient assets to finance its benefit obligations could increase, as the above modeling results show.

In developing a regulatory funding framework, government policy makers should be careful not to create incentives for pension plans to increase risk-taking. Such incentives could be detrimental to the long-term financial health of pension plans.

5. Plans with Non-Stationary Membership

Ontario regulations require a plan that is closed to new members to fund for a higher level of PfAD than an open plan, presumably on the grounds that such a plan would pose a greater funding risk. The modeling described in Section 3 looks at a plan with a steady stream of new members to keep the membership profile stable. In this section, we will extend the model in respect of a non-indexed DB plan to fit in with different membership profiles – *growing*, *stable* or *declining* – and provide a demonstration of its application to plans with growing and declining membership.

5.1 Extended Model

In developing the model formulas, we continue to use the notation introduced in Section 3. The expected liabilities at time $t+1$ were denoted as $\widetilde{L_{t+1,t}}$ therein. We will suppress the $\sim$ symbol above $L_{t+1,t}$, as we only consider a non-indexed DB plan in the model development hereunder.

Where there is a change in the membership profile over a valuation period $(t, t+1)$, we postulate that the accrued liabilities, calculated based on the discount rate i_t, would change according to a function $h(t)$ as follows:

$$L_{t+1,t} = h(t)L_{t,t}$$

The function $h(t)$ has the following defining properties:

- If the plan is young and its membership is growing, $h(t) = 1 + \mu_t$, where $\mu_t > 0$.
- If the plan has become matured with a stationary membership, $h(t) = 1$.
- If the plan is mature and closed, or is in a state where there are fewer new members to keep the membership profile stable, $h(t) = \frac{1}{1+\mu_t}$, where $\mu_t > 0$.

As noted in Section 3, the expected liabilities $L_{t+1,t}$ can be derived from the following formula:

$$L_{t+1,t} = L_{t,t}(1 + i_t) + (NC_t - B_t)(1 + i_t)^{1/2}$$

Combining this with the preceding formula, we obtain the following modified version of Equation (1):

$$NC_t - B_t = \frac{L_{t,t}(h(t) - 1 - i_t)}{(1 + i_t)^{1/2}} \tag{13}$$

Furthermore, Equations (6) and (10) can be rewritten, respectively, as follows:

$$F_{t+1} = \left(\alpha_t(1+r_t) + (h(t) - 1 - i_t)\left(\frac{1+r_t}{1+i_t}\right)^{\frac{1}{2}} + \left(\frac{max(0, 1 + PfAD_t - \alpha_t)}{n}\right)(1+r_t)^{\frac{1}{2}} \right) L_{t,t} \tag{14}$$

$$\alpha_{t+1} = \frac{F_{t+1}}{L_{t+1,t+1}} = \left(\alpha_t(1+r_t) + (h(t) - 1 - i_t)\left(\frac{1+r_t}{1+i_t}\right)^{\frac{1}{2}} + \left(\frac{max(0, 1 + PfAD_t - \alpha_t)}{n}\right)(1+r_t)^{\frac{1}{2}} \right)\left(\frac{1}{(1+\delta_{t+1})h(t)}\right) \tag{15}$$

The factor δ_{t+1} in Equation (15) is estimated using the following formula (Society of Actuaries/CIA, 2017):

$$\delta_{t+1} = \frac{L_{t+1,t+1}}{L_{t+1,t}} - 1 = exp\left[-(18 - 10.5p_{t+1}) \times (i_{t+1} - i_t) \times \left(1 - 8\left(\frac{i_t + i_{t+1}}{2} - 5.25\%\right)\right)\right] - 1 \tag{16}$$

where p_{t+1} is a variable representing the proportion of liabilities related to pensions in pay at time $t + 1$.

Starting with an initial funded ratio α_0 at time 0, we can apply Equation (15) to find future funded ratios α_t , $t = 1, 2, \ldots$

We now apply the extended model to two non-indexed DB plans: one with a *declining* membership and the other with a *growing* membership, over the 20-year period 2000–2019 inclusive. As in Subsection 4.3, we consider four (4) target asset mixes: 20/80 mix, 40/60 mix, 60/40 mix and 80/20 mix. We first project the funding outcomes in respect of the case where there is no PfAD included in the funding requirements, and then estimate the amount of PfAD required, under each investment policy, to attain a target funded ratio at the end of the projection period (1/1/2020).

5.2 Plan with a Declining Membership

We assume that the plan is closed to new members and the parameter μ_t in the function $h(t) = \frac{1}{1+\mu_t}$ has a constant value of 0.0175 for every t. Given this value, the initial liabilities of the plan at time 0 would decrease, as a result of the decline in membership, by approximately 30% after 20 years and 50% after 40 years. At the same time, we assume the proportion of liabilities related to pensions in pay would increase at the rate of 1.75% per year. Starting with $p_0 = 0.5$ at time 0, p_t would increase to approximately 0.7 after 20 years and to 1.0 after 40 years (at that time, the plan would be comprised solely of members with pensions in pay).

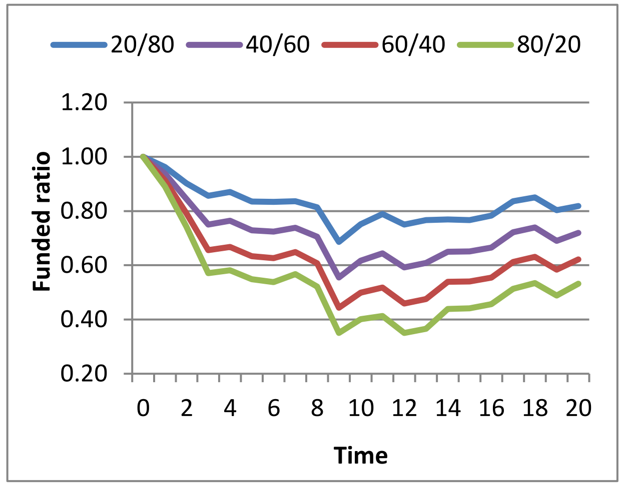
Funding without a PfAD

A summary of the projection results is given below.

Table 7: Statistics on funded ratios

	Asset mix			
	20/80	**40/60**	**60/40**	**80/20**
FR @ 1/1/2020	0.82	0.72	0.62	0.53
Mean	0.82	0.72	0.62	0.54
Standard deviation	0.07	0.11	0.14	0.17
Minimum	0.69	0.55	0.44	0.35
Maximum	1.00	1.00	1.00	1.00

Figure 7: Evolution of funded ratios



It can be observed that the projected funded ratio of the plan at 1/1/2020 and the average over the projection period, under various asset mixes, are both lower than the corresponding funded ratios pertaining to an open plan as shown in Table 5. The riskier the investment strategy, the larger the difference in the funding level between the two types of plans. An intuitive explanation of the observed funding pattern for the closed plan goes as follows. The unamortized part of the losses from economic sources in early years would become a larger proportion of the falling liabilities in later years, as the plan continues to operate with no new members. This would thus push down the funded ratio of the plan.

Funding with a PfAD

As in Subsection 4.3, we estimate what levels of PfAD would allow the plan to reach a target funded ratio of 0.85 at 1/1/2020. The required levels of PfAD are as follows: 2.5% for 20/80 mix, 8.5% for 40/60 mix, 13.5% for 60/40 mix and 18% for 80/20 mix.[11] The corresponding PfADs for an open plan are 2%, 7%, 11% and 14%, respectively; see Table 6.

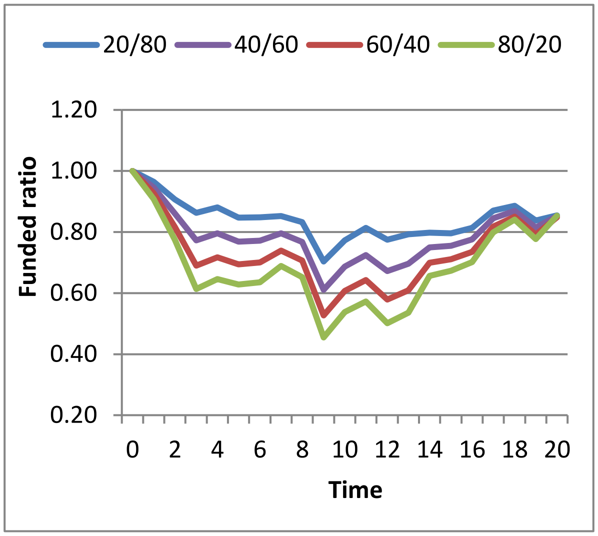
Below is a summary of the results for various asset mixes with PfADs at the indicated levels.

[11] Under the Ontario regulations, the required PfADs for a closed plan are 7%, 9%, 12% and 20%, respectively.

Table 8: Statistics on funded ratios

	Asset mix			
	20/80	**40/60**	**60/40**	**80/20**
PfAD	2.5%	8.5%	13.5%	18.0%
FR @ 1/1/2020	0.85	0.85	0.85	0.85
Mean	0.84	0.79	0.73	0.69
Standard deviation	0.07	0.09	0.12	0.14
Minimum	0.70	0.61	0.53	0.45
Maximum	1.00	1.00	1.00	1.00

Figure 8: Evolution of funded ratios



5.3 Plan with a Growing Membership

For the plan with a growing membership, we assume the parameter μ_t in the function $h(t) = 1 + \mu_t$ has a value of 0.0175 for every t. Given this value, the initial liabilities of the plan at time 0 would increase, as a result of the membership growth, by approximately 40% after 20 years. We further assume the initial proportion of liabilities related to members with pensions in pay (p_0) has a value of 0.35, and the growth rate of this proportion is 1.75% per year. The proportion p_t would increase to 0.5 after 20 years.

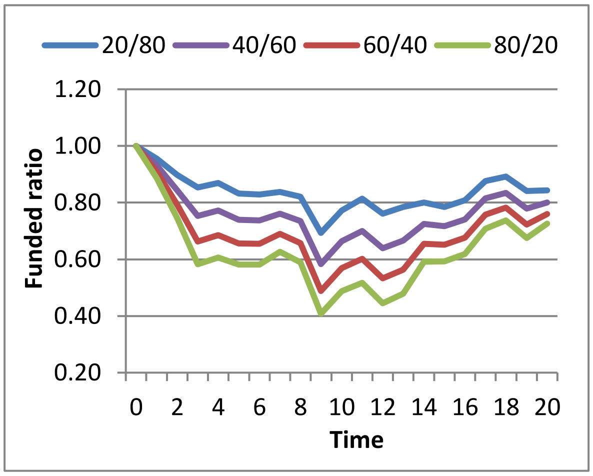
Funding without a PfAD

A summary of the projection results is given below.

Table 9: Statistics on funded ratios

	20/80	**40/60**	**60/40**	**80/20**
FR @ 1/1/2020	0.84	0.80	0.76	0.73
Mean	0.84	0.76	0.69	0.63
Standard deviation	0.07	0.09	0.12	0.14
Minimum	0.69	0.58	0.49	0.41
Maximum	1.00	1.00	1.00	1.00

Figure 9: Evolution of funded ratios



Note that the projected funded ratio of the plan at 1/1/2020 and the average over the projection period, under various asset mixes, are all lower than 1.0. However, they are higher than the corresponding funded ratios pertaining to a plan with either a stable or declining membership; see Tables 5 and 7. This suggests that a plan with a growing membership has a greater risk-bearing capacity than plans with the other two membership profiles.

Funding with a PfAD

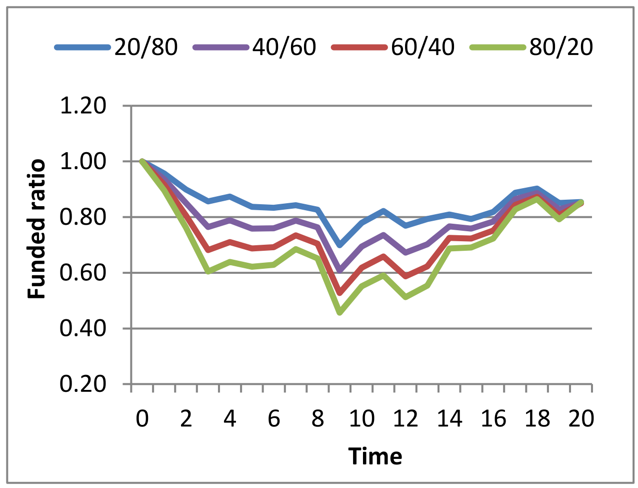
We estimate what levels of PfAD would allow the plan to reach a target funded ratio of 0.85 at 1/1/2020. The required levels of PfAD are as follows: 1% for 20/80 mix, 4.5% for 40/60 mix, 7.5% for 60/40 mix, and 10% for 80/20 mix. They are lower than the corresponding PfADs required for a plan with either a stable or declining membership profile.

Below is a summary of the results for various asset mixes with PfADs at the indicated levels.

Table 10: Statistics on funded ratios

	20/80	**40/60**	**60/40**	**80/20**
PfAD	1.0%	4.5%	7.5%	10.0%
FR @ 1/1/2020	0.85	0.85	0.85	0.85
Mean	0.84	0.79	0.74	0.69
Standard deviation	0.07	0.09	0.11	0.14
Minimum	0.70	0.61	0.53	0.46
Maximum	1.00	1.00	1.00	1.00

Figure 10: Evolution of funded ratios



5.4 Key Takeaways

Based on the above modeling results, we offer the following points of advice for government or plan policy makers:

1. Given an investment policy, a plan with a declining membership would pose a greater funding risk than a plan with a stable or growing membership. If a PfAD is used as a risk mitigation tool, it is justifiable for government policy makers to require a closed plan to fund for a higher level of PfAD than an open plan.
2. Plans with a declining membership profile have less risk-bearing capacity than plans with a stable or growing membership profile. If a plan is closed to new members, it should depend less on risky assets to meeting its funding obligations. The plan's policy makers should take reasonable steps to de-risk its investment strategy into assets matching the demographic profile of the plan as its membership runs off over time.

6. Areas for Further Investigation

As demonstrated in Sections 4 and 5, it is relatively straightforward to apply the mathematical model developed in this paper to assessing the long-term funding impacts associated with funding policy changes. The model may prove to be a convenient tool for government or plan policy makers as they deliberate on the design of a funding framework to balance the interests of various stakeholders.

We propose further investigation into the following aspects:

1. Apply the model to a stochastic economic environment (e.g., the one generated by an economic scenario generator) to gain insights into the uncertain nature of pension funding dynamics. There is no funding policy, regardless of how well it is designed, that will ensure benefits promised by a plan will always be fulfilled. In a voluntary employment-based pension system such as that of Canada, employers fund their plans on a best-effort basis. There is always a risk that members may not receive their promised benefits, unless their earned benefits are fully guaranteed by a third party (e.g., government).
2. Calculate a risk-based funding buffer (i.e., a PfAD) to meeting a stated long-term funding goal. For example, we can choose a time horizon of risk assessment and use the model to project the plan assets and liabilities under a given investment policy. We can then determine what level of PfAD will be required to meeting a target funded ratio at the end of the projection period, at a certain level of confidence. A riskier investment strategy will result in a higher level of PfAD. On the other hand, a higher confidence level will require a larger funding buffer.
3. Define reasonable metrics for assessing intergenerational equity and use the model to quantify the impacts of funding measures based on those metrics. Funding measures include the ones recommended by CAPSA (CAPSA, 2019). They also include the valuation methodology with which to calculate a minimum funding requirement. One of the most important factors is the discount rate used to measure the benefit obligations. The choice of the discount rate lies between the two extremes of the risk-free rate and the expected return on plan assets. It affects the extent to which risks are distributed among different generations of stakeholders[12] and therefore the perception of fairness.

[12] Stakeholders are taxpayers (or ratepayers) and/or members in the case of public-sector plans. In the case of private-sector plans, stakeholders are employer sponsors and/or members.

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Appendix A: Selected Economic Data

Below is an extract of the economic and investment data used for the modeling exercises in Sections 4 and 5. The figures in the last column of the table are annual bond yields in percent; other figures in the table are annual percentage rates of change/return.

Source:	CIA Report on Canadian Economic Statistics 1924–2019					Bank of Canada
	Table 1A			Table 3B		
YEAR	CONSUMER PRICE INDEX	CANADA COMMON STOCKS	US COMMON STOCKS IN CANADIAN $	FEDERAL BONDS	CORPORATE BONDS	Annualized (V122544) B14072
2000	3.20	7.41	-5.57	13.64	11.60	6.37
2001	0.72	-12.57	-6.40	3.92	6.08	5.80
2002	3.80	-12.44	-22.84	10.09	10.48	5.76
2003	2.08	26.72	5.76	8.06	14.86	5.57
2004	2.13	14.48	2.80	8.46	8.37	5.30
2005	2.09	24.13	1.51	15.05	10.65	4.80
2006	1.67	17.26	16.03	3.22	4.22	4.24
2007	2.38	9.83	-10.27	3.30	6.55	4.26
2008	1.16	-33.00	-22.59	13.65	-13.03	4.23
2009	1.32	35.05	9.12	-4.26	22.56	3.75
2010	2.35	17.61	8.89	11.45	15.10	4.00
2011	2.30	-8.71	4.41	18.79	15.89	3.79
2012	0.83	7.19	13.48	4.55	7.67	2.66
2013	1.24	12.99	41.53	-8.56	-5.02	2.59
2014	1.47	10.55	24.00	15.47	17.55	2.96
2015	1.61	-8.32	20.95	4.82	1.17	1.94
2016	1.50	21.08	8.62	-0.78	6.63	2.06
2017	1.87	9.10	13.83	3.54	7.90	2.47
2018	1.99	-8.89	3.98	2.59	0.13	2.37
2019	2.25	22.88	25.18	8.80	15.09	2.18
2020						1.46

Appendix B: Projection Results

For illustration, we provide detailed projection results in respect of the following funding option:

- Target asset mix: 60% equities/40% fixed income
- Amortization rule: 10 years; straight line and fresh-start
- PfAD design: as per Ontario regulations

The entries in the heading row of the table on the next page are explained below.

BDR	Ontario benchmark discount rate
ROR	Rate of fund return
ΔL	Change in liabilities due to change in BDR
F	Value of fund assets at beginning of year
L	Value of liabilities at beginning of year
$NC - B$	Calculated using Equation (1) in Section 3; amount payable at mid-year
$PfAD$	Ontario PfAD; 8% for 60/40 asset mix
UL	Unfunded liability at beginning of year
SP	Special payments payable at mid-year
FR	Funded ratio at beginning of year
Check FR	Check funded ratio value using Equation (10)

Initial funded status

Liabilities = 100

Assets = 100

Val Date	Time	BDR	ROR	ΔL	F	L	$NC-B$	$PfAD$	UL	SP	FR	Check FR
Reference				Eq. (8)	Eq. (5)	Eq. (9)	Eq. (1)		Eq. (3)	$= UL/10$	$= F/L$	Eq. (10)
2000 Jan	0	10.37%	5.60%		100.00	100.00	-9.87	8.00%	8.00	0.80	1.00	
2001 Jan	1	9.80%	-3.69%	4.53%	96.28	104.53	-9.78	8.00%	16.61	1.66	0.92	0.92109315
2002 Jan	2	9.76%	-6.47%	0.33%	84.76	104.88	-9.77	8.00%	28.51	2.85	0.81	0.80816973
2003 Jan	3	9.57%	14.33%	1.62%	72.58	106.58	-9.74	8.00%	42.52	4.25	0.68	0.68101188
2004 Jan	4	9.30%	8.55%	2.29%	77.12	109.02	-9.70	8.00%	40.63	4.06	0.71	0.70734194
2005 Jan	5	8.80%	12.83%	4.56%	77.84	113.99	-9.61	8.00%	45.27	4.53	0.68	0.68285232
2006 Jan	6	8.24%	11.48%	5.33%	82.43	120.08	-9.51	8.00%	47.25	4.73	0.69	0.68645993
2007 Jan	7	8.26%	1.84%	-0.20%	86.83	119.84	-9.52	8.00%	42.59	4.26	0.72	0.72456797
2008 Jan	8	8.23%	-16.55%	0.30%	83.12	120.19	-9.51	8.00%	46.69	4.67	0.69	0.69155003
2009 Jan	9	7.75%	16.91%	4.88%	64.94	126.07	-9.42	8.00%	71.22	7.12	0.52	0.51509355
2010 Jan	10	8.00%	13.26%	-2.43%	73.44	123.00	-9.47	8.00%	59.40	5.94	0.60	0.59705549
2011 Jan	11	7.79%	5.65%	2.18%	79.42	125.67	-9.42	8.00%	56.31	5.63	0.63	0.6319455
2012 Jan	12	6.66%	8.64%	12.87%	80.01	141.85	-9.14	8.00%	73.20	7.32	0.56	0.56399421
2013 Jan	13	6.59%	13.64%	0.81%	85.02	143.00	-9.12	8.00%	69.42	6.94	0.59	0.59453924
2014 Jan	14	6.96%	16.97%	-4.11%	94.29	137.12	-9.23	8.00%	53.80	5.38	0.69	0.68765674
2015 Jan	15	5.94%	4.99%	12.50%	106.13	154.27	-8.90	8.00%	60.48	6.05	0.69	0.68796879
2016 Jan	16	6.06%	10.08%	-1.44%	108.50	152.04	-8.95	8.00%	55.70	5.57	0.71	0.71362523
2017 Jan	17	6.47%	9.16%	-4.63%	115.90	145.00	-9.09	8.00%	40.71	4.07	0.80	0.79925958
2018 Jan	18	6.37%	-0.93%	1.06%	121.28	146.54	-9.06	8.00%	36.98	3.70	0.83	0.82761107
2019 Jan	19	6.18%	19.20%	2.27%	114.82	149.87	-8.99	8.00%	47.04	4.70	0.77	0.76611436
2020 Jan	20	5.46%		9.24%	132.18	163.73	-8.70	8.00%	44.65	4.46	0.81	0.80731464