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An international study

May 2021

Document rp221048

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AUTHORS

Doug Andrews
Steve Bonnar
Lori Curtis
Jaideep Oberoi
Aniketh Pittea
Pradip Tapadar

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Population aging, implications for asset values, and impact for pension plans: an international study

By Douglas Andrews, Stephen Bonnar, and Lori Curtis, University of Waterloo, and Jaideep Oberoi, Aniketh Pittea, and Pradip Tapadar, University of Kent

Abstract

This is the capstone paper from a multi-year research agenda entitled "Population Aging, Implications for Asset Values, and Impact for Pension Plans: An International Study." In this paper we:

- Outline demographic-economic scenario generators (DESGs) for Canada, the UK, and the US based on the approach developed by Oberoi et al. (2020);
- Identify the sensitivity of asset returns to demographic factors; and
- Illustrate the impact of varying the demographic factors on the finances of pension plans in Canada, the UK, and the US.

The pension plan risk measure that we analyze is based on the run-off of liabilities for current plan members. This is a very long-term measure.

Our observations of the results are summarized as follows:

- Asset returns in Canada are quite sensitive to variation in the future path of the demographic ratio;
- Returns in the UK and the US are not very sensitive to variation in this ratio;
- Similarly, variation in the future path of the demographic ratio has a material effect on the finances of the Canadian pension plan, but not for either of the UK or US pension plans; and
- We model a portfolio of country-specific equities and bonds for each of the country-specific pension plans. The risk exposure in the Canadian pension plan to varying future demographic paths could be mitigated by employing a portfolio that includes non-Canadian assets.

The bottom line is that the finances of Canadian pension plans are more exposed to the demographic effect on investment returns (older populations are associated with lower investment returns) than plans in the UK or the US. The reason for this is the differing paths of the demographic ratio in the three countries. In addition, Canadian pension plans may be exposed to the "double whammy" of any plan-specific longevity risk exposure.

1. Introduction

This is the capstone paper from a multi-year research agenda entitled “Population Aging, Implications for Asset Values, and Impact for Pension Plans: An International Study”.[1] The project was performed in multiple overlapping stages, and a number of papers and presentations have been made throughout the project. The objectives of the project are to determine if population aging has implications for asset values or returns, which we determine it does; and then, to establish a methodology to quantify this effect and be able to project its impact on asset values or returns in the future, for which the development of a demographic-economic scenario generator (DESG) is key; and to illustrate the impact of the projected asset values or returns on pension plans in three countries: Canada, the UK, and the US. Various literature reviews have been performed throughout the project.

Components of the overall literature directly relevant to a specific paper are included in that paper.

The effect of population aging on investment returns is the topic of a host of academic literature, although there is little consensus in the literature on the potential impact of population aging on asset values. Study conclusions range from catastrophic impact, described as “asset meltdown”, to a moderate effect on the markets but no meltdown, to a total rejection of the asset meltdown hypothesis. See Andrews and Bonnar (2018) for a guide to the relevant literature.

Bonnar et al. (2018) consider the impact of population aging on asset values in the context of a large-scale computable overlapping generations (OLG) model with endogenous labour supply, aggregate risk, and two asset classes, and find that asset prices are moderately lower with an older population. For this project it was not clear how to use the OLG model in pension plan assessment, so a different approach was developed as described below.

Generally, the papers analyzing the impact of demography on investment returns examine a single market only. Notable papers in this literature include seminal work by Bakshi and Chen (1994) regarding equities and bonds, and Mankiw and Weil (1989) regarding housing. Our research agenda included an investigation of the connection between population structure and bond yields (Andrews et al., 2020c) and between population structure and the returns on infrastructure investments (Andrews et al., 2020a). In contrast, this paper examines the effect of demographic change on a portfolio of country-specific equities and bonds, and illustrates the impact on pension plan financial status.

Many businesses incorporate demographic analysis into their long-term strategic plans. Similarly, many sophisticated and long-term investors use demographic analysis in their assessment of stock prices. Generally, these analyses are based on individual companies. The broad market view from a macro perspective is built up from these company-level

[1] Funding for this work has been received from the following organizations: the Canadian Institute of Actuaries, the Institute and Faculty of Actuaries, the Society of Actuaries, the Social Sciences and Humanities Research Council, the University of Kent, the University of Waterloo, and the National Pension Hub. In addition to the authors, the project team for the overall research agenda includes researchers Kathleen Rybczynski, University of Waterloo; Mark Zhou, Canada Mortgage and Housing Corporation; and Miguel Leon-Ledesma, University of Kent.

observations. However, future population structures are not known with certainty, and can potentially be quite different from expectations. This represents a financial risk that a pension plan should consider. In this research, we present a method to incorporate the impact of demographic change on asset values, in order to illustrate the risk this presents for the financing of pension plans.

Demographic change takes place over long time horizons. Accordingly, in selecting an appropriate risk measure to illustrate the impact of demographics on pension plans, we decided to work in an economic capital framework that considers the full run-off of a pension plan's benefit obligations from the time of assessment, approximately a 70-year period. See Andrews et al. (2019a) for a discussion of this risk measure. Although that work did not incorporate a demographic factor into the economic scenario generator (ESG), a demographic factor was part of the ESG for analysis performed for stylized Canadian pension plans in Andrews et al. (2019b). To our knowledge, that was the first instance of incorporating a demographic factor into any long-term economic projection.

In this paper we analyze pension plans for all three countries (Canada, the UK, and the US) using an ESG that includes a demographic ratio that has been defined consistently for all countries. In that regard the paper extends the literature and opens areas for future research.

In this paper we:

- Outline DESGs for Canada, the UK, and the US based on the approach developed by Oberoi et al. (2020);
- Identify the sensitivity of asset returns to demographic factors; and
- Illustrate the impact of varying the demographic factors on the finances of pension plans in Canada, the UK, and the US.

Our observations of the results may be summarized as follows:

- Asset returns in Canada are quite sensitive to variation in the future path of the demographic ratio;
- Returns in the UK and the US are not very sensitive to variation in this ratio;
- Similarly, variation in the future path of the demographic ratio has a material effect on the finances of the Canadian pension plan, but not for either of the UK or US pension plans; and
- We model a portfolio of country-specific equities and bonds for each of the country-specific pension plans. The risk exposure in the Canadian pension plan to varying future demographic paths could be mitigated by employing a portfolio that includes non-Canadian assets.

The bottom line is that the finances of Canadian pension plans are more exposed to the demographic effect on investment returns (older populations are associated with lower investment returns) than plans in the UK or the US. In addition, they may be exposed to the "double whammy" of any plan-specific longevity risk exposure.

The rest of this paper is organized as follows. Section 2 presents the demographic ratio used and shows how it varies by country. It describes the graphical model and discusses considerations regarding the choice of graphical structure for each country. It reviews the literature regarding the use of demographic factors in economic scenario generation. Section 3 outlines the impact of the demographic variable on pension plan finances in each country. Section 4 identifies areas for future research and summarizes the conclusions from the analysis. Further details on the graphical models and the distributions of pension plan financial results are contained in the Appendices.

2. Graphical Model

2.1 Demographic Factor

As discussed in Oberoi et al. (2020), a modeller can use a number of alternative criteria, like AIC, BIC, or simultaneous *p*-values, to arrive at potentially different plausible model structures. Typically, judgement is used to make a final model choice. Details of three possible alternative structures for each country are set out in Appendix A. Our recommended structures are summarized in the body of this report.

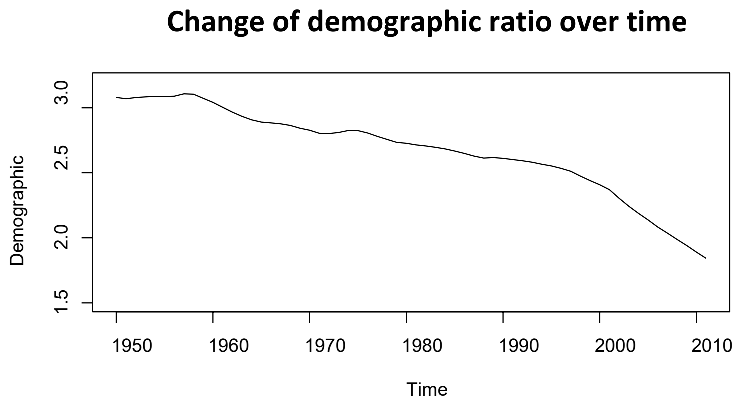
For this report, we adopt the following definition of demographic ratio:

$$\text{Demographic ratio} = \frac{\text{Individuals between ages 20 and 54}}{\text{Individuals between ages 55 and 85}}$$

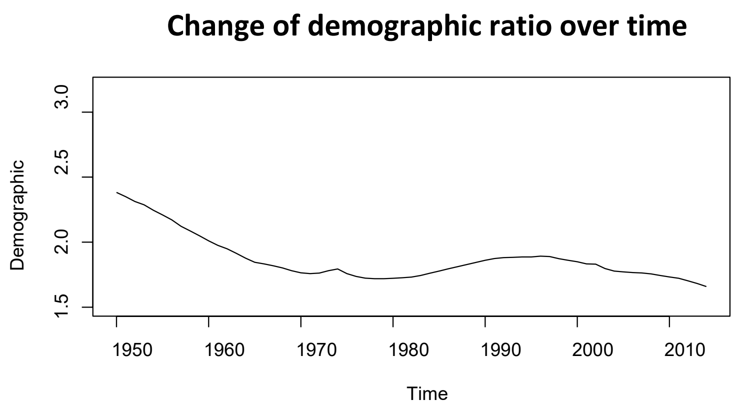
Alternative definitions of demographic ratio are possible. In fact, in our preliminary investigations, we considered two possible alternative cut-off ages: 54 and 64. We found that for the UK and the US, a cut-off age of 64 leads to graphical structures with weak and inconclusive links between the economic and demographic variables. In contrast, with a cut-off age of 54, both the UK and US graphical models incorporate robust links between the economic and demographic variables. For Canada, consistent structures were found for both the cut-off ages considered. For sake of consistency, here we use a cut-off age of 54, because this assumption produces consistent model structures for all three countries. Interested readers may refer to Andrews et al. (2019b) for an analysis of Canadian data, where a cut-off age of 64 was used.

Figures 1a, 1b, and 1c show the evolution of this demographic ratio over the period 1950 to 2012 for each country.

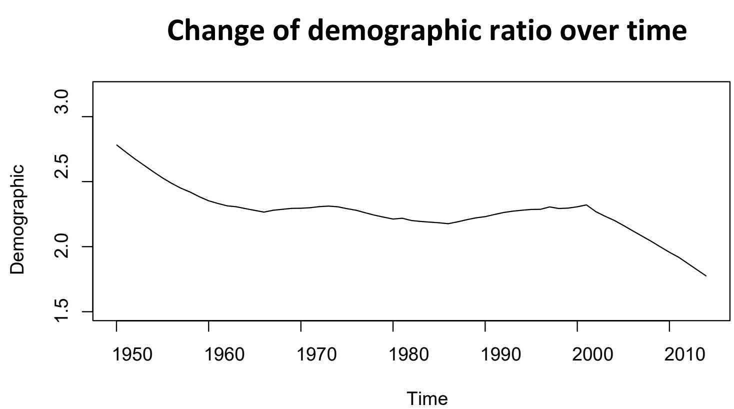
Figure 1: Change of population demographic — Canada, UK, and US



(a) Change of population demographic — Canada



(b) Change of population demographic — UK



(c) Change of population demographic — US

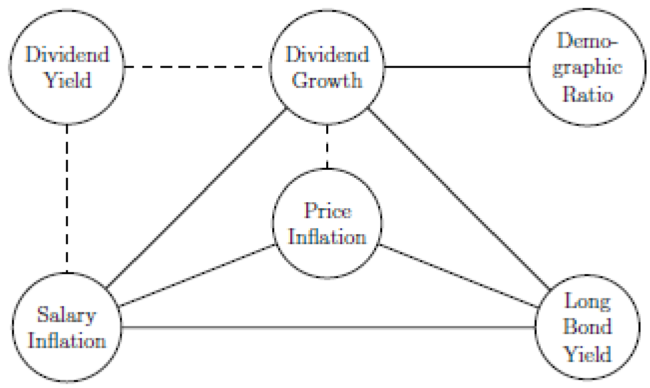
We can see from these summary statistics that the ratio commences its drop around the year 2000, when the oldest baby boomers reach age 55 and move from the numerator to the denominator of the ratio. Of particular note is that the ratio declined much more quickly in Canada and the US, relative to the UK. Canada and the US truly had a baby boom. The UK may be described as having had a “baby blip”. The ratio is more or less stable in the UK from 1970. This has an impact on both the structure of the model and the sensitivity of future investment returns to different future paths of the demographic ratio.

2.2 Model Structure for Each Country

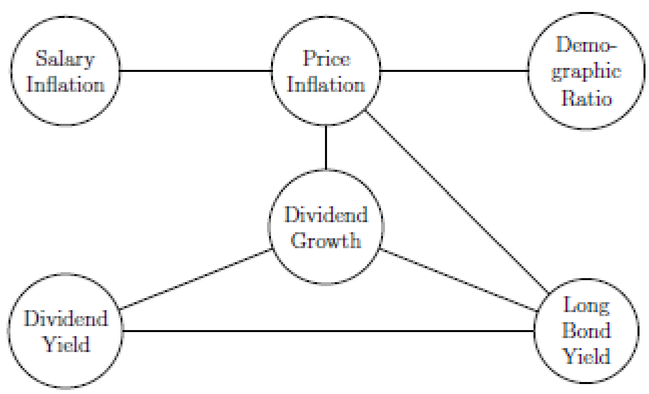
Figure 2 shows the recommended DESGs for the three countries. A quick note on the differences between the bold (non-dashed) and dashed links in the Canadian structure, which is based on simultaneous *p*-values: The bold links represent strong correlations between the corresponding variables. We have assumed that *p*-values of less than 0.1 indicate strong correlations. The dashed links indicate weaker or *intermediate* *p*-values, between 0.1 and 0.6 in this case. If there are no links between variables, then the *p*-values are greater than 0.6. The cut-off *p*-values of 0.1 and 0.6 are based on judgement.

Figure 2: Demographic-economic graphical model structures

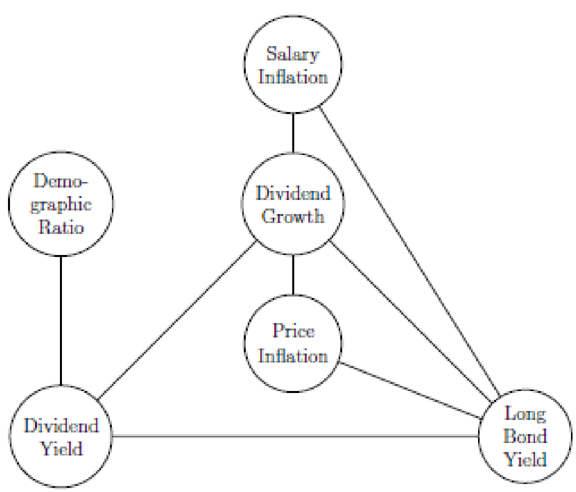
Canada – Simultaneous *p*-values



UK – AIC



US – BIC



While the different structures of the models are interesting, the important thing to determine is whether the demographic node has any material impact on future investment returns. Table 1 shows both expected returns and volatility for equities and bonds for each country using the 25th, 50th, and 75th percentile future paths of the demographic variable. Given how we have defined the demographic ratio, the 25th percentile represents an older population than the 50th percentile. The 75th percentile represents a younger population. In the table, the number in brackets is the projected value of the demographic variable in 50 years' time.

Table 1: Impact of different demographic paths using a 50-year projection

Canada		**Equity return**	**Bond return**
75th percentile	μ	11.03	5.41
(1.58)	σ	17.4	6.7
50th percentile	μ	10.48	5.28
(1.06)	σ	17.4	6.8
25th percentile	μ	9.97	5.13
(0.56)	σ	17.5	6.8
United Kingdom		**Equity return**	**Bond return**
75th percentile	μ	11.13	5.57
(1.85)	σ	7.85	4.80
50th percentile	μ	11.22	5.69
(1.45)	σ	7.84	4.81
25th percentile	μ	11.30	5.80
(1.07)	σ	7.84	4.82
United States		**Equity return**	**Bond return**
75th percentile	μ	8.27	5.00
(1.80)	σ	7.57	5.23
50th percentile	μ	8.52	5.11
(1.16)	σ	7.52	5.23
25th percentile	μ	8.77	5.21
(0.54)	σ	7.48	5.24

In Subsection 2.1 we pointed out that the downward trend in the demographic factor was much more severe for Canada and the US, relative to the UK. This observation is echoed in Table 1, where the projected inter-quartile range (i.e., spread from the 25th to the 75th percentile) of the demographic ratio in 50 years' time is 1.02 for Canada, 1.26 for the US, and .78 for the UK. This difference is reflected in the sensitivity of returns to the different paths of the demographic ratio. In Canada, average annual equity returns drop by over 100 basis points and bond returns drop by almost 30 basis points as the demographic ratio moves across the inter-quartile range. In the US, the average annual returns *increase* by 50 and 21 basis points for equities and bonds, respectively. Finally, in the UK the average annual returns also increase by 17 and 23 basis points.

The impact on investment returns across different paths of the demographic ratio is affected by both the steepness of the path and its variability. The steepness of the paths in the three countries is illustrated in Figure 1. The path is influenced by the drop-in fertility rates since the end of the baby boom – large in Canada and the US; and more modest in the UK. It is also influenced by the rate of mortality improvement that has been more modest in the US than Canada or the UK. Net migration also affects the ratio (both the amount and its age structure), and this has varied by country over the observation period. We have not analyzed the effect that the components have on the historical demographic ratio, and we do not project components when projecting the demographic ratio.

The variability of the future path of the demographic ratio is shown in Tables 3, 6, and 9 of Appendix B. For Canada and the UK, the standard deviation of the demographic ratio is 98 and 103 basis points respectively. For the US, the standard deviation is roughly 25% larger at 123 basis points.

While the impact of different paths of the demographic ratio is clearly material in Canada, the impact in the UK is barely significant statistically, and not at all significant economically, particularly given model uncertainty. The impact in the US may also not be economically significant. The true illustration of whether the impact is economically significant is shown in the following section that sets out the distribution of pension plan financial results. Further details of the models is provided in Appendices A and B.

3. Pension Financial Results

3.1 Economic Capital Approach

Given the long-term nature of pension risk, we examine the "run-off" liabilities of a plan. These reflect the current plan membership and continued accrual of benefits, but no new entrants to the plans. As such, the time horizon for this analysis is the time until the last of the current plan members dies, perhaps as much as 90 years. We examine the present value of the future surplus or deficit of the plans, where the present value is determined based on the future returns on the pension fund. Mathematically, this is expressed as follows:

$$V_0 = A_0 - \sum_{t=0}^{T} X_t D_{0,t}$$

In this equation, V_0 is the present value of the future surplus or deficit, A_0 is the current value of the assets of the pension fund, X_t represents each year's net cash flow (i.e., benefit payments less contributions), and $D_{0,t}$ is the discount factor based on the return on the pension fund assets over the period from today to time t.

Intuitively, V_0 represents the amount of money that could be taken out of (or deposited into) the pension fund so that the last dollar in the fund is used up when the last beneficiary of the plan receives the last payment.

We run 10,000 simulations[2] of the pension plan's finances based on the DESGs described in Section 2 in order to look at the distribution of V_0. So that this value does not depend on the size of the plan or the currency, we express V_0 as a percentage of the starting assets, A_0.

In line with the measures under Solvency II for financial institutions, we look at the 0.5th percentile of the distribution of the run-off deficit/surplus. Conceptually, this is the proportion by which the current assets would need to be increased (decreased) so that we have a 199-in-200 chance of having sufficient assets to pay all of the promised benefits. This is a stringent requirement, so we also look at the 10th and 50th percentiles of the distribution of V_0.

Details of the membership data and plan provisions for the plans that we have modelled are provided in Andrews et al. (2019b) for Canada and Andrews et al. (2019a) for the UK and the US. At a high level, the plans for Canada and the UK are fully sensitive to inflation (final average earnings or final earnings benefit, fully indexed to price inflation post-employment). The plan for the US is partially sensitive to inflation (final average benefit, no indexing post-employment).

The asset allocation that we have used to model the Canadian pension plan is 55% equities and 45% bonds, which is comparable to the policy allocation for the underlying plan. For the UK and the US, we use an allocation of 70% equities and 30% bonds.

[2] In each of these simulations, the contribution rate and the asset allocation are kept constant over time.

3.2 Results

In Table 2 the columns show *VaR* and *ES*, expressed as a percentage of starting assets. *VaR* is value at risk, which is the required amount of economic capital at various percentiles. So, for the 50th percentile demographic path, the median for the Canadian pension plan (Panel 1 in the table) shows there is an excess amount of assets of 45%. At the 0.5th percentile, the assets would need to be augmented by an additional 144% in order to provide for all of the promised benefits. *ES* is the expected shortfall, which is the average amount of deficit at the various percentiles. For example, the 10th percentile represents all of the scenarios where the shortfall is 13% of starting assets or more. On average, these scenarios have a shortfall of 55% of starting assets.

Table 2 confirms our suspicions at the end of Section 2. The impact on the economic capital of the pension plan of moving across the inter-quartile range (i.e., from the 75th to the 25th percentile path of the demographic variable) is material for Canada, but not for either the UK or the US. Focusing on the 10th percentile VaR, it worsens by 17% in Canada, but by only 8% in the UK, and it improves by 7% in the US. Similar observations can be made at the 0.5th percentile.

In Canada, the investment returns for both equities and bonds reduce as the demographic variable moves across the inter-quartile range, resulting in worse outcomes for the financial status of the pension plan. In the UK and the US, investment returns improve as the demographic variable moves across the inter-quartile range. Assuming that there is some positive correlation in future trends of the demographic variable across countries, this suggests that the risk exposure of Canadian pension plans to varying demographic paths could be mitigated by employing a portfolio that includes non-Canadian assets. However, all of this analysis is based on the risk measure that we have employed, which is very long-term. Interested readers can refer to Andrews et al. (2020b) for a discussion of the impact of time horizon on asset allocation decisions.

Changes in the financial position of the pension plan[3] are affected by the magnitude of changes in the investment returns on bonds and equities, and in salary and price inflation to the extent that they affect benefit payments. When examining the change in financial position of the pension plan over the inter-quartile range for each country we note the following. For Canada, the decline in investment returns contributes most to the decline in financial position (see Table 5). For the UK, the impact of the change in inflation on benefit payments is greater than the increase in financial position (see Table 8). In the US, the increase in investment returns contributes to the improvement in financial position (see Table 11).

The full distributions of economic capital in the various scenarios are provided in Appendix C.

[3] The changes in the financial position of the pension plans are exclusive of the impact of unexpected plan-specific excess mortality improvements.

Table 2: Summary of results

(V_0 as a percentage of A_0)

Canada						
Demographic percentiles	75th percentile		50th percentile		25th percentile	
Distribution percentiles	VaR	ES	VaR	ES	VaR	ES
50th percentile	48	14	45	8	43	2
10th percentile	-4	-41	-13	-55	-21	-65
0.5th percentile	-124	-182	-144	-210	-156	-227
United Kingdom						
Demographic percentiles	75th percentile		50th percentile		25th percentile	
Distribution percentiles	VaR	ES	VaR	ES	VaR	ES
50th percentile	29	-6	26	-9	23	-13
10th percentile	-27	-59	-31	-64	-35	-68
0.5th percentile	-125	-159	-130	-166	-137	-173
United States						
Demographic percentiles	75th percentile		50th percentile		25th percentile	
Distribution percentiles	VaR	ES	VaR	ES	VaR	ES
50th percentile	33	6	34	8	35	11
10th percentile	-9	-38	-6	-31	-2	-26
0.5th percentile	-97	-150	-87	-134	-77	-119

4. Concluding Observations

4.1 Conclusions

In this paper we develop DESGs for Canada, the UK, and the US. We show that asset returns in Canada are quite sensitive to variation in the future path of the demographic factor, while returns in the UK and the US are not. In examining the impact on pension plan finances, we see a similar pattern. The impact of varying the future path of the demographic factor is material on the Canadian pension plan (older populations are associated with lower investment returns), but not for either of the UK or the US pension plans. In addition, these plans may be exposed to the "double whammy" of any plan-specific longevity risk exposure.

Of note, we model a portfolio of country-specific equities and bonds for each of the country-specific pension plans. The risk exposure in the Canadian pension plan to varying future demographic paths could be mitigated by employing a portfolio that includes non-Canadian assets.

4.2 Areas for Future Research

There is a wide range of avenues to extend the research conducted in this capstone paper and the prior supporting research. Those topics include the following:

- Determining the demographic impact on a wider range of asset classes so that risk analysis can be performed using a wider variety of asset allocations. It might be that the demographically affected returns on different asset classes could be better modelled using different demographic factors;
- Due to the nature of the model that we use, we may not be capturing alternative equilibrium effects in the future scenarios. It may be that the lower sensitivity of future returns to different demographic paths in the US and the UK could be due to historical capital inflows to the financial markets.
- Economic capital is not commonly used as a risk measure for pension plans. Developing an approach to incorporate the demographic impact on asset values that could be used with alternative risk measures would increase interest in this research;
- Exploring different demographic factors that could be incorporated into an ESG, including analysis of when a DESG is preferable to an ESG, and vice versa;
- Illustrating the impact of demographic risk on investment returns on entities other than pension plans;
- Examining whether asset portfolios could be developed that diversify the demographic effect on asset returns or that do not include such an effect; and
- As the mortality experience of populations continues to improve, continuing research to update previous results, and perhaps discover new relationships.

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Appendix A: Demographic-Economic Graphical Models

A.1 Introduction

This appendix provides an overview of the demographic-economic scenario generators (DESGs) for Canada, the UK, and the US using a graphical modelling approach developed by Oberoi et al. (2020). The interested reader can refer to that paper for full details of this approach. To build our DESGs, we require data for consumer price inflation (I), salary inflation (J), dividend yield (Y), dividend growth (K), and long-term bond yield (C) over a long time horizon. In Section A.2, we provide an overview of the data we use. While the various data sources cover different time periods, the analysis covers the period from 1950 to 2014 due to constraints from the demographic data horizon. In Section A.3, we outline the modelling approach employed. Finally, in Sections A.4, A.5, and A.6 we describe how the graphical model structures were selected for Canada, the UK, and the US, respectively.

A.2 Data Sources

A.2.1 Canada

The demographic variables come from the Canadian census provided by Statistics Canada. For price inflation and salary inflation, we use data from two sources. Data between 1950 and 2000 come from Emmanuel Saez, who provides data for the retail price index and average wages. From 2001 onwards, we use inflation data and salary inflation data from the Federal Reserve Economic Data (FRED) database. For dividend yield, dividend growth and long term bond yield, we use data from Statistics Canada, which provides data for the Toronto Stock Exchange index, Toronto Stock Exchange dividend yield and 10-year government bond yield for the whole period.

A.2.2 United Kingdom

The demographic data has been taken from a UN database.[4] The economic data we use has been generously provided by David Wilkie, who has carried out a range of checks and matching exercises to construct all the relevant time series. We use the complete dataset provided by David Wilkie, which consists of annual values from 1926 to 2017 as at the end of June each year.

A.2.3 United States

The demographic data has been taken from the same UN database as for the UK. The economic data come from two sources. The first is Robert Shiller, who provides online data for the Consumer Price Index (CPI), S&P 500 price index, S&P 500 dividend index, and 10-year bond yield. The second is Emmanuel Saez, who provides online data for average wages in the United States. The data we use extend from 1913 to 2015.

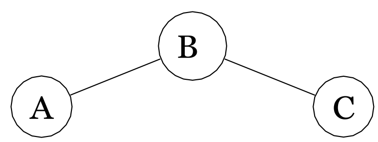
[4] UN. 2012. *World Population Prospects: The 2012 Revision*. Department of Economic and Social Affairs, Population Division, United Nations.

A.3 Modelling

A graph, $G = (V, E)$, is a structure consisting of a finite set of variables V (or vertices or nodes) and a finite set of edges E between these variables. The existence of an edge between two variables represents a connection or some form of dependence. The absence of this connection represents conditional independence.

For instance, if we have a set of three variables, $V = (A, B, C)$, where A is connected to B and not to C, but B is connected to C, then A is connected to C via B. A is then conditionally independent of C, given B. Such a structure can be graphically represented by drawing circles or solid dots representing variables with lines between them representing edges. The graphs we consider here are called undirected graphs because the edges do not have a direction (which would otherwise be represented by an arrow). Such graphs model association rather than causation. The graphical model described here with three variables, A, B and C, is shown in Figure 3.

Figure 3: Example of a graphical model



Graphical models enable us to represent the covariance structure with dimension reduction, by effectively capturing conditional independence between pairs of variables and shrinking the relevant bivariate links to zero while allowing for weak correlations to exist in the simulated data.

The aim of a graphical model DESG is to give importance to long-run stable relationships and to generate a distribution of joint scenarios. This takes the approach of estimating the joint distribution of the residuals of individual time series regressions and focuses on the dependence between the residuals. For each variable other than the demographic variable, a time series model is fitted independently, and then a graphical model is fitted to the time series residuals across variables. For each time series, X_t, the following AR(1) time series model formulation is used.

$$\mu_x = E[X_t]$$

$$Z_t = X_t - \mu_x$$

$$Z_t = \beta Z_{t-1} + e_t, \text{ where } e_t \sim N(0, \sigma^2)$$

From Figures 1a, 1c, 1b, we note the downward trend in the population demographic data in each country, though the trend is quite muted in the UK. Given that the data are not mean-reverting, we apply a first order differencing to the data and then model the differenced series as an AR(1). In other words, we model the demographic ratio as an ARIMA(1,1,0).

$$\mu_x = E[X_t]$$

$$Z_t = X_t - \mu_x$$

$$Z_t = Z_{t-1} + \beta_x (Z_{t-1} - Z_{t-2}) + e_{x,t} \text{ where } e_{x,t} \sim N(0, \sigma_x^2)$$

A.4 Graphical Model Structure for Canada

Figure 4 shows three possible graphical structures based on AIC, BIC, and simultaneous p-values. BIC typically produces more parsimonious models compared to AIC, and this is evident in the smaller number of links in the BIC model structure. The structure based on simultaneous p-values is intermediate between AIC and BIC.

A quick note on the differences between the bold (non-dashed) and dashed links in the structure based on simultaneous p-values: The bold links represent strong correlations between the corresponding variables. For Canadian data, we have assumed that p-values of less than 0.1 indicate strong correlations. The dashed links indicate weaker or intermediate p-values, between 0.1 and 0.6 in this case. If there are no links between variables, then the p-values are greater than 0.6. The cut-off p-values of 0.1 and 0.6 are based on judgement.

The three models presented are not very different. The model based on simultaneous p-values has two additional links compared to the BIC model, while the AIC model has one more link compared to the model based on simultaneous p-values.

We recommend using the model based on simultaneous p-values for Canada as it provides a good balance between parsimony and tractability in terms of ease of implementation for simulation purposes. Note that this model structure is different from the one proposed in Andrews et al. (2019b), as in that report a different cut-off age was used to define the demographic ratio.

A.5 Graphical Model Structure for the UK

Figure 5 shows the alternative models for the UK. Although, as seen previously for Canada, the BIC model is more parsimonious than the AIC model with fewer links, the AIC model for UK does not have too many links to begin with. Also note that in the BIC model, the demographic ratio is not linked to any of the economic variables.

As before, the simultaneous p-values model shows bold links for p-values less than 0.1 and dashed links for p-values between 0.1 and 0.6. In addition, we have added dotted links for p-values between 0.6 and 0.75. It shows that for the UK data, the demographic ratio's links to the economic variables are weak. The strength of the links worsens further if we define the demographic ratio with a cut-off age of 64.

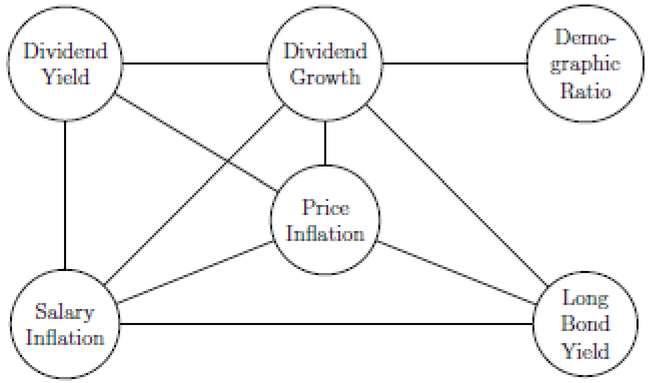
For the UK, the BIC model is not adequate for our purpose. The simultaneous *p*-values model, with *p*-values going up to 0.75, does include links between the economic variables and the demographic ratio. However, it is computationally inefficient because the model cannot be *triangulated*, a graphical model technique to ensure computational efficiency for simulation purposes. So, we recommend the AIC model for the UK.

A.6 Graphical Model Structure for the US

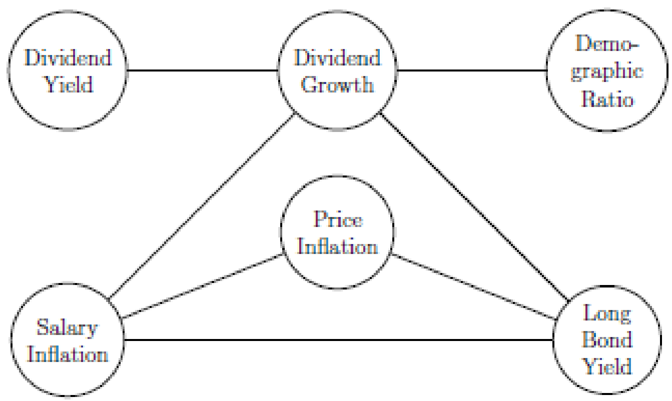
Figure 6 shows the alternative models for the US. As for the Canadian and UK models, the US BIC model has fewer links than the AIC model. The model for simultaneous *p*-values is intermediate. The BIC and simultaneous *p*-values models have the same links, except the link between price inflation and long bond yield in the BIC model is replaced by a new link between price inflation and dividend yield. For simultaneous *p*-values, we have used bold links for *p*-values less than 0.1, while the dashed links indicate *p*-values between 0.1 and 0.6. In terms of a recommendation for the US, either the BIC model or the simultaneous *p*-value model would be adequate for our purpose. However, a direct link between price inflation and long bond yields seems more appropriate, so we choose the BIC model.

Figure 4: Alternative graphical model structures for Canada

Canada – AIC



Canada – BIC



Canada – Simultaneous *p*-values

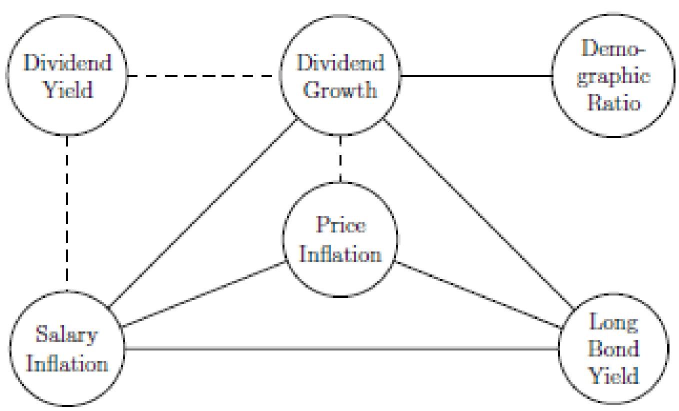


Figure 5: Alternative graphical model structures for the UK

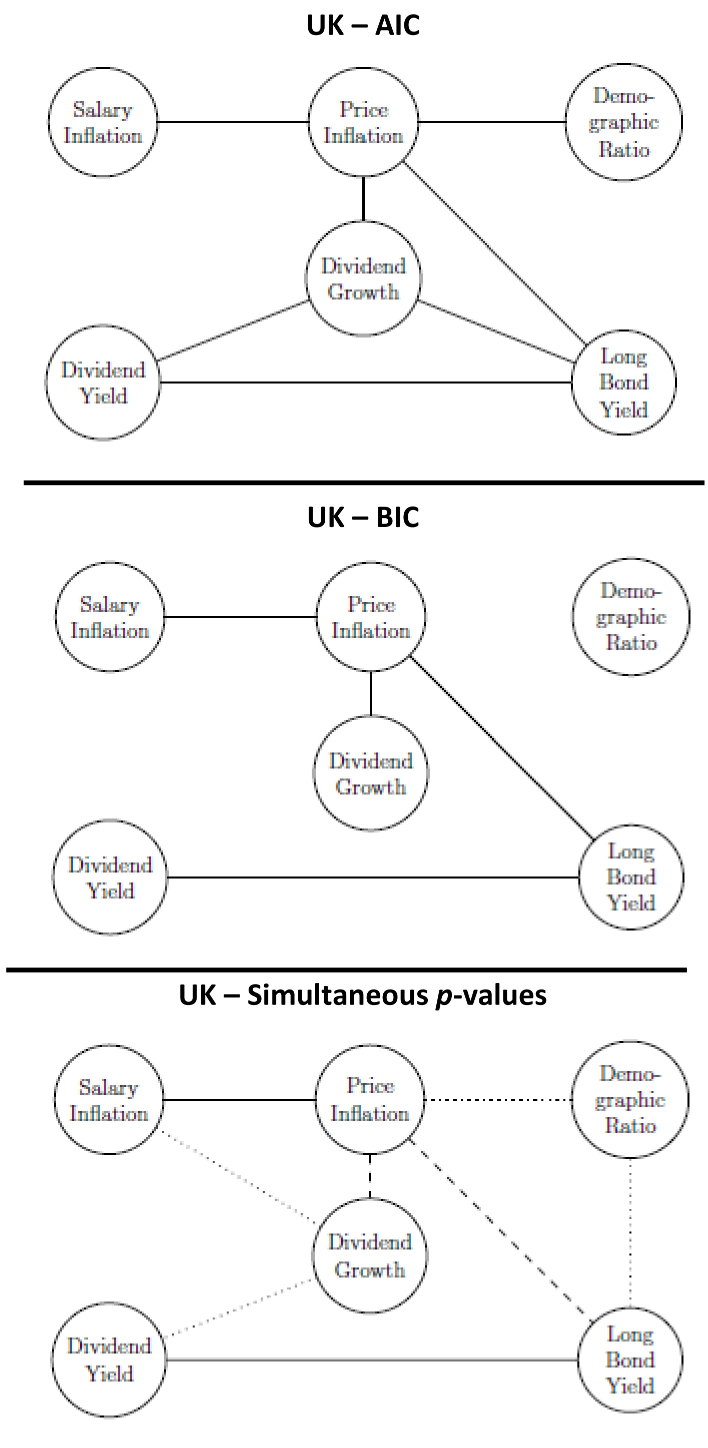
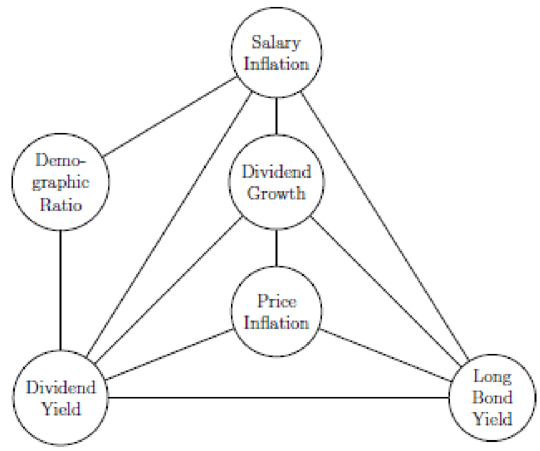
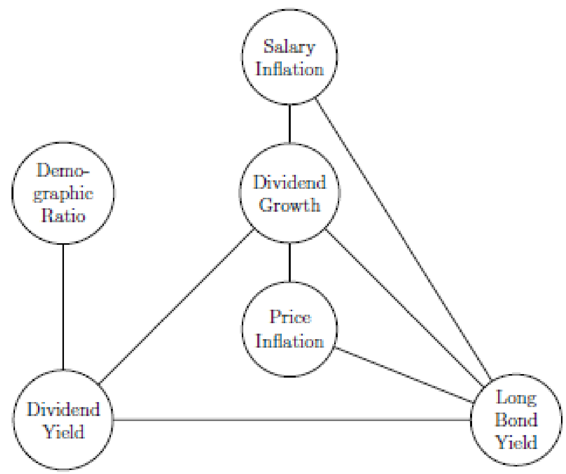


Figure 6: Alternative graphical model structures for the US

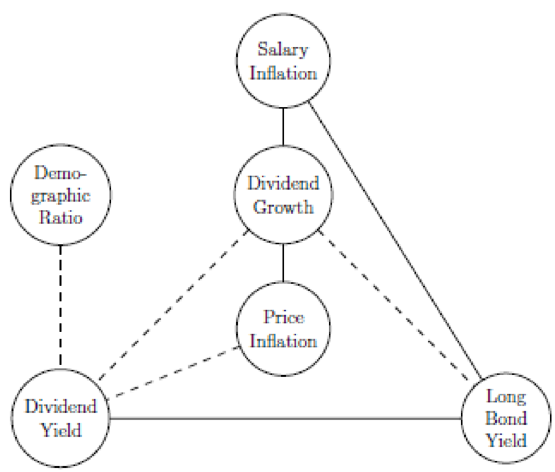
US – AIC



US – BIC



US – Simultaneous *p*-values



Appendix B: Parameters and Output from the Demographic-Economic Scenario Generators

B.1 Introduction

In this appendix, we set out for each country the following items:

- The time series parameter estimates;
- The partial correlations of the residuals;
- Fanplots from the simulations;
- Fanplots of the demographic ratio; and
- A summary of the change in model variables across different demographic paths.

B.2 Canadian Graphical Model

Table 3: Time series parameter estimates – Canada

	μ	β	σ
I_t	0.0361	0.7105	0.0225
J_t	0.0600	0.5358	0.0415
Y_t	0.0367	0.9112	0.0053
K_t	0.0684	0.1044	0.1755
C_t	0.0601	0.9699	0.0075
D_t	2.6576	0.9448	0.0098

Table 4: Partial correlations of residuals – Canada

	I_t	J_t	Y_t	K_t	C_t	D_t
I_t	1					
J_t	0.71	1				
Y_t	0.14	-0.22	1			
K_t	-0.25	0.38	0.27	1		
C_t	0.38	-0.33	0.17	0.42	1	
D_t	0.11	0.07	0.18	0.34	0.04	1

Figure 7: Fanplots of simulations – Canada

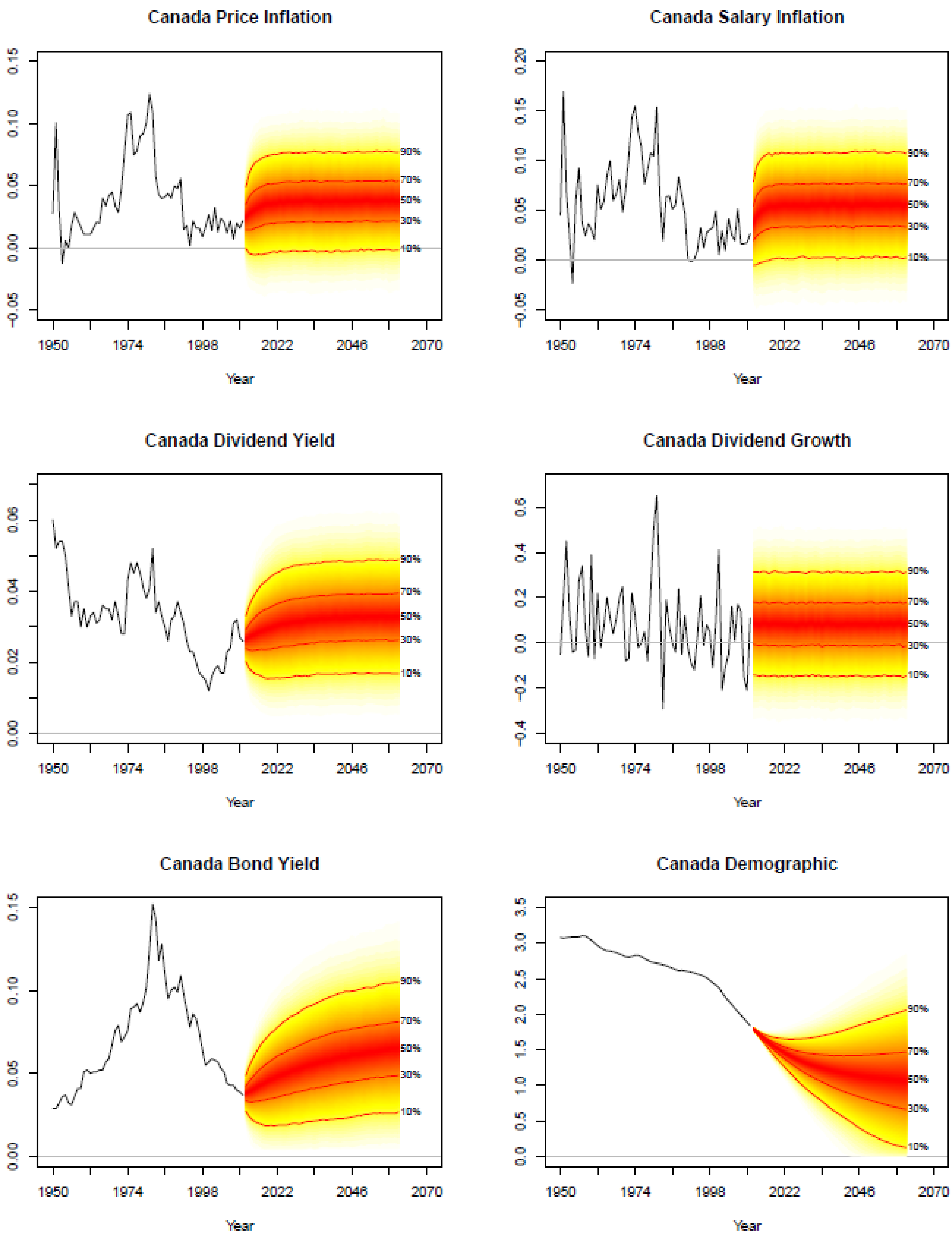
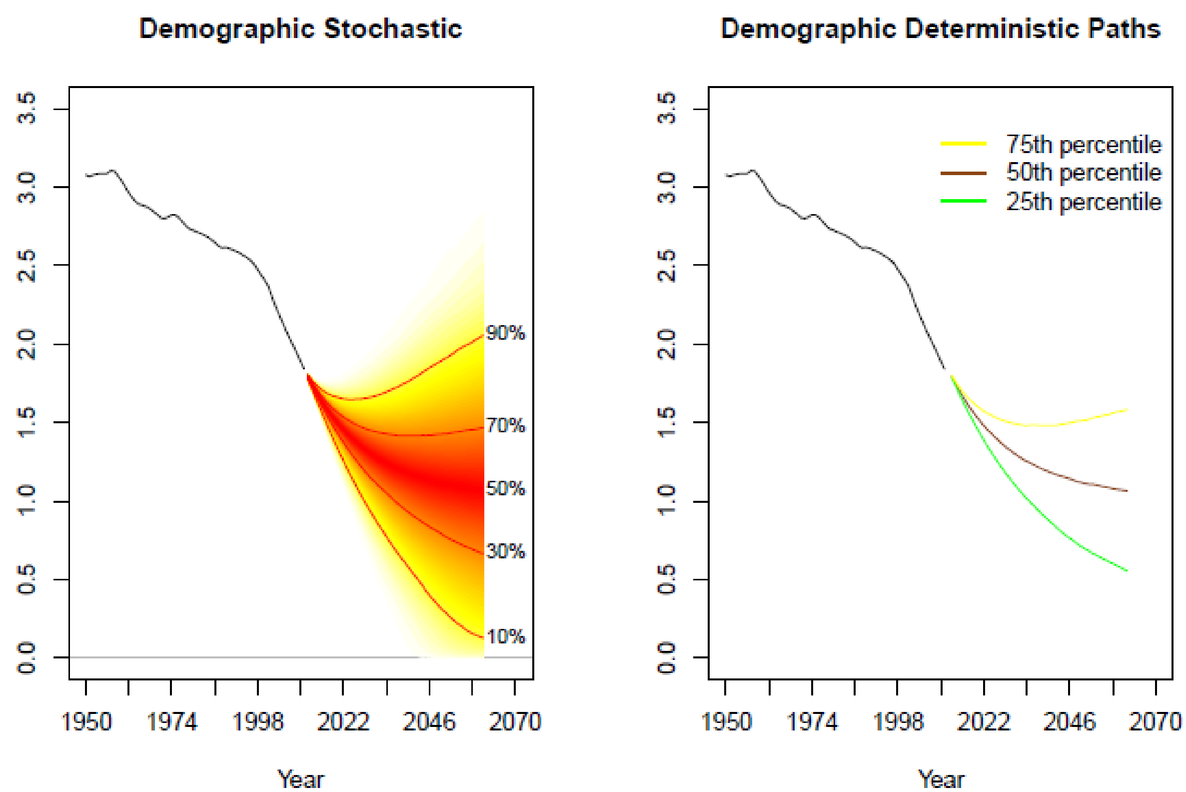


Figure 8: Fanplots of demographic ratio – Canada



The left-hand graph shows the full distribution of the projected demographic ratio. The right-hand graph shows selected paths from that distribution.

Table 5: Mean and standard deviation of simulations for different demographic paths using a 50-year projection – Canada

		I_t	J_t	Y_t	K_t	C_t	Equity Return	Bond Return
75th percentile	μ	3.69	5.53	3.19	8.82	5.71	11.03	5.41
(1.58)	σ	3.05	4.11	1.16	16.9	2.72	17.4	6.74
50th percentile	μ	3.65	5.48	3.15	8.28	5.56	10.48	5.28
(1.06)	σ	3.05	4.11	1.16	16.8	2.70	17.4	6.75
25th percentile	μ	3.61	5.43	3.10	7.79	5.42	9.97	5.13
(0.56)	σ	3.05	4.12	1.16	16.9	2.69	17.5	6.77

The number in brackets is the projected value of the demographic variable in 50 years' time.

B.3 UK Graphical Model

Table 6: Time series parameter estimates – UK

	μ	β	σ
I_t	0.0530	0.6437	0.0342
J_t	0.0651	0.7733	0.0289
Y_t	0.0457	0.7157	0.0080
K_t	0.0688	0.2755	0.0717
C_t	0.0739	0.9573	0.0097
D_t	1.8679	0.8922	0.0103

Table 7: Partial correlations of residuals – UK

	I_t	J_t	Y_t	K_t	C_t	D_t
I_t	1					
J_t	0.50	1				
Y_t	0.28	0.10	1			
K_t	0.37	0.32	0.24	1		
C_t	0.40	0.18	0.56	0.17	1	
D_t	-0.23	-0.21	-0.07	0.03	0.06	1

Figure 9: Fanplots of simulations – UK

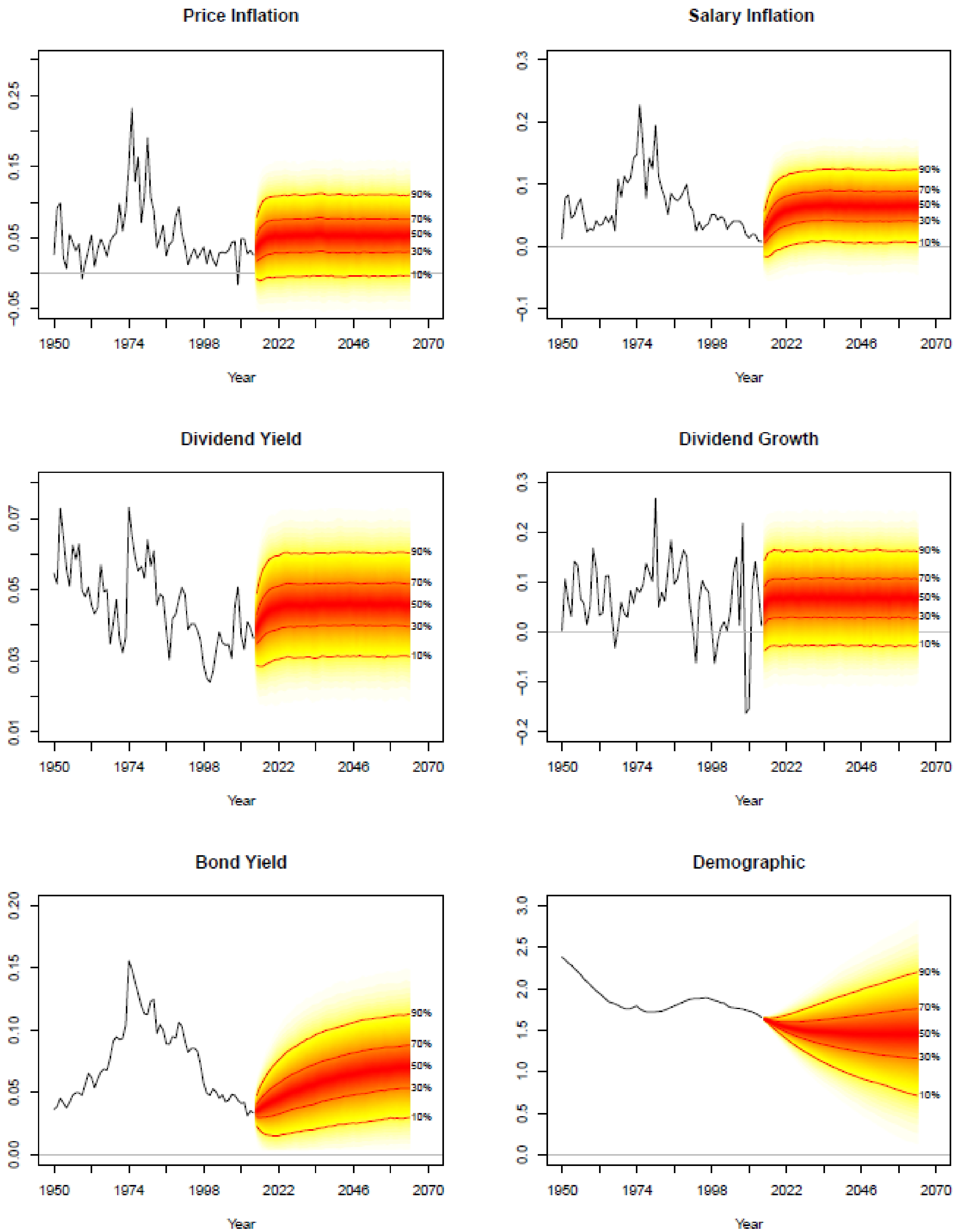
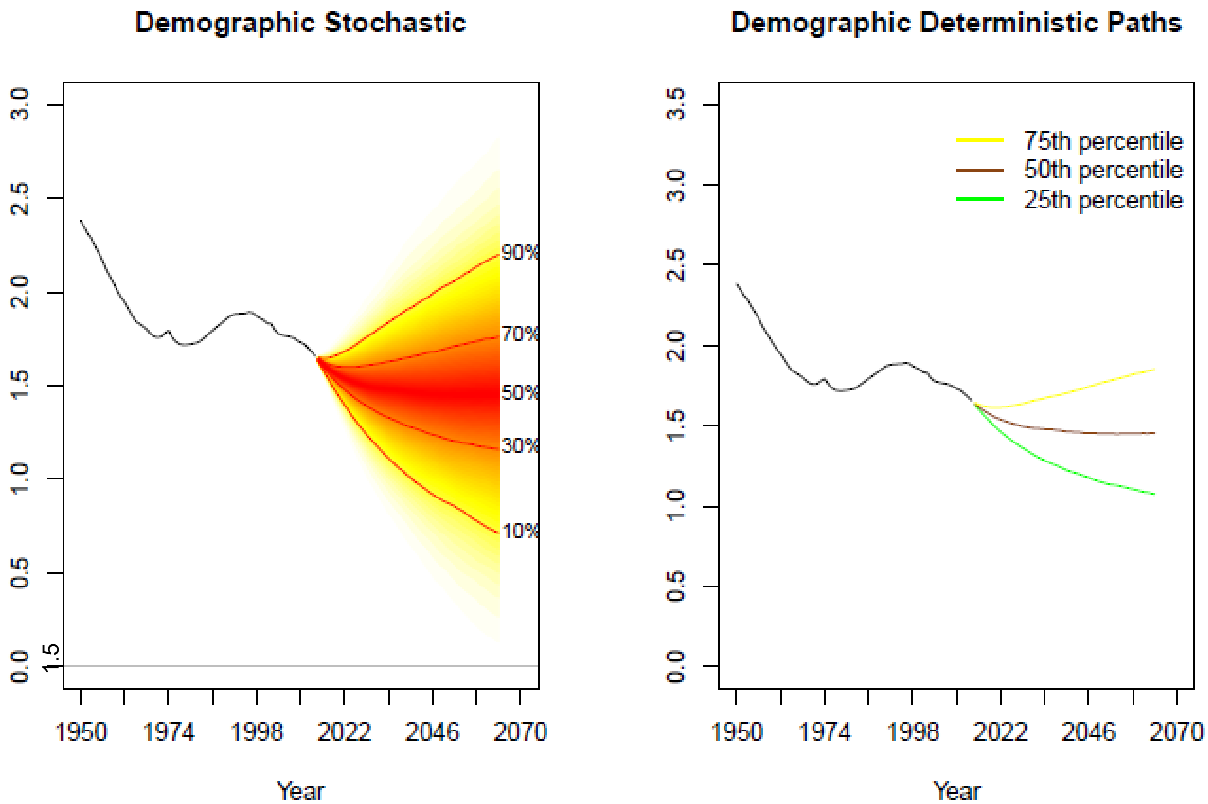


Figure 10: Fanplots of demographic ratio – UK



The left-hand graph shows the full distribution of the projected demographic ratio. The right-hand graph shows selected paths from that distribution.

Table 8: Mean and standard deviation of simulations for different demographic paths using a 50-year projection – UK

		I_t	J_t	Y_t	K_t	C_t	Equity Return	Bond Return
75th percentile	μ	5.01	6.00	4.51	6.76	5.81	11.13	5.57
(1.85)	σ	4.33	4.54	1.13	7.43	2.96	7.85	4.80
50th percentile	μ	5.21	6.13	4.52	6.84	5.93	11.22	5.69
(1.45)	σ	4.33	4.54	1.13	7.43	2.97	7.84	4.81
25th percentile	μ	5.39	6.25	4.54	6.91	6.04	11.30	5.80
(1.07)	σ	4.33	4.53	1.13	2.99	2.70	7.84	4.82

The number in brackets is the projected value of the demographic variable in 50 years' time.

B.4 US Graphical Model

Table 9: Time series parameter estimates – US

	μ	β	σ
I_t	0.0364	0.6529	0.0223
J_t	0.0480	0.4586	0.0282
Y_t	0.0329	0.9352	0.0057
K_t	0.0593	0.3328	0.0668
C_t	0.0592	0.9226	0.0113
D_t	2.2657	0.9288	0.0123

Table 10: Partial correlations of residuals – US

	I_t	J_t	Y_t	K_t	C_t	D_t
I_t	1					
J_t	0.29	1				
Y_t	0.41	0.07	1			
K_t	0.47	0.36	0.37	1		
C_t	0.30	0.37	0.37	0.10	1	
D_t	-0.02	0.15	-0.29	-0.19	0.08	1

Figure 11: Fanplots of simulations – US

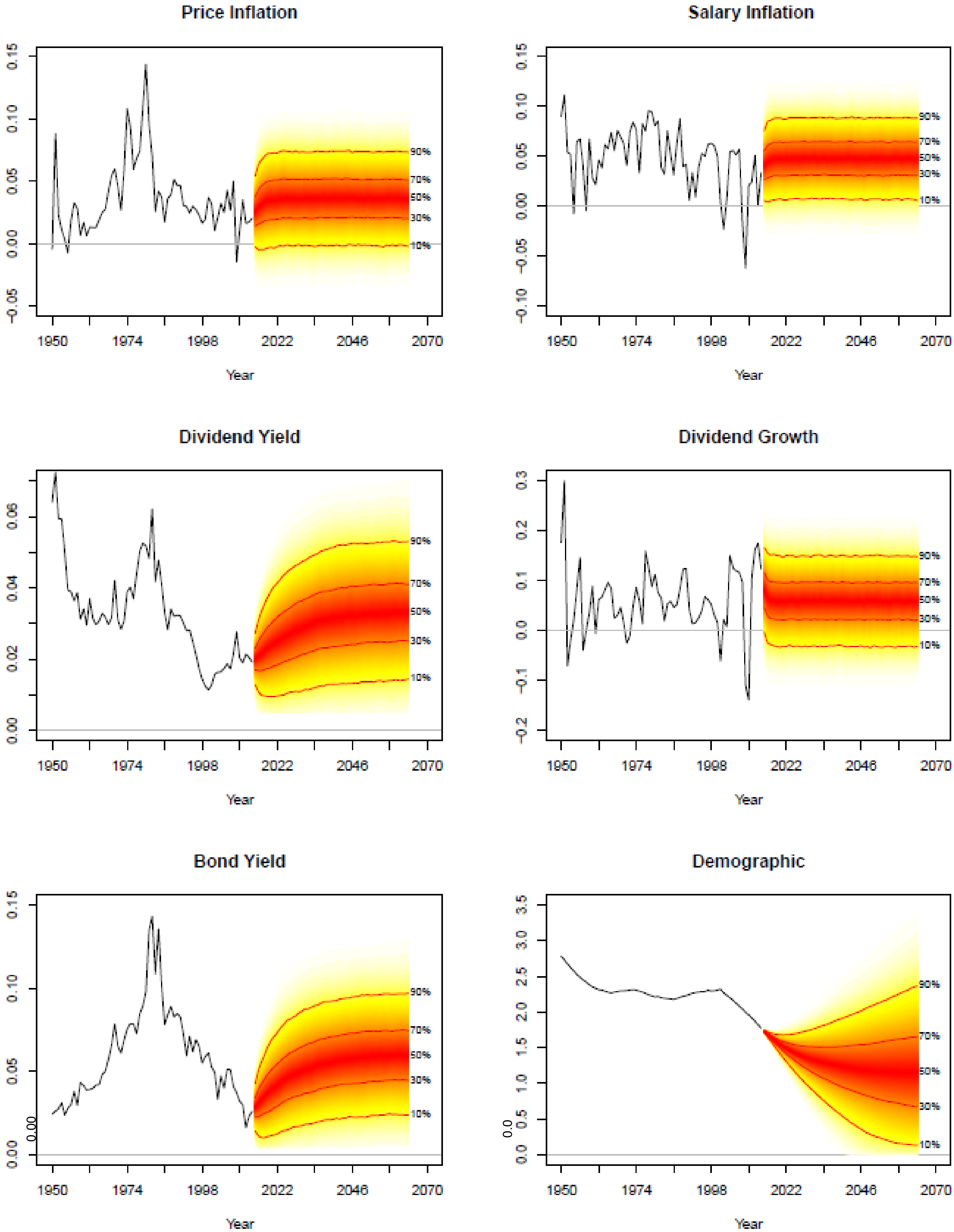
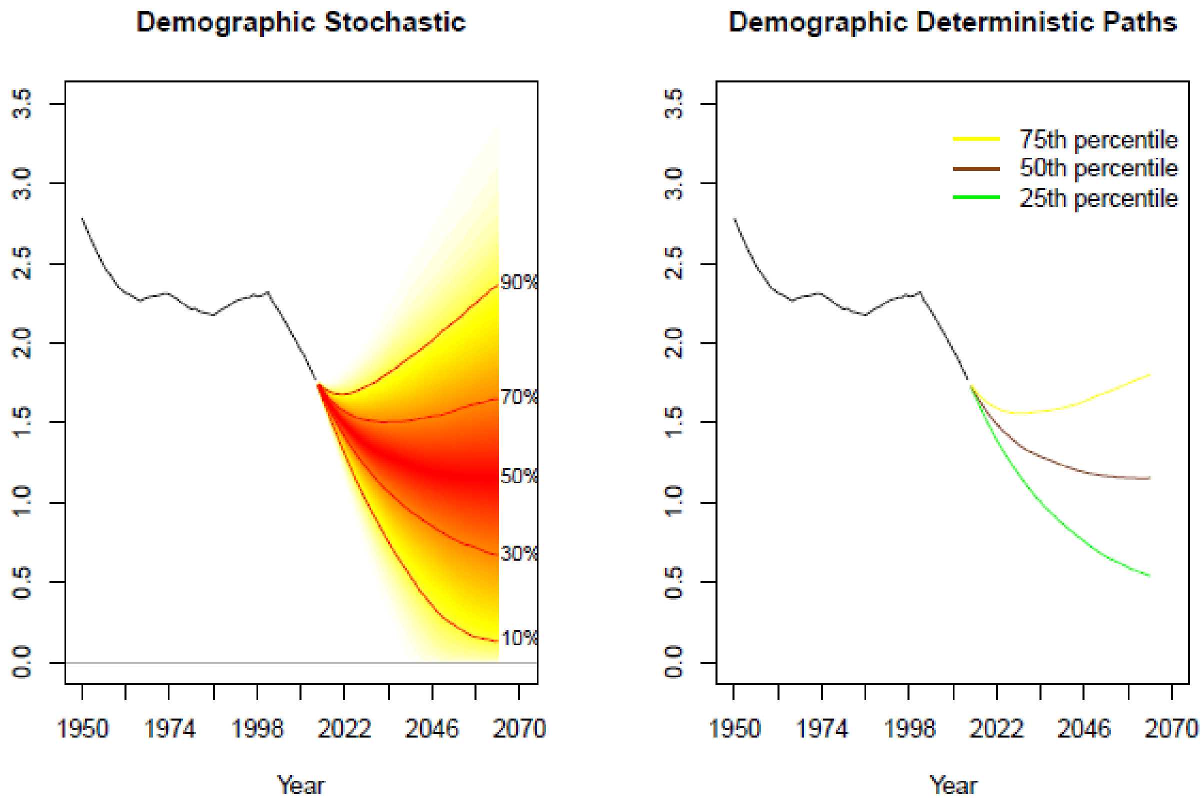


Figure 12: Fanplots of demographic ratio – US



The left-hand graph shows the full distribution of the projected demographic ratio. The right-hand graph shows selected paths from that distribution.

Table 11: Mean and standard deviation of simulations for different demographic paths using a 50-year projection – US

		I_t	J_t	Y_t	K_t	C_t	Equity Return	Bond Return
75th percentile	μ	3.53	4.75	2.83	5.89	5.19	8.27	5.00
(1.80)	σ	2.91	3.16	1.35	7.04	2.68	7.57	5.23
50th percentile	μ	3.57	4.78	2.99	5.98	5.30	8.52	5.11
(1.16)	σ	2.91	3.16	1.37	7.04	2.69	7.52	5.23
25th percentile	μ	3.61	4.81	3.14	6.07	5.40	8.77	5.21
(0.54)	σ	2.90	3.16	1.39	7.04	2.70	7.48	5.24

The number in brackets is the projected value of the demographic variable in 50 years' time.

Appendix C: Economic Capital Distributions by Country

Figure 13: Canada pension plan

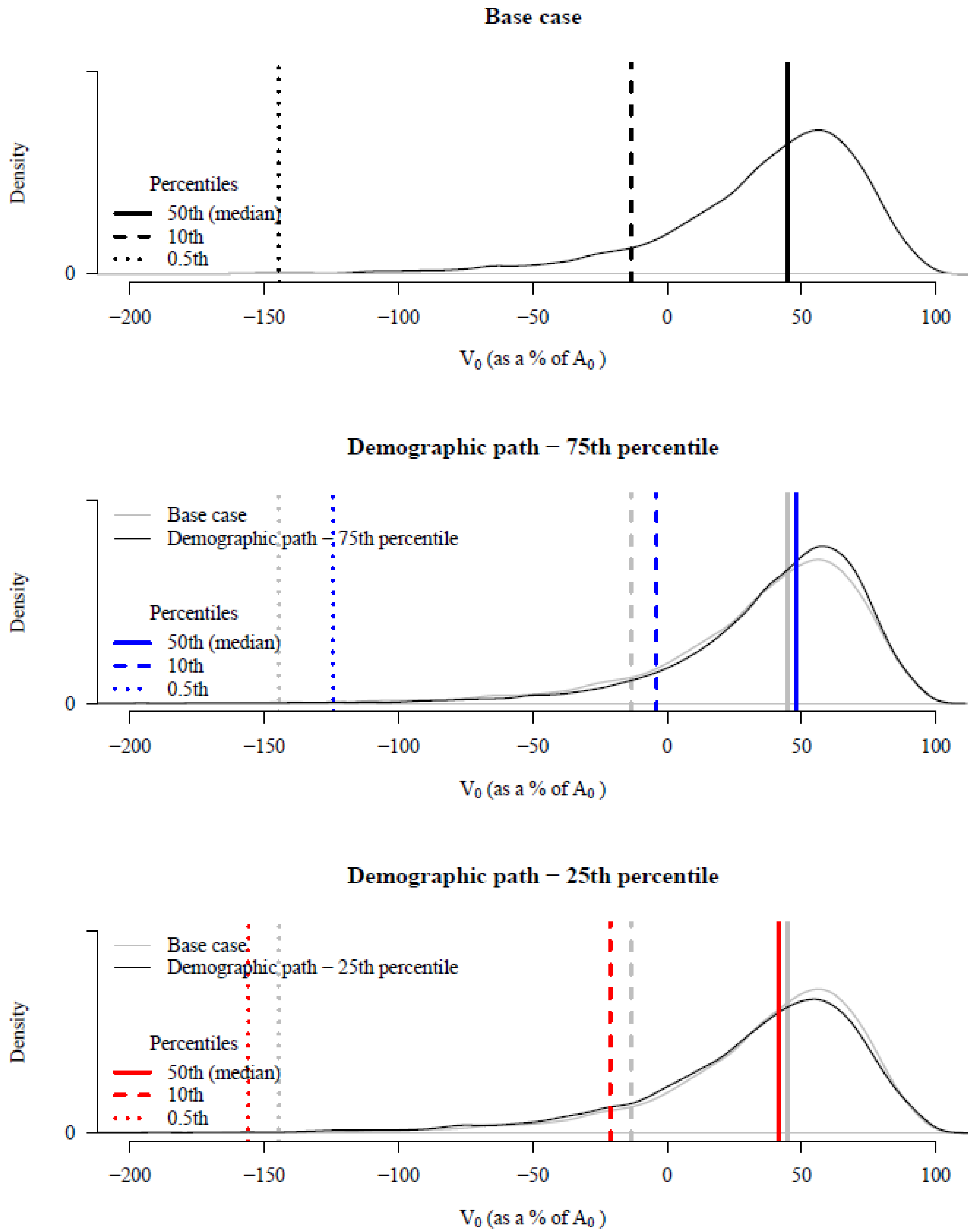


Figure 14: UK pension plan

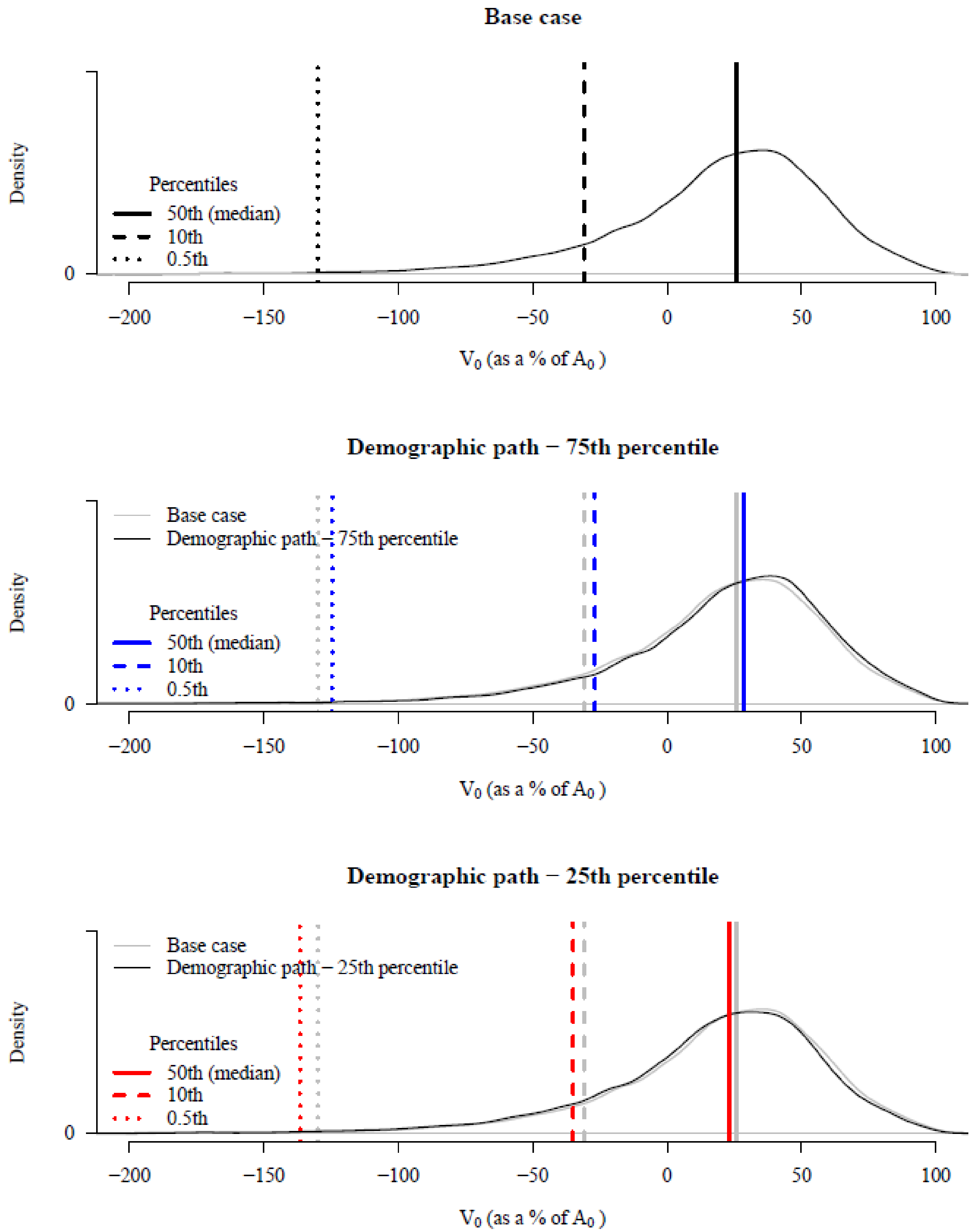


Figure 15: US pension plan