EDUCATIONAL NOTE

Educational notes do not constitute standards of practice. They are intended to assist actuaries in applying standards of practice in specific matters. Responsibility for the manner of application of standards in specific circumstances remains that of the practitioner.

EXPECTED MORTALITY: FULLY UNDERWRITTEN CANADIAN INDIVIDUAL LIFE INSURANCE POLICIES

COMMITTEE ON LIFE INSURANCE FINANCIAL REPORTING

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MEMORANDUM

TO: All Fellows, Associates and Correspondents of the Canadian Institute of Actuaries

DATE: July 8, 2002

FROM: Jacques Tremblay, Chairperson
Committee on Life Insurance Financial Reporting (CLIFR)

SUBJECT: **Educational Note on Setting Expected Mortality Assumption in CGAAP Life Insurance Valuation**

The Committee on Life Insurance Financial Reporting has developed the attached educational note on setting expected mortality assumptions for individual life insurance policies. It provides assistance in setting those assumptions for Canadian GAAP valuations, and speaks to matters covered by Sections 7.1.2, 7.2.1 and 7.2.5 of the Standards of Practice for the Valuation of Policy Liabilities of Life Insurers and to Sections 2350.05 and 2350.06 of the draft Consolidated Standards of Practice – Practice-Specific Standards for Insurers. This note applies to Canadian individual life insurance business that is fully underwritten. However, many of the concepts covered in this note will be useful to actuaries in establishing mortality assumptions for other types of business. It is expected that further notes (or amendments to this note) will be developed with respect to other life and annuity business.

The note exposes general principles and processes applicable to determining an expected mortality assumption. It also provides a practical overview on how to apply credibility criteria and blend industry and company/block specific experience data to construct an expected mortality assumption.

In accordance with the Institute's policy for Due Process, this Educational Note on Setting Expected Mortality Assumption in CGAAP Life Insurance Valuation has been approved by the Committee on Life Insurance Financial Reporting and has received final approval for distribution by the Practice Standards Council.

Educational notes will be covered under Section 1220 of the Consolidated Standards of Practice (CSOP) when it comes into effect. Although CSOP will only come into effect as of December 1, 2002 or such later date as of which the practice-specific standards applicable to the practice area concerned are adopted, and will, therefore, only apply from that date forward, in the opinion of CLIFR and the PSC, the substance of Section 1220 appropriately describes the status of this educational note as of the date of its publication.

Section 1220 prescribes that "The actuary should be familiar with relevant educational notes and other designated educational material."

It further explains that a “practice which the notes describe for a situation is not necessarily the only accepted practice for that situation and is not necessarily accepted actuarial practice for a different situation,” and that the “educational notes are intended to illustrate the application (but not necessarily the only application) of the standards, so there should be no conflict between them.”

We would like to thank the members of the working group who were primarily responsible for the development of this note: Barry Senensky, Wendy Harrison, Chris Denys, Scott McGaire, Jason Wiebe, and Micheline Dionne.

Questions should be addressed to me at my *Yearbook* address.

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100 INTRODUCTION

110 SCOPE

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3. Section 1220 prescribes that "The actuary should be familiar with relevant educational notes and other designated educational material."

4. It further explains that a "practice which the notes describe for a situation is not necessarily the only accepted practice for that situation and is not necessarily accepted actuarial practice for a different situation," and that the "educational notes are intended to illustrate the application (but not necessarily the only application) of the standards, so there should be no conflict between them."

5. This note concerns expected mortality for individual life insurance policies. It provides guidance in setting expected mortality assumptions for Canadian GAAP valuations, and is a supplement to Sections 7.1.2, 7.2.1 and 7.2.5 of the *Standards of Practice for the Valuation of Policy Liabilities of Life Insurers* and to Sections 2350.05 and 2350.06 of the *Consolidated Standards of Practice – Practice-Specific Standards for Insurers*.

6. This note restricts its guidance to Canadian individual life insurance business that is fully underwritten. However, many of the concepts covered in this note will be useful to actuaries in establishing mortality assumptions for other types of business.

120 DEFINITIONS

1. Besides ordinary dictionary meanings, when used in this note, "Mortality Table" means a table (or a set of tables) that reflect(s) the total mortality experience of a defined cohort of lives.[1]

2. "Simultaneous Deaths" means the occurrence of a second death within six months of the first death and as a result of the same event.

3. "Credible" means that the data is statistically reliable.

4. "Homogeneous" means of uniform structure or composition throughout.

130 GENERAL METHODOLOGY

1. Methods used in deriving an expected mortality assumption for valuation would have the following characteristics:

   a) the assumption derived is appropriate in aggregate for either the entire company, or the particular block of business;

   b) all relevant and material data is used, incorporating relevant component variation of mortality rates (e.g. sex, age, smoking status, amount, blood testing, duration, etc);

   c) the method results in an assumption that is unbiased; and

   d) the assumption derived is based on data that is homogeneous within each relevant class.

2. Where little credible experience is available (e.g., preferred underwriting situations), see section 610.

3. Where experience is 100% credible, companies may create mortality tables based on their own data[2]. The rest of this note would likely still be relevant in this situation.

THE REST OF THIS EDUCATIONAL NOTE FOCUSSES ON ASSISTING THE ACTUARY IN SETTING THE VALUATION EXPECTED MORTALITY ASSUMPTION WHEN CREDIBLE EXPERIENCE IS AVAILABLE, BUT IT IS NOT FEASIBLE TO CREATE A COMPANY TABLE.

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[1] A mortality table can be constructed to reflect as many variables as are needed. For instance, the CIA8692 table is made up of six tables covering gender and smoking status for standard underwritten Canadian Industry business.

[2] The mechanics of developing a mortality table based solely on company experience is outside the scope of this note.

200 ASSEMBLE DATA

Relevant mortality data consists of company and inter-company experience studies. Additional sources of data might be assembled with these two types of studies in developing expected mortality assumptions.

210 COMPANY EXPERIENCE

1. A company's own mortality experience for a particular block of business is usually the most relevant source of data. Often, company experience studies show mortality ratios for various time periods, and cross-sections of business, relative to an industry or internal table.

2. When a block of business is reinsured, the profile of the net retained business may be different than that of the gross or direct block. Therefore, it may be more appropriate to consider the net retained block when setting the valuation assumption.

220 INTER-COMPANY EXPERIENCE

1. Inter-company experience studies examine large volumes of insurance business taken across the industry. An industry study provides credible results of insured population mortality. The disadvantage is that the distribution of business may not parallel that of the business block a company is valuing. Whenever possible, the actuary would choose studies that closely resemble the company's business. If none exist, other available data may guide the actuary.

2. Both the CIA and SOA publish industry experience studies. Studies are also available on industry web sites, or in publications. (For example, web sites list the SOA large amount and older age studies; and actuarial organizations in the UK, Australia, and South Africa make mortality studies available. Publications include the *North American Actuarial Journal*, and the *Product Development News*. Other mortality research is provided on a fee-for-service basis by private firms.)

3. Care would be taken in using non-adjusted mortality data from other countries in developing Canadian expected valuation mortality assumptions. There are differences in population mortality, underwriting standards, socio-economic environment and product structures. The actuary would place less reliance on non-Canadian data especially when there is uncertainty about a table's derivation or underlying data.

230 OTHER SOURCES OF DATA

1. Other sources of information (besides company or industry experience) may be considered if available. This is particularly true if business is new or unique, or there is insufficient credible experience data.

2. One source of data is government or private sector population studies, examining large volumes of population experience. Insurance experience can differ substantially when considering the effects of underwriting, geographic location, and choice of market. Care would be taken in the use of population studies. However, population studies can isolate trends in population mortality, cigar usage, etc. Population mortality studies can be used to fill the gaps when insured mortality studies are not available.

3. A second source of data is medical studies, which have previously been instrumental in developing finer mortality classifications (e.g., the smoker/non smoker classifications introduced 20 years ago). These studies can be useful in understanding how levels of underwriting affect mortality experience.

4. There are other studies done by private organizations, reinsurers or actuarial organizations that may be available.

300 PREPARE DATA

310 COMPANY EXPERIENCE

Data Definition

1. The actuary would review available data, and scrutinise its applicability to the business being valued. Studies would be reviewed, giving attention to data sources and handling, assumptions, and methodology for developing results. The last step of the review includes assessing adjustments to the data to better reflect the business being valued. Such adjustments might apply different weightings, or adjust for mortality trends (improvement or deterioration) to update results to the valuation date. (See section 330 for further details.)

2. Other considerations include the need to:

   a) Identify information in order to define homogenous experience blocks. If data is incomplete, pinpointing causes for experience changes becomes extremely difficult. For instance, if relaxed underwriting programs are not separated from normally underwritten policies, there will be problems identifying sources for subsequent deterioration in mortality.

   b) Ensure relevant information is captured by the administration system. When considering new system designs, ensure essential information is captured. Older systems may not have all the required information. Since administrative practices may have invalidated some information, the actuary would check the existence of any "system workaround" procedures with the Policy Administration Department. The actuary would need to understand what fields contain needed information for the study. Ideally, field coding would be sampled to confirm. Additionally, caution would be taken with any system conversions, which create breaks in experience results.

   c) Ensure administrative procedures use the system consistently. If the administration department does not clearly understand the appropriate transaction for situations, the information may be corrupt. These problems sometimes are undetected. Underwriting and Cause of Death data are especially susceptible to miscoding given that it tends to be more informational than transactional.

   d) Document changes in underwriting methodology or class definitions as they occur. If changes are captured by the system, but not well understood at time of the experience study, insights may be lost.

Data Validation

1. Review the extract specifications with knowledgeable systems people.

2. Summarize data, and validate it against other sources (e.g. Are death benefits paid consistent with financial statements? Is the mix of business by size, underwriting class, etc. consistent with sales statistics?)

3. Review study results for reasonableness against past studies, as well as intuitive tests (e.g. non-smokers are expected to have better mortality experience than smokers).

4. Where inconsistencies in the data can be clearly identified, the data would be adjusted. The problem blocks of experience would be excluded from the study to remove any study bias if solutions to the inconsistencies are not evident, and results would be materially affected.

320 INTER-COMPANY EXPERIENCE

1. Normally, users of inter-company experience do not directly validate data. Any user validation focuses on the applicability of the inter-company study results to the company (e.g. if your company's business is all medically underwritten, then you would use inter-company results for medically underwritten business.).

2. In ensuring the data is appropriate, the actuary would review the methodology used in the study, and consider weightings adjustments of inter-company experience to more closely match the distribution of the company business[3]. Weighting by number of claims or by amount of claims is the usual practice.

330 ADJUSTING RESULTS TO VALUATION DATE

1. The data that emerges from both industry and company experience studies is often several years old at the valuation date. Study results could be adjusted to reflect trends in mortality (i.e., mortality improvement or deterioration) as discussed further below.

2. The current life standards of practice prohibit the application of mortality improvement after the valuation date. However, the actuary may apply improvement or deterioration to a table from observed mortality experience to that expected at the valuation date. With respect to such updates, the actuary would make an assumption about improvement (or deterioration) between the date of the observed experience (usually the mid-point of the experience study) and the valuation date. Current data on improvement may not be available.

[3] The actuary should be careful in making these adjustments. As an example, there is a school of thought that lowering premium rates for the better risks (and keeping them static for all others) will result in overall lower mortality by amount as most people buy an amount of insurance they can afford, rather than a fixed amount of insurance.

3. If appropriate to the circumstance of the company, the actuary may use historical trends to extrapolate mortality improvement up to the valuation date. However care would be taken using this methodology. The actuary would ensure that the data used in performing this analysis is homogeneous. Changes in business mix, or change in underwriting practices such as blood testing levels could give a false perception of mortality improvement, where no improvement actually exists. For this reason, the actuary would use caution in trending overall mortality ratios of recent Canadian Industry experience to the CIA8692 table.

4. The actuary would ensure that the data used to calculate mortality trends is credible. Reference to industry or population mortality trends may be appropriate even for large companies.

5. If homogeneous industry or company data cannot be obtained, the actuary may be able to get reasonable estimates of average mortality trends by reviewing population mortality. Population mortality tends to be based on a population of individual lives, whose composition changes slowly over time and therefore is reasonably homogeneous.

6. The actuary would consider whether apparent mortality improvement might have resulted from anomalies or inconsistencies with the benchmark mortality table. (For example, as the business ages, and in the absence of any mortality improvement, a benchmark mortality table with an unjustifiably steep slope automatically produces mortality ratios that decrease over time.)

7. The actuary would consider adjustments for recent known events that may affect mortality trends, but are not part of the trend data. An example of this is AIDS.

340 OTHER ADJUSTMENTS

The actuary might consider other adjustments to the data. Those include removing the anti-selective mortality present in the current experience (see section 620) and adjusting for any deficiencies in handling joint data in the construction of experience results (see section 630).

400 DETERMINE DIFFERENTIATION

410 GENERAL CONSIDERATIONS

1. The actuary would select factors for differentiating the mortality assumption (e.g. male/female, smoker/non-smoker). This selection process may be iterative as the desired differentiation may not be supported by available data (e.g., data is not split by the desired factor(s), or data post-split is not sufficiently credible). Section 420 outlines potential sources of differentiation.

2. The actuary's challenge is to determine predictive factors in differentiating mortality, and choose a subset that balances credibility and accuracy. A key decision involves the number, and identification of factors to use in differentiating the valuation mortality assumption.

3. To the extent that it makes a material difference to the policy liability, the actuary would not make the same assumption for two policies unless he expects their experience to be similar.

4. Differentiation choices can materially alter both current and projected levels of policy liabilities. In determining differentiation, the actuary would consider the following:

   a) the credibility of the information (exercise caution in differentiation if policy liabilities are sensitive, but credibility of the data supporting differentiation is low);

   b) the differentiation would make intuitive sense (the actuary would be able to explain the connection between factors and mortality results);

   c) the behaviour of differentiation over time (consider whether the effects wear off, remain level, or increase); and

   d) the correlation between factors (in crossing two or more factors, the possibility of double counting may lead to incorrect conclusions; for example, when the number of dimensions used in developing factors exceeds those for which raw mortality is available, the possibility of missing correlations between factors exists).

420 POTENTIAL FACTORS

1. Current industry mortality tables differentiate mortality by four basic factors: age, sex, smoking status, and duration. Large companies with sufficient experience to develop their own internal tables tend also to split their mortality assumptions by at least these four basic factors.

2. The annual CIA study of industry experience, relative to industry tables, can be used to assess how actual industry mortality has evolved, relative to the expected table over time. While mortality experience in aggregate may have improved since the construction of the table, the actuary would consider that the extent of improvement may differ among the four basic factors.

3. The actuary would also consider factors beyond these four basic factors, including but not necessarily limited to experience by face amount, type of underwriting, preferred risk classification and product type.

4. The annual CIA Mortality Study analyzes observed mortality by face amount band, and is a good source of information. The actuary would carefully interpret study results to adjust for the impact of inflation and underwriting changes over time.[4] The annual CIA experience study also provides mortality results split by type of underwriting - medical, paramedical, and non-medical. However, since levels of underwriting are largely driven by age and face amount, the correlation with these factors cannot be overlooked (to avoid double counting).

5. Section 610 provides background on considerations concerning preferred risk classification. Currently, no comprehensive public industry study exists. Studies that estimate protective value of enhanced testing can be used to estimate impacts of more rigorous preferred testing.

6. The annual CIA experience study provides mortality ratios split by product type. Some differentiation can be found where average face amounts of the product types differ. In addition, other product specific factors include those that impact policyholder anti-selection and overall lapse rates as well as the policyholder’s purpose for insurance.

7. Depending on the circumstances, the actuary may consider differentiating by other factors such as distribution type and geography. Reinsurers may consider differentiating by ceding company.

[4] Other studies are available to assess mortality by face amount – for example, the SOA large amount study (conducted on policies with face amounts of $1 million or more), and the SOA 90-95 experience study (which suggests face amount adjustment factors).

500 BLEND CREDIBLE DATA

510 OVERVIEW

1. The available company and industry data, suitably prepared and segmented, can now be blended using credibility weightings. This section discusses criteria for a good credibility method and summarizes several credibility methods. Appendices 2 and 3 provide additional guidance and examples for selecting a method, as well as discussion of each method's advantages and disadvantages.

2. The goal of credibility theory is to provide a framework for combining data from different sets of observations. These may be prior and current observations, industry and company mortality rates, or other sets. For the purpose of this section, we will consider the two sets to consist of:

   a) company data, which may not be fully credible; and

   b) industry mortality tables or data, which are assumed to be fully credible.[5]

3. The actuary needs to select an assumption for expected mortality taking into account the sample information, as well as the underlying statistical distribution.

4. The Normalized Method is the preferred credibility method and 3,007 is the suggested number of deaths needed for full credibility.

520 CRITERIA FOR A GOOD CREDIBILITY METHOD

The following are desirable characteristics of a "good" credibility method:

- the method is practical to apply;

- the sum of expected claims for the within-company sub-categories is equal to the total company expected claims;[6]

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[5] The basic assumption underlying the traditional use of credibility theory to set the valuation expected mortality assumption is that the industry mortality tables have 100% credibility. This assumption, however, may not hold if the actuary uses the industry data in its finer level of detail e.g. by gender, smoking status and year or duration. The actuary should review the amount of industry experience underlying the published data before attributing 100% credibility to the data.

[6] Please refer to the subsection entitled *Application of LFCT to Sub-categories of Business* in section 540.

- all of the relevant information is used;
- the results are reasonable in extreme or limiting cases; and
- the sub-category A/E ratios are reasonable relative to company and industry data (e.g. they fall within the range of corresponding industry and company experience A/E ratios).

530 TYPES OF CREDIBILITY THEORY

1. There are two major types of credibility theory: Limited Fluctuation Credibility Theory (“LFCT”), and Greatest Accuracy Credibility Theory (“GACT”)

2. While several approaches are discussed in this section and in Appendices 2 and 3, the Normalized Method, which is a type of LFCT, best meets the criteria for a good credibility method.

540 LIMITED FLUCTUATION CREDIBILITY THEORY[7]

1. Limited Fluctuation Credibility Theory (“LFCT”) provides a criterion for full credibility based on the size of the portfolio. Full credibility means it is appropriate to use only the portfolio’s own experience and to ignore the entire industry data.

2. In addition, LFCT provides an ad hoc methodology for the determination of partial credibility, where there is a weighting of the portfolio’s own experience and the industry experience.

3. The expected assumption for the aggregate amount of claims for a company for a year may be expressed as:

$$X_E = Z\overline{X} + (1 - Z)\mu$$

where

- X_E is the expected aggregate amount of claims,
- Z is the credibility factor, or weighting given to sample data,
- $\overline{X}$ is the mean and is calculated from the company experience data $\mathbf{X}=\{X_1, X_2, \ldots, X_n\}$,
- μ is the mean of the underlying distribution and is equal to the expected number or amount of death claims (or ratio of actual to expected death claims), based on the industry data for the same portfolio, and
- n is the number of years of data.

[7] Limited Fluctuation Credibility Theory is also known as “American Credibility”

In practice, the credibility factor is usually applied to the applicable ratio of actual to expected mortality (A/E), rather than to expected claims.

While this weighted average credibility formula has intuitive appeal, LFCT does not provide an underlying theoretical model for distribution of the X_i's which is consistent with the formula.

In LFCT, one calculates X_E by selecting a range parameter r (r > 0) and a probability level p (0<p<1) such that the difference between X_E and its mean μ is small.

4. The criterion can be written as

$$\Pr\{|X-\mu| \le -r\lambda\} \ge p$$

where r is the error margin, and p is the confidence level. Parameter values of p = 90% and r = 3% are interpreted as a 90% probability of being correct within a 3% margin of error.

5. In other words, X_E is a good estimate of future expected mortality if the difference between X_E and its mean μ is small relative to μ with high probability.

Poisson Model

1. Although the theoretical distribution for mortality is binomial, when the probabilities of the event (death, represented by the random variable X in the above formulas) are small, the Poisson distribution provides a reasonable approximation to a binomial distribution.

2. In the simple Poisson model, the only random variable is the number of claims, which is assumed to be Poisson.[8] Variations in claim size are ignored. If there is significant dispersion in the net amount at risk for each policy in the block under consideration, the use of a simple Poisson model may be inappropriate. The Compound Poisson model incorporates the effect of variation in claim size, and would normally result in a higher threshold of claims needed to reach the same credibility level. The Compound Poisson model is discussed in Appendices 1 and 2.

3. Parameter values p = 90% and r = 5% are frequently cited as the minimum levels required for full credibility; however, there is no theoretical basis for determining these parameter values. When setting the expected mortality assumption for valuation purposes, one may want to use a higher threshold for full credibility, such as p = 90% and r = 3%. These parameters were the subject of many discussions within the Task Force and within CLIFR. The consensus was that a minimum of 3007 deaths would be recommended for 100% credibility. We expect that

[8] See Loss Models: From Data to Decisions Example 5.20 or Introductory Credibility Theory Example 3.2.2.

this issue will be revisited periodically as new literature and research emerges in this area. The actuary would justify the use of parameters p and r different than p = 90% and r = 3% for valuation purposes.

4. For p = 90% and r = 3%, the factor for partial credibility is defined by $Z = \min\left\{\sqrt{\dfrac{n}{3{,}007}}, 1\right\}$

   Where n = number of claims in experience data and 3007 is taken from the standard normal table.[9]

Number of claims	30	120	271	481	752	1083	1473	1924	2436	3007
Z	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00

5. The parameters defined in Step 4 above are suggested for use in most situations. A significant dispersion in net amount at risk in the inforce block will increase volatility and could result in the need to use a higher number of deaths.

6. The use of the blending methodology set out in this section assumes that there exists relevant industry basis for blending. If there is no industry table or study that corresponds to the company's business mix, then it may be appropriate to assign a higher credibility factor to the company data than otherwise.

7. The number of claims needed for full credibility under other values of p and r are set out in the Standard Normal Table in Appendix 2.

8. The Poisson application can be extended to include data from more than one period or year. However, the number of years would be limited so that the mix and material risk characteristics of the portfolio are homogeneous over time.

Application of LFCT to Sub-categories of Business

1. If the actuary wants to reflect experience split by sub-category (perhaps by sex, product, or duration) but the experience in those sub-categories is not 100% credible, the actuary would decide either to use the overall credibility factor, or the lower credibility for the amount of experience in that sub-category.

2. One can pool disparate distributions within the aggregate data under certain conditions. Several approaches are discussed in Appendix 2. The Normalized Method is summarized below.

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[9] The credibility factors set out in the previous CIA standard VTP 6 Expected Mortality Experience for Individual Insurance were based on LFCT using a simple Poisson distribution. The factors incorporate a conservative bias that depended, in part, upon whether industry or company data is better. Therefore the credibility factors in VTP 6 are different than those obtained using the above formula. Since the objective is to select the expected valuation assumption, a conservative bias is not appropriate.

550 NORMALIZED METHOD – LFCT

1. The “Normalized Method” uses the credibility and the A/E ratios of the subcategories. However, the blended A/E ratios are adjusted to reproduce the expected claims level generated by the total company blended A/E ratio. The total expected claims for the company is the same as that produced using total company credibility, but the A/E ratios of the subcategories are used to allocate these deaths to the subcategories.

2. The Normalized Method has the following strengths:

   - The sum of expected claims for the sub-categories matches total expected claims, based on a blended A/E ratio, calculated at the total company level (i.e., the number of sub-categories selected does not affect the overall result.)
   - All of the information is used: both total company and sub-category A/E ratios and credibility factors.
   - The results are reasonable in extreme or limiting cases.
   - The sub-category A/E ratios fall within the original range (or very close to the range).
   - Interactive effects between sub-categories may be captured.
   - It is simple to apply in practice.

Normalized Method

Step 1: Calculate the A/E mortality ratios and credibility factors for the total company (or block) and for each of the subcategories.

Step 2: Calculate total company blended expected mortality ratio and corresponding expected claims using the total company credibility factor and total company mortality ratio from Step 1.

Step 3: Calculate total company blended expected mortality ratio and corresponding expected claims using the credibility factors and the A/E mortality ratios from the subcategories.

Step 4: Adjust or “normalize” the A/E ratios of the subcategories by the ratio of the total expected claims from Step 2 to the total expected claims from Step 3.

Although this method does not have a strong theoretical base, it is pragmatic, and satisfies the criteria for a good credibility method.

The following example sets out the Normalized Method using a simple Poisson model for claims. The compound Poisson model could also be used. Additional description of the Poisson and compound Poisson models are set out in Appendices 1 and 2.

Step 1 is completed in the followed table:

MORTALITY EXPERIENCE DATA						
	MORTALITY RATIOS					
Industry Data	**Male**	**Female**	**Total**			
Medical	71.0%	75.0%	71.9%			
Non-Medical	84.0%	83.0%	83.8%			
Para-Medical	73.0%	85.0%	74.3%			
Total	**74.5%**	**78.7%**	**75.32%**			
	MORTALITY RATIOS			**NUMBER OF CLAIMS**		
Company Data	**Male**	**Female**	**Total**	**Male**	**Female**	**Total**
Medical	59.0%	47.0%	56.2%	63.8	15.4	79.2
Non-Medical	85.9%	90.1%	86.9%	43.7	14.5	58.2
Para-Medical	75.0%	101.2%	77.8%	54.0	8.6	62.6
Total	**69.9%**	**67.1%**	**69.3%**	**161.5**	**38.5**	**200.0**
	COMPANY EXPECTED CLAIMS ASSUMING INDUSTRY MORTALITY AT 100%			**CREDIBILITY FACTORS WITH p = 90% and r = 3%**		
Company Data	**Male**	**Female**	**Total**	**Male**	**Female**	**Total**
Medical	108.1	32.8	140.9	.15	.07	.16
Non-Medical	50.9	16.1	67.0	.12	.07	.14
Para-Medical	72.0	8.5	80.5	.13	.05	.14
Total	**231.0**	**57.4**	**288.4**	**.23**	**.11**	**.26**

The credibility factor for the total company is .26 (calculated as $\min\left\{\sqrt{\frac{200}{3{,}007}}, 1\right\}$), where 200 is the total number of claims for the company, and 3,007 is the factor from the normal table corresponding to p = .9 and r = .03.

Step 2: Calculate the total company blended expected mortality ratio and corresponding expected claims using the total company credibility factor and total company mortality ratio from the above table. The blended expected mortality ratio is 73.8% (calculated as .26 × 69.3% + .74 × 75.32%). The corresponding expected claims are 212.8 (calculated as 73.8% blended mortality ratio × 288.4 total expected claims using industry mortality table at 100%).

Step 3: Calculate the expected number of claims for the total company, using the claims and credibility of the sub-categories.

	BLENDED EXPECTED MORTALITY RATIOS – Sub-category Credibility			EXPECTED NUMBER OF CLAIMS		
	Male	**Female**	**Total**	**Male**	**Female**	**Total**
Medical	69.3%	73.0%	70.1%	74.9	23.9	98.8
Non-Medical	84.2%	83.5%	84.0%	42.8	13.5	56.2
Para-Medical	73.3%	85.9%	74.6%	52.8	7.3	60.1
Total	**73.8%**	**77.8%**	**74.6%**	**170.4**	**44.7**	**215.1**

Here, the ratio for each sub-category is the weighted average of the company and industry ratios; for example, the male medical expected ratio = $0.15 \times 59.0\% + (1-0.15) \times 71.0\% = 69.3\%$. Further, the total medical ratio is 70.1%, calculated as follows. First, the expected number of claims is calculated for each sub-category; for example, male medical = $69.3\% \times 108.1 = 74.9$. Second, the total expected number of medical claims is the sum of the expected claims for each sub-category = 74.9 + 23.9 = 98.8. Finally, the total medical ratio = $98.8 \div 140.9 = 70.1\%$, where 140.9 is the expected number of medical claims at 100% of industry data.

Step 4: "Normalize" the A/E ratios from Step 3 by multiplying them by the ratio of the total expected claims from Step 2 to the total expected claims from Step 3.

The A/E ratios and corresponding expected number of claims by sub-category under the Normalized Method are set out in the following table:

	BLENDED EXPECTED MORTALITY RATIOS – Normalized Method			EXPECTED NUMBER OF CLAIMS		
	Male	**Female**	**Total**	**Male**	**Female**	**Total**
Medical	68.5%	72.2%	69.4%	74.0	23.7	97.7
Non-Medical	83.3%	82.6%	83.0%	42.4	13.3	55.7
Para-Medical	72.5%	84.9%	73.8%	52.2	7.2	59.4
Total	73.0%	77.0%	73.8%	168.6	44.2	**212.8**

Here the expected mortality ratio for medically underwritten male lives is 68.5%, which is equal to the blended ratio from Step 3 multiplied by the ratio of expected claims from Step 2 to that from Step 3 (calculated as $69.3\% \times 212.8 \div 215.1 = 68.5\%$).

The Normalized Method allows for the calculation of credibility factors by sub-category, but then produces the same number of expected claims in total for the company as if there was only one aggregate category.

The use of the blending methodology set out in this section assumes that there exists relevant industry basis for blending. If there is no industry table or study that corresponds to the Company business mix, then it may be appropriate to assign a higher credibility factor to the company data than otherwise.

560 BUHLMANN OR GREATEST ACCURACY CREDIBILITY THEORY

1. The Greatest Accuracy Credibility Theory (GACT) or "European credibility" is based on work by Buhlmann. GACT has a better theoretical basis than LFCT, and ensures that results are "balanced", so normalization is obviated. Greatest Accuracy Credibility Theory allows one to estimate within and between sub-category sources of variation.

2. GACT is theoretically complete, and meets the criteria for a credibility method with one shortcoming. That shortcoming is that additional information about industry experience (beyond what is customarily collected and published) is required. Without these practical difficulties, GACT would likely be the preferred credibility method to use in determining the expected valuation mortality assumption.

3. There are several versions of GACT. The simpler Buhlmann model, and the slightly more complex Buhlmann-Straub model are outlined in Appendix 3.

570 SUMMARY

1. From a theoretical point, the GACT method is preferable since it is theoretically complete. However, current industry data is not sufficiently detailed to support the use of GACT.

2. The Normalized Method, a variant of LFCT, is then the favoured approach. Except for the theoretical shortcomings, the Normalized Method meets all criteria for a good credibility method.

3. 3,007 is the recommended number of deaths needed for full credibility. Dispersion in net amount at risk and the absence of credible industry data are two significant factors that would be considered when determining the number of deaths needed for full credibility.

580 Sources of Information

For a more detailed explanation of the credibility theory, please refer to Appendices 1, 2 and 3, and to the sources listed below:

- "Loss Models: From Data to Decisions" text by Klugman, Willmot and Panjer published by John Wiley and Sons
- "Introductory Credibility Theory" by Gordon E. Willmot published by IIPR.
- "A Credibility Approach to Mortality Risk" by Mary R. Hardy and Harry H Panjer published by IIPR
- "Introduction to Credibility Theory" by Thomas N. Herzog

600 OTHER ADJUSTMENTS

Following the steps outlined in sections 200 through 500 results in a base mortality table. Other adjustments can now be made to reflect factors anticipated to influence the mortality experience. Some of the adjustments outlined below are specific to a particular segment of the business.

610 NEW UNDERWRITING TECHNIQUES

Overview

1. Changes to underwriting techniques and testing thresholds are occurring all the time (for example, blood and urine testing have recently become more prevalent at lower amounts of insurance). Until sufficient credible insurance experience accumulates, the actuary would have to use estimates for the impact that these changes may have on mortality experience.

2. The actuary utilizes knowledge of mortality levels based on the old underwriting basis, together with the anticipated impacts of changes on the underwritten population, to derive the new mortality table. One method used to reflect improvements in underwriting involves the following formula:

   $Q(NEW) = Q(OLD) \times [1 - A - B - C \times (A + B)] \div (1 - A - B)$

   Where Q(NEW), Q(OLD), A, B, and C are defined as :

   Q(NEW) : the new mortality rate anticipated due to the underwriting changes.

   Q(OLD) : the old mortality rate, or current mortality, based on the old underwriting approach. If industry mortality has been used in developing these rates, the actuary would adjust for existing differences between the company and industry underwriting to avoid double counting.

   A: the impairment frequency, or frequency that the underwriting technique will screen otherwise undetectable medical impairments. Medical laboratories often can estimate how frequently their tests detect impairments. If possible, the actuary would examine the company's own data on blocks where the test has been applied.

   B: the sentinel frequency, or frequency that prospects with those impairments will avoid the company because of the underwriting change. This is difficult to estimate since by definition it represents a segment of the insured population that the company is not tracking. The risk is likely to occur whenever the prospective life insured is aware of a significant impairment, such as HIV or cocaine use, and has alternatives for obtaining insurance without screening. The nature and sophistication of the distribution system will also significantly impact this factor.

   C: the additional mortality, or average amount of increased mortality that can be expected to occur in the impaired group defined by A and B. Estimates can often be obtained by discussion with the underwriting area and/or the medical director. The actuary would carefully examine the evidence supporting this assumption.

3. Application of this formula would consider the following:

   - **Variations by Age:** The mortality Q, the frequency of the impairment, and the average excess mortality represented by the impairment can be expected to vary by issue age. The calculation would be split into several age groups and interpolated.

   - **Variations by Duration Since Issue:** In the absence of reliable experience, the actuary could reasonably assume that mortality differentials would disappear over time with perhaps some residual differentials remaining.

   - **Multiple Underwriting Technique Changes:** If the actuary is reviewing more than one change in underwriting technique at a time, care would be taken if the impairments uncovered by the techniques are not independent. Any correlation would be accounted for in setting the factor values.

   - **Reliance on Assumptions:** A number of key assumptions of this formula are difficult to estimate with confidence. The actuary would consider the credibility of these assumptions, particularly if they have a material impact on policy liabilities.

4. Underwriting changes do not always improve mortality. In some cases, companies may remove a requirement to simplify the underwriting process, or save costs. This formula can be utilized in reverse to address these situations.

Preferred Underwriting/ Changes to Underwriting Classifications

1. The primary challenge in developing mortality assumptions for new underwriting classifications, such as “preferred” is the time needed for credible experience to develop. Industry experience studies may not exist and, even if they do, company differences in class criteria may jeopardize the applicability of results to any one company.

2. The lack of credible homogenous data does not diminish the importance of industry experience in evaluating company preferred mortality experience.

3. Even in the presence of credible early duration experience split by class, the actuary may still need to estimate the impact of the new underwriting classes on mortality over time. It is reasonable to assume that mortality rates for preferred and non-preferred risks would revert over time towards overall standard regular underwriting mortality rates.

4. In the absence of reliable and relevant experience, the actuary would review the length of time that effects of preferred underwriting persist. In these circumstances, it would be reasonable to assume that the effects of preferred underwriting wear off over the select period, that the effects of preferred underwriting wear off linearly between the last duration for which the insurer has reliable experience, and the duration at which the effects are expected to completely wear off.

5. At time of writing, the United States was the only potential source of preferred experience data with sufficient credibility and duration to evaluate longer term impacts[10]. If the actuary has access to this data, reviews of this experience for valuing Canadian business would consider differences by product, underwriting criteria, testing requirements, population mortality, or frequency of impairments. Mortality results from such sources will rarely be applicable without adjustment. Specifically, U.S. experience reflects changes in underwriting practices as well as the introduction of preferred products and therefore would not be applied in Canada without modification.

6. In the absence of any credible and relevant mortality data for the preferred class, the actuary may use approaches similar to that outlined for new underwriting techniques. The actuary may calculate the mortality for the preferred class as if he/she were introducing tougher underwriting requirements or new underwriting techniques. Both situations involve estimating the impact of removing a segment of the insured population or class on the mortality of the remaining population or class. "B" in the formula below can be determined as [Q(OLD) – Q(NEW)] ÷ Q(OLD).

7. A formula for splitting one class (say NS Standard) into two (NS Preferred, NS Residual) by applying tougher underwriting standards (eg. blood pressure) is as follows:

   $Q(\text{NS Preferred}) = Q(\text{NS Standard}) \times (1 - B)$

   $Q(\text{NS Residual}) = Q(\text{NS Standard}) \times (1 - A + B \times A) \div (1 - A)$

   Where Q(NS Preferred), Q(NS Standard), A, and B are defined as:

   Q(NS Preferred): is the preferred mortality rate, or mortality anticipated for applicants who qualify under the tougher underwriting standard.

   Q(NS Standard): is the standard mortality rate, or mortality currently experienced for the aggregate standard class undifferentiated by the underwriting criteria.

   A: is the preferred proportion, or frequency that a normally standard client will be accepted for preferred classification due to the tightened requirements. This can be a difficult task since most companies often do not store lab test results for future analysis. Medical directors, lab testing companies, and reinsurers can be valuable resources in determining estimates. If available, the actuary would examine the company's own data.

   B: the mortality differential for the preferred class relative to the old standard class. Estimates for this figure can often be obtained by discussions with the underwriting area and/or the medical director. The actuary would carefully examine the evidence

[10] Even then, the data is company specific with no publicly available industry studies. Furthermore, experience by duration will also reflect trends in mortality that may be more due to different generations of product, market, and underwriting conditions than to the ageing of the preferred underwriting effect.

supporting this assumption. The preferred underwriting class will normally be defined by more than one underwriting criterion. The estimates for A and B can be developed with the aggregate impacts of the various criteria in mind. The actuary would reflect any correlation between the health criteria in the determination of these assumptions. If the underwriting criteria are independent, qualifying percentages and mortality ratios can be developed for each independently, and then multiplied to obtain the final result. A thorough review of underwriting files can assist the development of these assumptions.

8. When dealing with multiple underwriting classes, repeat the procedure as many times as required. Start with the class having the strictest underwriting requirements, and successively refine each class in each further step.

9. If a new underwriting technique is added at the same time new classifications are developed, the change in overall mortality due to the new underwriting technique would first be quantified, before examining the relationship of preferred to standard mortality.

10. There are various considerations for applying this formula in practice:

    - **Variations by Age:** The additional mortality and qualifying percentages can be expected to vary by issue age, so the calculation would be split into several age groups and interpolated.

    - **Variations by Duration:** It is reasonable to assume that effects of preferred underwriting wear off over the select period.

    - **"Reverse Sentinel Effect":** Competitors' criteria for preferred classes may differ, so one company may lose the best risks of its classes to its competitors, and another may gain. The net result may concentrate poorer risks in each of its classes, but it is difficult to estimate. The importance of examining actual to expected mortality experience as credible experience develops becomes even more important if a company's underwriting classes differ considerably from the industry.

    - **Reliance on Assumptions:** Key assumptions are difficult to estimate with confidence. The actuary would consider how much confidence he/she has in the assumptions, particularly if they materially impact policy liabilities. Where confidence does not exist, the actuary could value all new risk classifications that comprise the original standard classification using one aggregate mortality assumption, or increase the level of MFADs.

    - **Reinsurance:** Care would be taken in using the reinsurer's premium rates as a proxy for the valuation expected mortality assumption. Despite the insurer substituting their mortality cost for a fixed rate basis, an independent assessment of the underlying mortality may be required. A reinsurer's rates are usually simple multiples of a standard industry table, for ease of sale and comparison. Actual experience on new risk classifications will likely vary by age and duration.

Example: Suppose a company is simultaneously introducing tougher underwriting testing and splitting standard non-smoker classes into two, based on a set of underwriting criteria. Suppose further that:

- the condition detected by the new underwriting tests is present in 2% of insurance applicants on average
- the company is late relative to industry in adding this test to its requirements
- it is estimated that there are an additional 1% of applicants with the condition that seek insurance because of the company's weaker underwriting regime
- the mortality experience used as a base reflects the additional mortality costs associated with this discontinuity
- the mortality of these applicants is 500% of the normal standard underwritten case (so the <u>additional</u> mortality is 400%)
- the current duration 1 mortality rate assumed for a female non-smoker aged 60 is $1/1000.

To determine the new aggregate mortality rate once the test is implemented:

Qnew = $1/1000 x (1 - .02 - .01 - (.02+.01)*400%)/(1-.02-0.01)

= $1/1000 x .85/.97 = $0.88/1000

Next, assume the company splits the new non-smoker class into two, based on a set of qualifying criteria they assume will separate risks into the top 40% and residual 60%. The preferred risks are anticipated to have 15% lower mortality than anticipated in the otherwise aggregate class. In this case, the preferred and residual class mortality can be calculated as:

Qnew (preferred) = $0.88/1000 x (1 - 0.15) = $0.748/1000

Qnew (residual) = $0.88/1000 x (1 - .4 + .15 x .4) / (1 - .4) = $0.968/1000

As a check on the results, the actuary can perform a test to show that the new assumptions produce the same overall mortality as the old assumptions.

[Qnew (preferred)] x .4 + [Qnew (residual)] x (1 - .4) = .748/1000 x .4 + .968/1000 x .6 = $0.88/1000

620 SELECTIVE LAPSATION

1. Selective lapses are defined as lapses whose mortality experience would be identical to that of newly selected lives.

2. The actuary would consider effects of selective lapsation when setting the expected mortality assumption even though selective lapsation is difficult to observe (as the health of lapsed policyholders is unknown). Selective lapsation is typically modelled as an explicit adjustment to the base/expected mortality.

3. Lapse rates on renewable term insurance products can be expected to show a temporary increase when premium rates rise at a renewal date. In general, healthy lives are more likely to lapse their policies at renewal than unhealthy lives, the net effect being a deterioration in mortality for the remaining lives.

4. Policies with low ultimate lapse rates such as Term to 100 and Level Cost of Insurance Universal Life may exhibit "reverse selective lapsation" as more healthy lives than average persist.

5. The following factors would be considered when determining the Selective Lapse Rate assumption(s):

    - **Size of Premium Rate Increase:** Large increases are more likely to result in higher selective lapses.
    - **Period Between Premium Increases:** Selective lapse rates are likely to be higher if the period between increases is longer.
    - **Duration:** Selective lapse rates are likely to be lower at higher durations for the same attained age. For instance, the selective lapse rate at age 45 for a policy issued at age 25 will probably be less than the rate at age 45 for a policy issued at age 35.
    - **Policy Size:** Larger polices are likely to experience higher selective lapse rates.
    - **The Distribution System Used:** High replacement activity and/or operation in an upscale market may lead to higher selective lapse rates.
    - **Heaped Renewal Commissions:** Higher commissions at premium renewal dates are likely to result in lower selective lapse rates.
    - **External Market Conditions**: At the time of renewal, if lower cost alternatives are available, more healthy clients will consider leaving the block.
    - **Proportion of Healthy Lives Remaining:** In the extreme, if no healthy lives remain, the selective lapse rate will be zero. Similarly the selective lapse rate would not be greater than the proportion of lives, just before renewal, who are still healthy.
    - **Conversion Activity:** High conversion rates at later durations may improve mortality for the remaining lives.

6. Selective lapses may occur at times other than at renewal. For example, high replacement activity may be an indication healthy lives are leaving for cheaper alternatives, even on policies other than renewable term. If lapse rates are high on a product, some of those lapses would be assumed to be selective lapses.

7. The actuary would examine the degree to which the effects of selective lapsation (or "reverse selective lapsation" in the case of products such as term to 100 and level cost of insurance universal life) are already included in the mortality experience data. Specific considerations include the type of business underlying the mortality experience data, the level of lapse rates, and the pattern of mortality ratios by duration. The actuary may consider adjusting the selective lapse rates, and the expected mortality assumption, to recognize that there may be lag time for some policyholders between the date the premium increases, and the date some healthy lives will lapse.

8. See Appendix 4 for detailed formulae to determine mortality rates for remaining lives, assuming no effects of selective lapsation are in the underlying mortality assumption. To use these formulae, the actuary would adjust mortality experience data to remove the effects of selective lapsation. Alternatively, if satisfied the mortality experience data fully reflects effects of selective lapsation, the actuary can use this data as the base for the expected mortality assumption without adjustment for selective lapsation.

630 MULTIPLE LIFE POLICIES

Approximation Methods

1. Determining the mortality assumption for policies with multiple lives can be complex and calculation intensive, especially if death benefits are paid on the last death. The Equivalent Single Age (ESA) and Joint Equal Age (JEA) are two common approximation approaches that are used to reduce this calculation effort. Unfortunately, both may generate a mortality curve significantly different from actual joint mortality calculated from first principles. The approximations in these cases would at best be appropriate only for a short period, and diverge from the exact result over time.

2. The actuary would calculate expected mortality using multiple life contingencies, with exact ages, and gender information, whenever possible.

Equivalent Single Age (ESA)

3. Under the ESA approach, the joint mortality is approximated using the mortality of a single age that would have roughly the same present value of death benefits. A set of rules is defined for converting from the actual ages of the joint lives to an ESA.

4. Single life mortality has a very different slope than joint mortality. For joint last survivor ("JLS"), the mortality rates under an ESA approach are significantly higher in early durations than exact mortality developed via first principles. At later durations, the relationship reverses, and JLS mortality is higher than the ESA mortality. An ESA calculated at issue will understate the policy liabilities beyond the first duration.

5. The actuary could improve the approximation by recalculating the ESA at each valuation date, but this requires complete knowledge of each life making up the ESA, which may not be practical. Alternatively, the actuary could estimate a set of factors for each future valuation date to apply to the ESA policy liabilities to produce a more accurate joint policy liability by first examining the ratio of the joint policy liability to the ESA policy liability for various ages, sexes, and smoking statuses.

   The following example helps to illustrate the size of this difference:

Policy type:	Joint Last Survivor
Life 1:	Male NS, Age 45
Life 2:	Female NS, Age 40
Mortality:	86.5% 86-92 CIA, Age nearest birthday
Interest rate:	6%
Equivalent Single Age:	Male NS, Age 30

	Present Value of Future Death Benefits		
Duration	Joint	ESA	Difference
0	.0671	.0676	.0005
20	.2130	.1881	-.0249
40	.5620	.4573	-.1047

6. The opposite relationship occurs with joint first to die (“JFS”) mortality. Early duration ESA mortality rates are lower than actual mortality rates, calculated from first principles, while later duration mortality rates are higher.

Joint Equal Age (JEA)

7. Under the JEA approach, mortality rates are approximated using joint mortality of the same number of multiple lives, with a single age and underwriting class. The JEA is selected to approximate the same present value of death benefits. Rules are defined for converting from actual ages to the JEA.

8. The JEA approach is superior to the ESA since the mortality slope better matches an exact age approach. However, the actuary would ensure the present value of future mortality using the approximation is still appropriate.

Mortality Studies Involving Joint Lives

9. Mortality studies on joint life business are rarely credible. The actuary would ensure that any mortality studies performed on joint life policies are conducted, and interpreted, correctly. The following issues would be considered:

- **First Death Reporting:** The most accurate method to study mortality on multiple life policies is to compare deaths on each life. This is relatively easy on JFS policies, since the reporting of deaths is the same as for single lives. However, this approach may be impractical for JLS policies if material numbers of first deaths are not reported.

- **Choice of Expected Mortality:** The choice of expected mortality for a study of multiple life policies presents unique challenges. The lack of any multiple life industry studies necessitates the use of single life mortality tables. The actuary would ensure that the table selection is appropriate. For example, the actuary might choose the expected mortality for single lives that most closely matches the average underwriting characteristics for multiple lives, as multiple life policies may be larger on average.

- **Incidence of Substandard Lives:** Since a significant number of JLS policies are issued with one life substandard, JLS policies generally have a higher incidence of substandard lives than the single life portfolio. Some companies adjust ESAs rather than apply a rating, which may make tracking substandard experience difficult.

- **Credibility:** Refining data into credible subgroups is more difficult for JLS than for single life business. The early duration credibility for JLS business is significantly lower than a similarly sized block of single life policies due to the low probability of claim. So, larger in-force blocks are needed relative to single life policies. In addition, the number of policy combinations grows exponentially.

- **Use of Approximations:** The actuary would exercise caution when using an expected table developed using the ESA or the JEA method. For example, the expected basis for a JLS block of business calculated using an ESA approach will show very favourable early duration expected claim experience. However, this expected claim experience will deteriorate for later durations.

- **Application of Mortality Improvements:** The actuary would use caution in any application of single life mortality improvement factors to JLS claim experience.

Simultaneous Deaths on JLS Policies

For two totally unrelated lives with no regular interaction in their day to day lives, the probability of simultaneous death is remote. However, people who have a reason to buy a JLS life insurance policy will often have regular interactions increasing the risk of simultaneous death. If this risk is not considered, the mortality assumption may be understated.

Suggested Readings

The following is a list of articles published by the SOA on the topic of multiple life mortality:

- The Actuary, January 1994, Jack Bragg, Jack Luff and Bob Vose;
- Last Survivor Insurance Antiselection – SOA Product Development News, February 1994, Craig W. Reynolds;
- Second-to-Die with Possibility of Simultaneous Death – SOA Product Development News, June 1994, Harry H. Panjer

640 AIDS

1. When the AIDS epidemic first emerged, there was no data on the effect of AIDS on insured life mortality. The CIA promulgated a general theoretical methodology for reflecting the level of AIDS mortality in the policy liabilities for individual life insurance. This general methodology used an AIDS model based on population mortality.

2. The population mortality was to be adjusted to represent insured mortality using a number of factors. These factors are set out later in this section.

3. While the factors are still relevant, it is important to recognize the degree to which AIDS mortality is already included in the experience data. An explicit AIDS provision is no longer required if the actuary considers that AIDS claims are fully included in the experience. When determining the extent to which AIDS is included in the experience, the actuary would consider the following:

   - AIDS claims as a percent of total claims for own company experience relative to comparable industry experience or population experience;
   - the degree to which AIDS deaths are reflected in experience may vary by issue date and issue age, since AIDS has emerged relatively recently;
   - target markets; and
   - historical underwriting standards and testing limits.

4. In addition, the actuary would consider medical changes with respect to the treatment of AIDS and the impact that these changes will likely have on mortality experience.

5. To the extent that the actuary believes that AIDS is not fully included in the experience data, the actuary would adjust the expected mortality, taking into account the following factors:

- **Assumed Level of Ownership:** A lower portion of the at-risk group than the overall population will own individual life insurance. The minimum recommended assumed proportion for pre-1984 extra AIDS mortality was 40%. This proportion would be updated to the current valuation year assuming selective lapsation.

- **Assumed parameters of the AIDS Epidemic:** The AIDS models described in the CIA guidance notes reflect a number of assumed parameters, including the pattern of future infections, the incubation time, the development of clinical AIDS to death, and the distribution of AIDS cases in the population by age.

- **Effect of HIV testing:** At many life insurance companies, specific testing for HIV was introduced in the late 1980's or early 1990's. For some companies, the threshold for testing was reduced at a later date. These testing levels may be important to consider when reviewing company experience.

- **Effect of Selective Lapsation:** It is reasonable to assume that people who know they have AIDS will be unlikely to surrender their policies. This assumption may also apply for those who are HIV positive and to a lesser degree for those in a high-risk group. The selective lapsation methodology set out elsewhere in this note could be applied.

- **Geographic Differences:** The incidence of AIDS may vary by territory and within territory by region.

- **Company Characteristics:** Different companies may have different AIDS experience, depending upon the target market (urban versus rural), age and sex distribution, and underwriting limits.

APPENDIX 1 – PROBABILITY AND STATISTICAL CONCEPTS

PROBABILITY CONCEPTS

Poisson Distribution

The number of claims of a portfolio of business may be described as a Poisson model.

If X and Y are independent Poisson random variables with Poisson parameters a and b respectively, then

E[X] = Var[X] = a,
E[Y] = Var[Y] = b, and
W = X + Y is also Poisson with parameter c = a + b (and E[W] = Var[W] = c = a + b).

We will refer to such a distribution as an aggregate Poisson distribution.

Also, if we know W is a Poisson random variable with parameter c, then we know that W may be decomposed into 2 or more Poisson random variables with respective Poisson parameters summing to c.

These are the addition and decomposition properties of the Poisson model.

Although the theoretical distribution for mortality is binomial, when the probabilities of the event (death) are small, Poisson is a reasonable approximation to binomial.

Compound Poisson Distribution

The total claims of a portfolio of business can be described as a compound Poisson model, which reflects both the number and amount of claims.

Let N be a random variable representing the number of claims from an insurance company and assume that N has a Poisson distribution with mean and variance λ. The observed number of claims is n.

For k = 1,2,3,…n, let Y_k be the random variable representing the amount of the kth claim.[11]

Assume that Y_k's are independent and have a distribution with mean μ_y and variance σ_y^2.

Assume that the number of claims N is independent of the claim size Y_k.

The total claims $X = Y_1 + Y_2 + Y_3 + \ldots + Y_N$ has a compound Poisson distribution.

[11] The "random" claim size comes in this way: given that a claim from the portfolio occurs, what are the probabilities of the various possible amounts that this claim can be? The answer is: for any amount, it is the sum of the mortality rates for all policies of that amount, all divided by the sum of the mortality rates for all policies in the portfolio. The mean and the variance of this distribution can be easily calculated for a portfolio from the amounts and mortality rates for the policies in the portfolio.

Using conditional expectation on N, it can be shown that

$$E[X] = \mu = \lambda\mu_y \text{, and}$$
$$Var[X_i] = \sigma^2 = \lambda\left(\mu_y^2 + \sigma_y^2\right)$$

In summary, X_i has a compound Poisson distribution with Poisson parameter λ and claims size distribution with mean μ_y and variance σ_y^2.

Estimators

Consider a portfolio of n life insurance policies numbered 1, 2, 3, …, n with corresponding one-year mortality rates q_1, q_2, q_3, …, q_n and corresponding net-amounts-at-risk (net of reinsurance and policy liabilities) b_1, b_2, b_3, …, b_n.

For a one-year period, the mean and standard deviation of the total number of deaths and aggregate death claims are:

	Numbers of Deaths	Aggregate Death Claims
Expected :	$\lambda = \sum_{i=1}^{n} q_i$	$\mu = \sum_{i=1}^{n} q_i b_i$
Std Deviation:	$\sqrt{\sum_{i=1}^{n} q_i(1-q_i)}$	$\sqrt{\sum_{i=1}^{n} q_i(1-q_i)b_i^2}$

For large values of n, the distribution of the numbers of claims and the aggregated claims are approximately normally distributed with the above means and standard deviations.

The Poisson distribution with mean λ can also be used as an approximation to the number of deaths. The standard deviation of the Poisson distribution is $\sqrt{\lambda}$ which is slightly larger than the true standard deviation given above.

The corresponding compound Poisson distribution can be used to approximate the distribution of aggregate claims. It has mean μ and standard deviation $\sqrt{\sum_{i=1}^{n} q_i b_i^2}$ which is slightly larger than the true standard deviation given above.

STATISTICAL CONCEPTS

Summary Statistics

Define the mean amount of aggregate claims per year as

$$\overline{X} = \frac{\sum_{i=1}^{m} X_i}{m}$$

where m represents the number of years of experience for the company.

Then [12]

$E\left[\overline{X}\right] = \mu = \lambda\mu_y$ and $Var\left[\overline{X}\right] = \sigma^2 = \lambda(\mu_y{}^2 + \sigma_y{}^2)/m$

Central Limit Theorem

According to the Central Limit Theorem, if the amount of experience is "large", then the random variable

$$\frac{\left(\overline{X} - x\right)}{\sqrt{Var\left(\overline{X}\right)}}$$

is approximately distributed as a normal random variable with mean zero and standard deviation one (*x* is the true value of X).

[12] See Introductory Credibility Theory Example 2.2.3.

APPENDIX 2 – LIMITED FLUCTUATION CREDIBILITY THEORY[13]

1. Limited Fluctuation Credibility Theory (LFCT) provides a criterion for full credibility based on the size of the portfolio. Full credibility means it may be appropriate to use only the portfolio's own experience and to ignore the industry data.

2. In addition, LFCT provides an ad hoc methodology for the determination of partial credibility, where there is a weighting of the portfolio's own experience and the industry experience.

3. The expected assumption for the aggregate amount of claims for a company for a year may be expressed as

$$X_E = Z\,\overline{X} + (1-Z)\mu$$

where

- X_E is the expected aggregate amount of claims,
- Z is the credibility factor, or weighting given to experience data,
- $\overline{X}$ is the observed mean and is calculated from the experience data $\mathbf{X}=\{X_1, X_2, \ldots, X_n\}$,
- μ is the mean of the underlying distribution and is equal the expected number or amount of death claims, based on the industry data for the same portfolio, and
- n is the number of years of data.

While this weighted average credibility formula has intuitive appeal, LFCT does not provide an underlying theoretical model for distribution of the X_i's which is consistent with the formula.

In LFCT, one calculates X_E by selecting a range parameter r (r > 0) and a probability level p (0<p<1) such that the difference between X_E and its mean μ are small.

The criterion can be written as

$$\Pr\left\{\left|X-\mu\right| \le -r\lambda\right\} \ge p$$

where r is the error margin, a "small" number and p is the confidence interval, a "big" number.

In other words, X_E is a good estimate of future expected mortality if the difference between X_E and its mean μ is small relative to μ with high probability.

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[13] American Credibility

Parameter values p = 90% and r = 5% are frequently cited as the minimum levels required for full credibility; however, there is no theoretical basis for determining these parameter values. When setting the expected mortality assumption for valuation purposes, one may want to use a higher threshold for full credibility, such as p = 90% and r = 3%.

In many situations, it is reasonable to approximate the distribution of $\overline{X}$ by a normal distribution. In these cases, the number of claims corresponding to state values of p and r may be taken from the standard normal tables. For p = 90% and r = 3%, the value from this distribution is 3,007 claims.

The following table sets out the number of claims needed for full credibility for various values of p and r.

Standard Normal Table – Range and Probability Parameters					
Number of Claims Needed for Full Credibility					
Probability Parameter p	**Range Parameter r**				
	5%	**4%**	**3%**	**2%**	**1%**
90%	1,082	1,691	3,007	6,765	27,060
95%	1,537	2,401	4,268	9,604	38,416
99%	2,654	4,147	7,373	16,589	66,538
99.9%	4,331	6,767	12,030	27,068	108,274

There are various models for the underlying distribution of claims. Poisson and Compound Poisson are discussed here.

Poisson Model

Although the theoretical distribution for mortality is binomial, when the probabilities of the event (death) are small, the Poisson distribution provides a reasonable approximation to a binomial distribution.

In the simple Poisson model, the only random variable is the number of claims, which is assumed to be Poisson.[14] Variations in claim size are ignored.

For p = 90% and r = 3%, the factor for partial credibility is defined by $Z = \min\left\{\sqrt{\frac{n}{3{,}007}}, 1\right\}$,

where n = number of claims in experience data.[15]

---

[14] See Loss Models: From Data to Decisions Example 5.20 or Introductory Credibility Theory Example 3.2.2.

[15] The credibility factors set out in the previous CIA standard VTP 6 Expected Mortality Experience for Individual Insurance were based on LFCT using a simple Poisson distribution. The factors incorporate a conservative bias that depended, in part, upon whether industry or company data was

Number of claims	30	120	271	481	752	1083	1473	1924	2436	3007
Z	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00

In other words, for p = 90% and r = 3%, one gets full credibility if the number of claims in the exposure period is greater than or equal to 3,007. The credibility formula can be viewed as the square root of the ratio of the number of claims in the data to the number of claims needed for full credibility.

Below is an example of the application of the Poisson Model.

Example 1	**Source of Data**	**Mortality Ratio**	**Observed Number of Claims**	**Credibility Factor**	**Blended A/E Claims Ratio**
Industry Data	From industry mortality study	75.3%	Not needed	1.00	
Company Data	From company study for same period	69.4%	200	$\{200/3{,}007\}^{.5}$ = .26	.26 x 69.4% +.74 x 75.3% = 73.8%

The Poisson application can be extended to include data from more than one period or year. However, the number of years would be limited so that the mix and material risk characteristics of the portfolio are homogeneous over time.

Compound Poisson Model

The total claims of a portfolio of business can be described as a compound Poisson model, which reflects both the number and amount of claims. When the additional consideration is given to the variability of claims sizes, the threshold for full credibility is increased relative to the Poisson model.

Let N be a random variable representing the number of claims from an insurance company and assume that N has a Poisson distribution with mean and variance λ. The observed number of claims is n.

For k = 1, 2, 3, …n, let Y_k be the random variable representing the amount of the kth claim.

---

better. Therefore the credibility factors in the previous CIA standard VTP 6 are different than those obtained using the above formula. Since the objective is to select the expected valuation assumption, a conservative bias is not appropriate.

Assume that Y_k's are independent and have a distribution with mean μ_y and variance $\sigma_y{}^2$.

Assume that the number of claims N is independent of the claim size Y_k.

The total claims $X = Y_1 + Y_2 + Y_3 + \ldots + Y_N$ has a compound Poisson distribution.

Using conditional expectation on N, it can be shown that

$$E[X] = \mu = \lambda\mu_y, \text{ and}$$

$$Var[X_i] = \sigma^2 = \lambda\left(\mu_y{}^2 + \sigma_y{}^2\right)$$

In summary, X_i has a compound Poisson distribution with Poisson parameter λ and claims size distribution with mean μ_y and variance $\sigma_y{}^2$.

Using the compound Poisson model with r = 3% and p = 90%, it may be shown[16] the number of deaths needed for full credibility is given by

$$C = \left\{ 3007 \times \frac{\left(\sum_{i=1}^{n} q_i b_i{}^2\right)\left(\sum_{i=1}^{n} q_i\right)}{\left(\sum_{i=1}^{n} q_i b_i\right)^2} \right\}$$

where b_i =net amount at risk for policy i

q_i =one-year mortality rate for policy i

i = 1, 2, 3, …, n

In order to calculate Z for compound Poisson, one needs to calculate mean μ_y and standard deviation σ_y of the claim size distribution. These may be calculated from the total exposure, or may be estimated using the actual claims.

For full credibility, the number of deaths in the portfolio's own experience must exceed this number C.

When the criterion for full credibility is not met, partial credibility can be applied. For p = 90% and r = 3%, the "square root" rule gives the credibility factor Z as

$$Z = \min\left\{ \sqrt{\frac{X}{C}}, 1 \right\}$$

where C is the criterion for full credibility and X is the observed number of deaths in the experience of the portfolio. If only number of deaths are considered, C = 3007.

[16] Appendix 1, Estimators

The compound Poisson example can be extended to include data from more than one period or year, where

> N_j is a random variable representing the number of claims during period j, j = 1, 2, 3, …, m.
>
> $Y_{j,k}$ is the random variable representing the amount of the k^{th} claim in period j.
>
> X_j is the random variable denoting the aggregate claims amount for the company in period j and is defined by $X_j = Y_{j,1} + Y_{j,2} + \ldots + Y_{j,n}$.

However, the number of years would be limited so that the portfolio is homogeneous over time.

The details of the derivations of all of the formulas are given in "Loss Models: from Data to Decisions", Klugman, Panjer and Willmot , (1998) John Wiley and Sons.

Below is an example of the application of the Compound Poisson Model.

Example 2	Source of Data	Mortality Ratio	Number of Claims N	Claim Rate q_i	Claim Size b_i	Credibility Factor	Blended A/E Clams Ratio
Industry Data	From industry mortality study one year	75.3%	Not needed			1.00	
Company Data	From company study for same period	69.4%	200	.001	50 at 50,000 50 at 100,000 50 at 150,000 50 at 200,000	0.24	.24 x 69.4% +.76 x 75.3% = 73.9%

The key assumptions underlying the compound Poisson formula for credibility are:

- The distribution of claim amounts Y_k are independent and have a distribution with mean μ_y and variance $\sigma_y{}^2$.
- The number of claims N is independent of the claim size Y_k.
- Central Limit Theorem is used to approximate the distribution of $\left((X_i - x) / \sigma_x \right)$ as a normal random variable with mean zero and standard deviation one[17].

[17] See page 193 of Hogg and Craig [1978]

Application of LFCT to Sub-categories of Business

If the actuary wants to reflect experience split by sub-category (perhaps by sex, product, or duration) but the experience in those sub-categories is not 100% credible, the actuary would decide either to use the overall credibility factor, or the lower credibility for the amount of experience in that sub-category.

One can pool disparate distributions within the aggregate data under certain conditions. Several approaches are discussed below.

Assume that the portfolio comprises six different sub-categories: male and female; split by medical, non-medical, and paramedical. For each sub-category, the distribution of number of claims is Poisson with a different parameter (for this purpose, assume that the random variable under consideration is the A/E ratio calculated for each category).

If the relative proportions of the sub-categories are stable over time, then an actuary can use the credibility factor based on the aggregate distribution of these heterogeneous Poisson distributions (i.e. the total company credibility factor) for each of the sub-categories.

Below are examples of using credibility weights based on total claims, in aggregate.

DATA FOR EXAMPLES 3 TO 5						
	MORTALITY RATIOS					
Industry Data	**Male**	**Female**	**Total**			
Medical	71.0%	75.0%	71.9%			
Non-Medical	84.0%	83.0%	83.8%			
Para-Medical	73.0%	85.0%	74.3%			
Total	74.5%	78.7%	**75.32%**			
	MORTALITY RATIOS			**NUMBER OF CLAIMS**		
Company Data	**Male**	**Female**	**Total**	**Male**	**Female**	**Total**
Medical	59.0%	47.0%	56.2%	63.8	15.4	79.2
Non-Medical	85.9%	90.1%	86.9%	43.7	14.5	58.2
Para-Medical	75.0%	101.0%	77.8%	54.0	8.6	62.6
Total	69.9%	67.1%	69.3%	161.5	38.5	**200.0**
	COMPANY EXPECTED CLAIMS AT 100% OF INDUSTRY DATA			**CREDIBILITY FACTORS**		
Company Data	Male	Female	Total	Male	Female	Total
Medical	108.1	32.8	140.9	.15	.07	.16
Non-Medical	50.9	16.1	67.0	.12	.07	.14
Para-Medical	72.0	8.5	80.5	.13	.05	.14
Total	231.0	57.4	288.4	.23	.11	.26

Example below uses credibility weights based on the credibility of the total company. The industry and company data are from the above table “Data for Examples 3 to 5”.

Example 3 – LFCT Total Company Claims Credibility

	BLENDED EXPECTED MORTALITY RATIOS			EXPECTED NUMBER OF CLAIMS		
	Male	**Female**	**Total**	**Male**	**Female**	**Total**
Medical	67.9%	67.8%	67.9%	73.4	22.2	95.6
Non-Medical	84.5%	84.8%	84.6%	43.0	13.7	56.7
Para-Medical	73.5%	89.2%	75.2%	52.9	7.6	60.5
Total	73.3%	75.7%	73.8%	169.4	43.5	**212.8**

In this example, the expected mortality ratio for medically underwritten male lives is 67.9% (calculated (1-.26) x 71.0% + .26 x 59.0%). The calculation uses the credibility factor for the entire company, with the expected mortality ratios from the sub-categories. (Note that the total number of expected claims is 212.8, which is greater than the company's actual claims. This is reasonable, since the blended ratio reflects industry experience, which is not as good as company experience in this example.) The expected number of claims for male medical is 73.4 (calculated as 67.9% mortality ratio x 108.1 expected claims at 100% from the table Data for Examples 3 to 5).

The requirement that the portfolio mix be stable with respect to the major sub-categories over time may limit the applicability of this result. The portfolio mix may be regarded as stable over time if the proportions of the relevant sub-categories are constant (both for the study period, and the projection period for the future). If the study is based on smoking distinct business, but new preferred underwriting classes are added to the portfolio, the assumption of stability of the portfolio mix may not hold.

If the relative proportions of the subgroups are not stable over time, and hence the assumptions do not hold, it may be appropriate to reflect the credibility of the sub-categories in the determination of the expected mortality assumption. Assessing whether the conditions hold requires the actuary's judgment.

Example 4 below uses credibility weights based on the credibility of the sub-categories. The industry and company data are the same as was used in the previous example.

Example 4– LFCT Sub-Category Claims Credibility

	BLENDED EXPECTED MORTALITY RATIOS			EXPECTED NUMBER OF CLAIMS		
	Male	**Female**	**Total**	**Male**	**Female**	**Total**
Medical	69.3%	73.0%	70.1%	74.9	23.9	98.8
Non-Medical	84.2%	83.5%	84.0%	42.8	13.5	56.2
Para-Medical	73.3%	85.9%	74.6%	52.8	7.3	60.1
Total	73.8%	77.8%	74.6%	170.4	44.7	**215.1**

In this example , the expected mortality ratio for medically underwritten male lives is 69.3%, (calculated as (1-.15) x 71.0% + .15 x 59.0%, where 15% is the company credibility for the male medical mortality category, and is equal to the square root of 63.8/3007. The calculation uses the credibility factor for the sub-category, with the expected mortality ratios from the sub-category.

Continuing the example, the total medical ratio is 70.1%, calculated as follows. First, the expected number of claims is calculated for each sub-category; for example, male medical = 69.3%×108.1 = 74.9. Second, the total expected number of medical claims is the sum of the expected claims for each sub-category = 74.9 + 23.9 = 98.8. Finally, the total medical ratio = 98.8÷140.9 = 70.1%, where 140.9 is the expected number of medical claims at 100% of industry data.

Note that the total expected claims in this example is greater than in the example before. The overall level of expected claims under this method depends on the choice of sub-categories. The greater the division of sub-categories, the smaller the company credibility in each cell, and the closer results will be to industry experience.

The choice of sub-categories can significantly affect the final mortality assumption. In the extreme, even if overall company experience significantly differs from industry experience, the actuary could derive an expected assumption equal to or close to the industry table by selecting enough sub-categories such that credibility for each is close to zero.

Furthermore, if there is interaction between company sub-categories, the LFCT may not capture it. (For example, a company's underwriting of smokers may have recently been liberalized. One would expect an interactive effect between smoking status and duration, but this interaction would not be reflected in the results under LFCT.)

"Normalized Method" – A Variation of LFCT

The Normalized Method is currently the preferred approach.

The "Normalized Method" uses the credibility and the A/E ratios of the subcategories. However, the blended A/E ratios are adjusted to reproduce the expected claims level generated by the total company blended A/E ratio.

The Normalized Method has the following strengths:

- the sum of expected claims for the sub-categories matches total expected claims, based on a blended A/E ratio, calculated at the total company level (i.e., the number of sub-categories selected does not affect the overall result);
- all of the information is used: both total company and sub-category A/E ratios and credibility factors;
- the results are reasonable in extreme or limiting cases;
- the sub-category A/E ratios fall within the original range (at least in these examples);
- interactive effects between sub-categories may be captured; and
- it is simple to apply in practice.

Although this method does not have a strong theoretical base, it is pragmatic, and satisfies the criteria for a good credibility method. Below is an example of the Normalized Method. The industry and company data is the same as was used in the previous examples.

Example 5 – LFCT Normalized Method

From the first example, the total company blended A/E ratio (based on total company experience and credibility) is 73.8%. From the table *Data for Examples 3 to 5*, the total expected number of claims, using 100% of industry mortality, is 288.4. Therefore, the total expected number of claims for the company is 212.8 (calculated as 73.8% of 288.4).

In the Normalized Method, the A/E ratios for each of the subcategories is adjusted or "normalized" by multiplying by the ratio of the total expected claims from Example 1 to that from Example 4.

	BLENDED EXPECTED MORTALITY RATIOS			**EXPECTED NUMBER OF CLAIMS**		
	Male	**Female**	**Total**	**Male**	**Female**	**Total**
Medical	68.5%	72.2%	69.4%	74.0	23.7	97.7
Non-Medical	83.3%	82.6%	83.0%	42.4	13.3	55.7
Para-Medical	72.5%	84.9%	73.8%	52.2	7.2	59.4
Total	73.0%	77.0%	73.8%	168.6	44.2	**212.8**

In the above example #5, the expected mortality ratio for medically underwritten male lives is 68.5%, (calculated as the blended ratio from example #4 x multiplied by the ratio of expected claims from example #1 to that from example #4: 69.3% x 212.8 / 215.1 = 68.5).

Note that the Normalized Method allows for the calculation of credibility factors by sub-category, but then produces the same number of expected claims in total for the company as if there was only one total category. The sensitivity of the overall result to the choice of sub-category is eliminated. While the total number of claims is the same as in example #3 the distribution among sub-categories differs, reflecting the known experience in these sub-categories.

APPENDIX 3 – GREATEST ACCURACY CREDIBILITY THEORY/ BUHLMANN METHOD[18]

Overview

The Greatest Accuracy Credibility Theory allows one to estimate within and between sub-category sources of variation.

GACT is theoretically complete, and meets the criteria for a credibility method with one shortcoming. That shortcoming is that additional information about industry experience (beyond what is customarily collected and published) is required. Without these practical difficulties, GACT would likely be the preferred credibility method to use in determining the expected valuation mortality assumption.

There are several versions of GACT. One of the simplest, the Buhlmann model, is discussed here. A more complex model, the Buhlmann-Straub is outlined in latter sections of this appendix.

Buhlmann Model[19]

1. Assume for a particular policyholder or risk class, we know the past claim experience X={X_1, X_2,..., X_n } and that it is distributed with the same mean and variance, conditional on θ. For our purpose, assume that X is the experience of a particular company. The industry data comprises experience from many companies.

2. The policyholder has been categorized by underwriting characteristics and we have a "manual" rate μ that reflects these underwriting characteristics. The rating class is viewed as homogeneous with respect to the underwriting characteristics, but even within this rating class there is some heterogeneity (good risks and bad risks) since no rating classification can be detailed enough to capture all information.

3. Assume that this residual variation in the risk level of each policyholder in the portfolio may be characterized by a parameter θ (possibly a vector), but that θ for a given policyholder cannot be known.

4. Assume further that the cumulative distribution function $B(\theta) = \Pr\{\Theta \le \theta\}$ is known. $B(\theta)$ represents the probability that a policyholder picked at random from the rating class has a risk parameter less than or equal to θ.

5. Assume that the claims experience of a policyholder can be expressed as the following conditional distribution:[20]

[18] European Credibility

[19] Loss Models: From Data to Decisions Section 5.4.3 or Introductory Credibility Theory Section 4.3

6. $f_{X_j|\Theta}\left(x_j\middle|\theta\right),\ j=1,2,\ldots,n,n+1$. Assume that the past claims experience **X**={$X_1, X_2, \ldots, X_n$} is distributed with the same mean and variance, conditional on a risk parameter which is not known for a particular policyholder.

7. Define

$u(\theta)=E\left(X_j\middle|\Theta{=}\theta\right)$	(hypothetical mean)
$v(\theta)=Var\left(X_j\middle|\Theta{=}\theta\right)$	(hypothetical variance)
$u=E\{u(\theta)\}$	(pure premium)
$v=E\{v(\theta)\}$	(expected value of process variance [or variability within company])
$a=Var\{u(\theta)\}$	(variance of hypothetical means [or variability between companies])

8. It can be shown that the credibility factor is of the form:

$$Z=\frac{n}{n+k}$$

where

$$k=\frac{\text{expected value of process variance}}{\text{variance of hypothetical mean}}=\frac{v}{a}$$

9. As a (the variance of means across companies) decreases, k increases, and credibility factor Z decreases (if there is little difference between companies, more weight would be given to the industry experience, which will be less subject to random fluctuation).

As v (the expected value of the variability within the company) decreases, k decreases, and Z increases (if there is little fluctuation within the company, its own experience is more representative of the expected future experience).

10. For example, if {$X_j|\Theta$; j = 1,2,3,…,n] are independently and identically Poisson with given mean Θ, and Θ is Gamma with parameters a and b, then $Z=\dfrac{n}{n+1/b}$[21].

The amounts v and a may be estimated using non-parametric estimators of the form[22]:

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[20] Loss Models: From Data to Decisions Section 5.4 and also Chapter 4 of Introductory Credibility Theory

[21] Loss Models: From Data to Decisions Example 5.36 and Introductory Credibility Theory Example 4.3.2

[22] Loss Models: From Data to Decisions Section 5.5.1 and Introductory Credibility Theory Chapter 5

$$\hat{v}_i = \frac{1}{n-1}\sum_{j=1}^{n}\left(X_{ij} - \overline{X}_i\right)^2 \qquad \hat{v} = \frac{\sum_{i-1}^{r}\sum_{j=1}^{n}\left(X_{ij} - \overline{X}_i\right)^2}{r(n-1)} \qquad \hat{a} = \frac{1}{(r-1)}\sum_{i=1}^{r}\left(\overline{X}_i - \overline{X}\right)^2 - \frac{\hat{v}}{n}$$

11. Example #6 uses parametric estimators described in more detail in Appendix 2.

Example 6 – Buhlmann

A/E Mortality Ratios by Year			
Experience Study Year	**Company 1**	**Company 2**	**Total**
1	70.0%	70.0%	
2	75.0%	85.0%	
3	80.0%	100.0%	
Company Mean $\overline{X_i}$ and $\overline{X}$	75.0	85.0	80.0
Expected Value of Process Variance v_i	.0025	.0225	
Variance of Hypothetical Mean a			.00417
$K_i = v_i/a$	.00250/.00417 = .60	.0225/.00417 = 5.40	
$Z_i = n/(n+k)$	3/(3+.60) = 83.33%	3/(3+5.4) = 35.7%	
$X_{Ei} = Z_i\overline{X} + (1 - Z_i)\mu$	75.7%	81.8%	

1. Note that company 1, which has much less variation in A/E mortality ratios over the study period, has a lower expected value of process variance, and therefore higher credibility.
2. The GACT approach attempts to obtain the variance components based on either model considerations (requiring similar information as in LFCT) or historical data (from which the variance components can be calculated without assumptions about models).
3. The difficulty with this approach lies in the data currently available for the industry. While companies can track their own A/E ratios by over time by sub-category, the problem of estimating the "between company" variation remains (no company has access to another's data)[23].
4. Many companies group mortality data for submission to experience studies, and grouped data is not sufficiently detailed to support the calculation of the parameter estimates. If seriatim data is not supplied to the experience study, modifications to the parameter estimates would be needed.

[23] One solution might be to have the CIA specify sub-categories, and periodically publish the variation across companies by sub-category (using experience studies with specified sub-categories).

Buhlmann -Straub[24]

1. The Buhlmann model provides a simple, theoretically consistent formula but does not allow for variations in exposure or claim size. Buhlmann-Straub is a generalization of Buhlmann model that allows for variations in exposure or size.

2. Introduce the amount $m_{j,}$ a known constant that measures exposure i.e. m_j is expected claims (in $).

3. Assume $X_1, X_2, \ldots, X_n$ are independent conditional on Θ with common mean (as before). Then the hypothetical mean is given by

   $$u(\theta) = E(X_j | \Theta = \theta)$$

   as before, but the conditional variances are

   $$v(\theta) = Var(X_j | \Theta = \theta) = v(\theta)/m_j$$

   and

   Z = m / (m+k)

   where k is same as under the Buhlmann model above and m = sum of all exposure amounts m_j

4. This formula reflects variations in exposure and allows for between company effect and within company effect.

5. Development of the non-parametric estimators for the Buhlmann-Straub model is given in Loss Models: From Data to Decisions section 5.5.1.

   $$\hat{v}_i = \frac{\sum_{j=1}^{n_i} m_{ij}\left(X_{ij} - \overline{X_i}\right)^2}{n_i - 1}$$

   $$\hat{v} = \frac{\sum_{i-1}^{r} \sum_{j=1}^{n_i} m_{ij}\left(X_{ij} - \overline{X_i}\right)^2}{\sum_{i=1}^{r} (n_i - 1)}$$

   $$\hat{a} = \left(m - m^{-1} \sum_{i=1}^{r} m_i^2 \right)^{-1} \left[\sum_{i=1}^{r} m_i \left(\overline{X_i} - \overline{X}\right)^2 - \hat{v}(r-1) \right]$$

[24] Loss Models: From Data to Decisions Section 5.4.4 and Introductory Credibility Theory Section 4.4

APPENDIX 4 – SELECTIVE LAPSATION

It is necessary to divide the lapse into three mutually exclusive components.

They are:

1. "Underlying" lapses are the part of the lapses comparable to what was experienced in the exposure underlying the construction of the select mortality table.
2. "Average" lapses are the part of the additional lapses which will exhibit mortality experience identical to that expected for the group of lives who persisted at least to the beginning of the current policy year.
3. "Selective" lapses are the remaining part of the additional lapses which will exhibit mortality identical to that of newly selected lives.

Since the mortality of all the lapses taken together is not likely to be better than that of the third group nor worse than that of the second group, all three components will be positive or zero.

Using the division given above, it is possible to develop a recursive formula which defines the expected mortality of the persisting group of lives.

In order to come up with a workable formula, it is necessary to make an idealizing assumption that the average and selective lapses occur only at policy anniversaries. Thus, the average and selective lapse rates are applied to the population persisting just prior to the anniversary and acted on instantaneously at the anniversary. This presents no great hardship for the actuary since average and selective lapses will normally be assumed to occur only at renewal dates.

The underlying lapse rate, like the mortality rate, is assumed to apply continuously. Consequently the total lapse rate for a policy year will not be simply the sum of the three component lapse rates.

However, since at renewals the underlying lapse rate will almost always be quite small compared to the average and selective lapse rates, and since it is not likely possible to predict the average and selective lapse rates with a high degree of accuracy, likely little harm will be done by approximating the total lapse rate as the sum of the three component lapse rates.

In the following development, the underlying lapse rate is assumed to continue to apply to those who have departed as average and selective lapses. In the experience underlying the construction of the select mortality table, average and selective lapses would not have occurred. Hence the underlying lapse rate can be assumed to continue to apply to the (now hypothetical) persisting population in an environment in which the underlying lapse rate is the only lapse rate.

It is important to note that the following formula relies entirely on a tracheotomy of lapse rates that cannot in fact be made. No experience studies on renewable term will ever yield underlying, average and selective lapse rates. Fortunately reasonable numbers are not too hard to develop. The underlying lapse rates, if not known, can often be determined or can be estimated fairly accurately. The additional lapses will show up in lapse studies. Most companies will have statistically significant data.

Mortality deterioration can be adjusted in future valuations as lapse experience emerges. The main element of judgment lies in subdividing the additional lapse into average and selective.

In the following formulas:

s is the duration at which the average and selective lapses occur

$\mathbf{q}_{[x]+t}$ represents mortality rates from the standard select mortality table

$\mathbf{q'}_{[x]+t}$ represents mortality rates from the table appropriate to the group of lives persisting to duration s, but just prior to the average and selective lapses. There may have already been some mortality deterioration prior to duration s. For the sake of determining this table, there is assumed to be no average or selective lapse rates at duration s or at any later duration. The mortality rates of this table are identical to those of the above table until the first selective lapses occur.

$\mathbf{q''}_{[x]+t}$ represents mortality rates from the table appropriate to the persisting lives after the average and selective lapses at duration s, assuming no further average or selective lapses. Mortality rates prior to duration s, are taken from the above table.

$\mathbf{q}^{u}_{[x]+t}$ represents the underlying lapse rate applicable to policy year **t**

$\mathbf{q}^{a}_{[x]+t}$ represents the average lapse rate which acts at exact duration **t**

$\mathbf{q}^{s}_{[x]+t}$ represents the selective lapse rate acting at exact duration t. (Note that the average lapse rate and the selective lapse rate must be 0 for duration 0.)

The probability that a life in force just prior to the average and selective lapses at duration **s** will die between durations **s + t** and **s + t + 1** is:

$$\left[\prod_{r=0}^{t-1}\left(1-q'_{[x]+s+r}\right)\left(1-q^{u}_{[x]+s+r}\right)\right]q'_{[x]+s+t}\left(1-.5\,q^{u}_{[x]+s+t}\right)$$

The probability that one of these lives will be an average lapse at duration s and then die in the above year is:

$$q^{a}_{[x]+s}\left[\prod_{r=0}^{t-1}\left(1-q'_{[x]+s+r}\right)\left(1-q^{u}_{[x]+s+r}\right)\right]q'_{[x]+s+t}\left(1-.5\,q^{u}_{[x]+s+t}\right)$$

The probability that one of these lives will be a selective lapse at duration s and then die in the above year is:

$$q^{s}_{[x]+s}\left[\prod_{r=0}^{t-1}\left(1-q_{[x+s]+r}\right)\left(1-q^{u}_{[x]+s+r}\right)\right]q_{[x+s]+t}\left(1-.5\,q^{u}_{[x]+s+t}\right)$$

The probability that one of these lives will persist until dying in the above year is:

$$\left(1-q^{a}_{[x]+s}-q^{s}_{[x]+s}\right)\left[\prod_{r=0}^{t-1}\left(1-q''_{[x]+s+r}\right)\left(1-q^{u}_{[x]+s+r}\right)\right]q''_{[x]+s+t}\left(1-.5\,q^{u}_{[x]+s+t}\right)$$

This last probability is by definition equal to the first probability less the sum of the second and third probabilities. Therefore, eliminating the factors in the underlying lapse rate which are obviously common.

$$\left(1-q^{a}{}_{[x]+s}-q^{s}{}_{[x]+s}\right)\left[\prod_{r=0}^{t-1}\left(1-q''{}_{[x]+s+r}\right)\right]q''{}_{[x]+s+t}$$
$$=\left(1-q^{a}{}_{[x]+s}\right)\left[\prod_{r=0}^{t-1}\left(1-q'{}_{[x]+s+r}\right)\right]q'{}_{[x]+s+t}-q^{s}{}_{[x]+s}\left[\prod_{r=0}^{t-1}\left(1-q{}_{[x+s]+r}\right)\right]q{}_{[x+s]+r}$$

$$\therefore \quad {}_{t|}q''{}_{[x]+s}=\frac{\left(1-q^{a}{}_{[x]+s}\right)\ {}_{t|}q'{}_{[x]+s}-q^{s}{}_{[x]+s}\ {}_{t|}q{}_{[x+s]}}{1-q^{a}{}_{[x]+s}-q^{s}{}_{[x]+s}}$$

Assuming the select period is 15 years, the above mortality table will follow the underlying ultimate beginning 15 years after the last selective lapses. An actuary who believes the mortality deterioration to run off over a longer period can construct an underlying table with a longer select period. It would not be proper to use a select period of less than 15 year, particularly for the higher ages.

Approximation Methods

a) Single Scale Products

The approach that follows is simpler and produces mortality rates higher than those developed using the formula above. It may be used as an approximation, however, caution would be followed with respect to death supported business.

x is the issue age;

s is years since issue.

$\mathbf{q_x}$ is the valuation mortality table appropriate for the original group of insured lives.

$\mathbf{q'_{x+s}}$ is the mortality rate of all insured lives just prior to the renewal date. At the first renewal date, $\mathbf{q'_{x+s} = q_{[x]+s}}$

$\mathbf{q''_{x+s}}$ is the mortality rate of the remaining lives after the renewal date.

SL is the additional Selective Lapse Rate by healthy (select) lives at a renewal date.

AL is the additional lapse rate at renewal by policyholders whose mortality experience is the same as that of all policyholders just before renewal

Then: $\mathbf{q'_{x+s} = SL\ q_{[x+s]} + AL\ q'_{x+s} + (1 - SL - AL)\ q''_{x+s}}$

$\mathbf{q''_{x+s} = ((1 - AL)\ q'_{x+s} - SL\ q_{[x+s]}) / (1 - SL - AL)}$ **(formula A)**

This formula provides the mortality rate of the remaining lives immediately after a renewal date. It is now necessary to consider the future mortality experience of this group. The mortality rate of the total group of lives immediately before renewal would eventually (at the end of the select period) become the ultimate mortality rate. The mortality rate of the selective lapses, although initially select, will eventually become the ultimate mortality rate too. Consequently, the mortality rate of the remaining lives must eventually become the ultimate mortality also.

Define K such that $\mathbf{(q''_{x+s}) / q''_{[x]+s} = (100 + K)/100}$

Assume the select period is 15 years.

Then $\mathbf{q''_{x+s} = ((100 + K)\, q_{[x]+s})/100}$

$\mathbf{q''_{x+s+1} = ((100 + 14K/15)\, q_{[x]+s+1})/100}$

$\mathbf{q''_{x+s+2} = ((100 + 13K/15)\, q_{[x]+s+2})/100}$

$\mathbf{q''_{x+s+5} = ((100 + 10K/15)\, q_{[x]+s+5})/100}$

Assuming there is a five year period between renewal, $\mathbf{q''_{x+s+5}}$ would be set equal to the $\mathbf{q'}$ factor in formula A above at the next renewal date, and a new value of $\mathbf{q''_{x+s+5}}$ (mortality of the remaining lives after the next renewal date) would be determined.

b) Re-Entry Products

In valuing re-entry products it is necessary to make an assumption either about the mortality of those who do not re-qualify for select rates, or about the percentage of policyholders who will re-qualify for select rates at each renewal date - the “Re-entry Proportion”.

It is easier to choose the Re-entry Proportion than to make an assumption about the mortality rate for those who do not re-qualify, because the Re-entry Proportion is related to current and expected future underwriting practice. Many companies will have statistically significant data which they can monitor.

It is also possible to value the select and ultimate groups separately. However, there is little to recommend this approach since it is difficult even under ideal conditions, and nearly impossible if conditions are less than ideal. It is necessary to make separate mortality assumptions for the select and ultimate groups. Furthermore it will be necessary to study separately the mortality experience of each group of lives that fails to re-qualify for at least 15 years. If the underwriting at re-qualification is the same as at issue, the mortality assumption for re-entry group can be normal select mortality, and the mortality assumption for the ultimate group can be determined by treating the re-entry as a selective lapse. If, as may well be the case, the underwriting is less strict at re-entry, then the mortality for both the select and ultimate groups will be higher than it would otherwise be, but there is no good way of determining an appropriate mortality assumption for either group.

It should be noted that the choice of the Re-entry Proportion leads to an implicit assumption about the mortality of those who do not re-qualify, and vice versa. If the mortality rate for those who do not re-qualify is chosen, the corresponding Re-entry Proportion can be calculated and rationalized. That Re-entry Proportion would also be used in blending premium rates. If the Re-entry Proportion is chosen, the corresponding mortality rates for those who do not re-qualify would be calculated as an overall test of reasonableness.

These mortality rates may be very high (e.g. in excess of 1,000% of standard).

If the Re-entry Proportion approach is used, the valuation method can assume that the mortality for the group of policyholders as a whole resembles the normal select/ultimate mortality one would expect from a single scale product given the same Selective Lapses.

For example, the mortality rate in the sixth policy year for the total group will be $\mathbf{q}_{[x]+5}$.

The gross premium, however, will be:

RP x GPS + (1 - RP) x GPU

where

RP = Re-entry Proportion

GPS = Select (preferred) renewal rate

GPU = Ultimate (guaranteed) renewal rate

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[0]  RATMRT←RATLAP MORTALITY_DETERIORATION DTARSK;AGEISS;CLSRSK;DNR;DURADJ;DURLST;DURREQ;DURSEL;I;NMR;
PERSEL;RATMRTBAS;RATMRTSEL;RATSEL;WGTBAS;WGTSEL;WGTTOT
[1]  →START_
[2]  δ PURPOSE : Provides mortality rates reflecting the mortality deterioration as a result of
[3]  δ          selective lapsation
[4]  δ INPUT:
[5]  δ    RATLAP – Lapse Rates
[6]  δ      two dimensional matrix (2 x #durations)
[7]  δ      RATLAP[1;] – base lapse rates
[8]  δ      RATLAP[2;] – selective lapse rates
[9]  δ      note that Total Lapse Rate for duration T is RATLAP[1;T] + RATLAP[2;T]
[10] δ
[11] δ    DTARSK – Risk Data
[12] δ      one dimensional (length = 2)
[13] δ      DTARSK [1] – risk class (eg, 1 for Male Non-Smoker)
[14] δ      DTARSK [2] – issue age
[15] δ
[16] δ    rATMRTSEL – Select Mortality Rates
[17] δ      three dimensional (#risk classes x #issue ages x #select durations)
[18] δ      (eg, rATMRTSEL[1;36;3] corresponds to Male Non-Smoker, Issue Age 35, Duration 3)
[19] δ
[20] δ
[21] δ    rATMRTULT – Ultimate Mortality Rates
[22] δ      two dimensional (#risk classes x #ages)
[23] δ      (eg, rATMRTULT [1;36] corresponds to ultimate mortality for Male Non-Smoker with
[24] δ      attained age 35)
[25] δ
[26] δ OUTPUT:
[27] δ    RATMRT – Mortality Rates
[28] δ      one dimensional (length = #durations)
[29] δ      mortality rates by duration for given risk class and issue age adjusted for
[30] δ      mortality deterioration
[31] δ      (eg, RATMRT[22] is mortality rate for duration 22)
[32] δ
[33] START_:
[34] DURREQ ← ¯1↑ρRATLAP                      δ Let DURREQ = the #durations until there are no more lapses as given
[35] DURLST ← ι+/∨\φRATLAP[2;]≠0              δ Let DURLST = last duration where selective lapse is non-zero
[36]                                          δ
[37] CLSRSK ← DTARSK[1]                       δ Let CLSRSK = code for given risk class (eg, 1 for Male Non-Smoker)
[38]                                          δ
[39] AGEISS ← DTARSK[2]                       δ Let AGEISS = given issue age
[40] PERSEL ←  ¯1↑ρrATMRTSEL                  δ Let PERSEL = select period of the given table
[41] DURSEL ← PERSEL ⌊ DURREQ                 δ Let DURSEL = lower of PERSEL or DURREQ
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RATSEL ← rATMRTSEL[CLSRSK;AGEISS+DURLST;ιDURSEL]              δ Let RATSEL = the (sub)table of relevant select mortality rates
                                                              δ         with respect to the given Issue Age (AGEISS) and
                                                              δ         risk class (CLSRSK).
                                                              δ         RATSEL is two dimensional (DURLST x DURSEL)
                                                              δ         (eg, RATSEL[4;10] is select mortality at duration 10
                                                              δ         for issue age AGEISS+3)
                                                              δ
RATMRT ← rATMRTSEL[CLSRSK;AGEISS+1;]                          δ Let RATMRT = set of select mortality rates
RATMRT ← RATMRT,(AGEISS+ρRATMRT)↓rATMRTULT[CLSRSK;]           δ         and ultimate mortality rates prior to adjusting for
RATMRT ← DURREQ↑RATMRT                                        δ         selective lapses.
                                                              δ         there should be as many mortality rates as given lapses
                                                              δ

I ← ιDURREQ                                                   δ For each duration I
LOOP_DURATION_:                                               δ     where
→(0=ρI←1↓I)/END_                                              δ         I is less than or equal to DURREQ and
→(RATLAP[2;I[1]]=0)/LOOP_DURATION_                            δ         the selective lapse rate is non-zero
                                                              δ         adjust RATMRT as follows:
DURADJ ← (DURSEL⌊ρI)↑I                                        δ             Let DURADJ = the “adjustment period” where DURADJ is the
                                                              δ                      lower of the select period (DURSEL) or # of
                                                              δ                      remaining durations
RATMRTBAS ← RATMRT[DURADJ]x1,x\1-RATMRT[ ¯1↓DURADJ]           δ             Let RATMRTBAS = set of mortality rates for each duration in
                                                              δ                         the adjustment period. RATMRTBAS[T] is the
                                                              δ                         probability of death (based on RATMRT) in
                                                              δ                         duration I+T-1 given survival to duration I.
                                                              δ

RATMRTSEL ← RATSEL[Z[1];ιρZ]x1,x\1-RATSEL[Z[1];ι ¯1+ρZ←DURADJ] δ             Let RATMRTSEL = set of mortality rates, same as RATMRTBAS
                                                              δ                         but based on RATSEL instead of RATMRT.
                                                              δ                         (ie, assuming select period begins at
                                                              δ                         duration I)
WGTBAS ← 1-RATLAP[1;DURADJ[1]]                                δ             Let WGTBAS = 1 less the base lapse rate for duration I
WGTSEL ← RATLAP[2;DURADJ[1]]                                  δ             Let WGTSEL = the selective lapse rate for duration I
WGTTOT ← 1-+\RATLAP[;DURADJ[1]]                               δ             Let WGTTOT = 1 less the total lapse rate for duration I


NMR ← ((RATMRTBASxWGTBAS)-RATMRTSELxWGTSEL)÷WGTTOT             δ             For each duration T in the adjustment period:
                                                              δ               Let NMR[T] (numerator)
                                                              δ                 = (WGTBAS[T] X RATMRTBAS[T]) + (WGTSEL[T] X RATMRTSEL[T])
                                                              δ                                           WGTTOT[T]
DNR ← 1-+\ ¯1↓0,NMR                                           δ               Let DNR[T] (denominator)
                                                              δ                 = 1 for T=1
                                                              δ                 = 1 less the sum of  NMR[1] to NMR[T-1] for T>1

RATMRT[DURADJ] ← NMR÷DNR                                      δ               Let RATMRT[T] = NMR[T] ÷ DNR[T]

→LOOP_DURATION_
END_:
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