EDUCATIONAL NOTE

*Educational notes do not constitute standards of practice. They are intended to assist actuaries
in applying standards of practice in specific matters. Responsibility for the manner of application of standards
in specific circumstances remains that of the practitioner.*

SELECTION OF INTEREST RATE MODELS

COMMITTEE ON LIFE INSURANCE FINANCIAL REPORTING

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MEMORANDUM

TO: All Fellows, Associates and Correspondents of the Canadian Institute of Actuaries

DATE: December 2003

FROM: Jacques Tremblay, Chairperson
Committee on Life Insurance Financial Reporting (CLIFR)

SUBJECT: **Educational Note on Selection of Interest Rate Models**

The Committee on Life Insurance Financial reporting (CLIFR) has developed the attached educational note for Selection of Interest Rate Models. CLIFR would like to point out that this educational note was derived from previous fall letters and that the substance of the text remains essentially unchanged.

Interest rate modelling requires a thorough understanding of stochastic methods and statistical techniques, and demands the use of higher mathematics and sophisticated algorithms. There are some desirable attributes of an interest rate model (or models) for Canadian GAAP valuation. The educational note provides guidance on the appropriate use of a Monte Carlo interest rate simulation model.

In accordance with the Institute's policy for Due Process, this "Educational Note on Selection of Interest Rate Models" has been approved by the Committee on Life Insurance Financial Reporting, and has received final approval for distribution by the Practice Standards Council.

Section 1220 of the Consolidated Standards of Practice prescribes that "The actuary should be familiar with relevant educational notes and other designated educational material." It further explains that a "practice which the notes describe for a situation is not necessarily the only accepted practice for that situation and is not necessarily accepted actuarial practice for a different situation." As well, "educational notes are intended to illustrate the application (but not necessarily the only application) of the standards, so there should be no conflict between them."

We would like to thank Geoffrey Hancock, Christian-Marc Panneton and Jacques Potvin who were primarily responsible for the development of this educational note.

Questions should be addressed to me at my *Yearbook* address.

JT

Selection of Interest Rate Models

If the liabilities were determined using stochastic simulation, the actuary would adopt policy liabilities in the range defined by CTE (60%) and CTE (80%).

The CSOP Practice Specific Standards for Insurers describes the elements of an interest rate scenario (risk-free interest rates, credit spreads, default rates, inflation, investment strategy), and suggests that the assumed terms of interest rates should permit assumption of changes in the shape and steepness of the yield curve. That implies a minimum of short, medium and long-term rates.

Notwithstanding any definition for a plausible range on Canadian default-free interest rates, the above provides little guidance in the selection, fitting and use of a stochastic interest rate model. The CIA (CLIFR) wants to promote narrowing of the range of practice, and believes additional guidance would clearly be helpful to the actuary.

Interest rate modelling requires a thorough understanding of stochastic methods and statistical techniques, and demands the use of higher mathematics and sophisticated algorithms. Ignoring technical specifications, there are some desirable attributes of an interest rate model (or models) for Canadian GAAP valuation.

At the outset, the actuary must recognize the differences between scenarios created under the real-world and risk-neutral probability measures (P-measure and Q-measure, respectively). Real-world scenarios are used for projection; risk-neutral scenarios are used for market pricing (i.e., fair value determination). Real-world (P-measure) models give sample *distributions*, while risk-neutral (Q-measure) models provide single measurements of value (typically the mean). Canadian GAAP valuation requires projection under real-world interest rates. Whether a risk-neutral pricing model is required within this framework depends on:

1. the assets under consideration;
2. the strategy for covering negative cash flows; and
3. the re-investment strategy.

Monte Carlo simulation is a common technique for projection, but other models (e.g., lattice models) can be used when market values are required.

The following general conditions offer some guidance on the appropriate use of a Monte Carlo interest rate simulation model.[1] However, some of the points may not be relevant or even desirable for a given application. Indeed, certain attributes may be in conflict for some models; by itself, this would not invalidate the use of the model for valuation. The actuary must determine which features are most appropriate to the risks being valued.

- The random number generator is robust. (The generated sequences need to pass standardized statistical tests for randomness. This generally means that the generator would: 1) exhibit long periodicity; and 2) not suffer from serial correlation.)
- Variance reduction techniques (e.g., low discrepancy sequences) can be effectively used for pricing or market valuation.

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[1] Note: some of the desirable characteristics are described in academic and professional literature as "stylized facts."

- The model reflects the correlation among yields of varying maturities.

- Various yield curve shapes are permitted, consistent with historical observation. (This would ordinarily necessitate modelling directly, or indirectly, at least 3 points on the yield curve: short, medium, and long. The frequency and severity of curve inversions need to be reasonable.)

- Generally, nominal yields remain non-negative. (If permitted by the model, negative rates occur rarely, and are not persistent. The actuary runs additional testing to ensure that the inclusion of negative yields does not materially affect the results of the valuation).

- Interest rates do not increase without bound. (The maximum rates produced by the model are consistent with history.)

- The projections start from the conditions prevailing at the valuation date (e.g., the starting yield curve).

- Ideally, the model captures the tendency of interest rates to experience reasonably long periods of relative stability, interspersed with periods of instability. This does not necessarily imply the need for a regime-switching or stochastic volatility model, but could suggest the inadequacy of single-factor models for certain applications.

- Ideally, interest rates movements are correlated with other economic factors, such as equity returns. (At the very least, rates of inflation would bear a logical relationship to interest rates.)

- Ideally, the interest rate model does not permit the earning of material profits at no risk, nor positive profits at zero net cost - i.e., "arbitrage free." (The actuary confirms that any admittance of arbitrage opportunities does not materially distort the valuation results.) However, it is important to note that the "no arbitrage" condition may not be relevant for many applications where the assumed re-investment policy is static or does not involve a 'trading' strategy.

- Parameter estimation is based on sound statistical methods. (While judgement may be applied in setting the valuation parameters, the actuary is aware of the "most probable" parameters suggested by the data - e.g., those obtained through maximum likelihood estimation.)

- Enough scenarios are generated such that the result derived therefrom is not materially different from the result that would be produced if more or many more scenarios had been generated. (To achieve this objective, the required number of scenarios will vary by application, but would typically exceed 100). As a general rule, more scenarios are required whenever the net asset-liability cash flows are sensitive to changes in the economic environment (e.g., contain embedded options).

Sometimes only a single rate (e.g., the short rate) is needed for certain applications. In that case, some of the above conditions might not apply, and the actuary may decide to adopt a simpler model that would be reasonable and adequate for the circumstances. These models are relatively easy to construct, and parameter estimation is straightforward.[2]

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[2] One example of such a model is that proposed by Cox-Ingersoll-Ross. There are many others.